Seismic source deghosting
Application to the Delta 3 source array

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Abstract

Here I investigate two practical approximations of the marine source array signature deconvolution operator: 1) at low frequency and zero take-off angle and 2) at all frequencies and small take-off angles. These are general, i.e. applicable to any source array, and they are based on the expression of the far field signature of a source array, which involves the array geometry, the air guns’ notionals and the eventual firing synchronisation of the sources. The second approximation replaces the source signature with a user defined target wavelet and also handles to some extent the source ghost, directivity and the residual bubble effects.

The small take-off angle approximation leads to the design of a spatially compact filter, which limits the smearing of shot to shot perturbations in common receiver gathers while providing designature functionalities.

Thus, this approach is further investigated and tested for a particular synchronised multi-depth source array, the Delta 3 source.

First, synthetic data is successfully filtered with the present method and the limitations of the approximation are analysed (range of validity that roughly extends to ±35° below 90 Hz and to ±20° beyond this frequency for the Delta 3 source).

Then, the present approach is tested on real data under further approximations (1D earth, average notionals). The results show the expected small - but nevertheless observable - improvements of source designature at the target level: improved event continuity, dephasing and sharpening of the wavelet.

Finally, the outcome is compared with data processing best practice results.

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First and foremost, I want to express my gratitude to my supervisors, Philippe Caprioli and Dirk-Jan van Manen. Their guidance and encouragement, their useful comments and their involvement throughout this research project have been of capital importance.

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Acronyms

CMP  Common Midpoint
CMS  Calibrated Marine Source
CRG  Common Receiver Gathers
CSG  Common Source Gathers
DUT  Delft University of Technology
ETH  Swiss Federal Institute of Technology
FFS  Far Field Signature
FX   Frequency-Space
LFNA Low Frequency Noise Attenuation
NFH  Near Field Hydrophone
NMO  Normal moveout
PB   Peak-to-Bubble Ratio
RMS  Root-Mean-Square
RWTH Aachen University
SBM  Sea-Bottom Multiple
SBR  Sea-Bottom Reflection
SNR  Signal-to-Noise Ratio
STASD Small Take-off Angle Source Deghosting and Designature
With the ultimate goal of obtaining an image of the subsurface, seismic data acquisition - either onshore (land) or offshore (marine) - and data processing are essential, specially at the target depths frequent in oil and gas exploration.

Given that many oil and gas field are situated in water-covered areas [Drijkoningen, 2003], marine exploration has been of capital importance for the oil business during the last decades. The main difference with respect to the land operations is the pace of operation, [Sheriff, 1978]. Although the costs of marine surveys are much higher than those on land (US $4.2 million gross versus US $1.2 million gross on average throughout 115 3D surveys, as discussed by Aylor [1995]), advances on technology and the emphasis on efficiency on marine acquisition have been so successful that the cost per kilometer is much lower in offshore surveys, despite very large per-day costs [Sheriff, 1978], [Dobrin, 1981].

But the quality of the final image is deteriorated by several negative effects that leave a strong imprint on the raw data. The nature of these effects is different on land than in marine environments and various technological and processing solutions have been suggested in the literature to counteract them. Nevertheless, some aspects still need to be understood.

In particular, conventional marine data is limited by the imprint of the so-called source and receiver ghosts, spurious reflections of the wave field at the sea surface. The effect of the source and receiver ghosts is a reduction of the useful frequency bandwidth, leading to a poorer seismic resolution [Parkes and Hatton, 1986].

Acquisition and processing strategies exist to partially remove the effect of both source and receiver ghosts. Deghosting schemes are usually applied at an early stage in the processing flow so that the subsequent steps benefit from the improved bandwidth. Yet, the angular dependency of ghost effects, the sampling limitations of conventional acquisitions and the moderate signal-to-noise ratio (SNR) at such an early stage mandates further investigation, specially on the source side.

For this reason, I focus on (pre-stack) source-side deghosting of marine data in this report. Current processing and acquisition source-side deghosting solutions exist. For instance, several pre-stack source deghosting processing methods focus on the over-under source [Halliday,
These approaches require either the over-under sources to be fired separately and then combined [Posthumus, 1993; Hopperstad et al., 2008a,b] or fired simultaneously and then separated before processing [Parkes and Hegna, 2011]. But the latter require the data to be acquired with the over-under source.

Alternatively, Robertsson et al. [2008] suggest to acquire data using airguns conveniently distributed in time and space such that they can be used to compute the vertical gradient of the seismic wave field.

On the other hand, the multi-depth synchronised source arrays represent an acquisition source deghosting solution. But these source configurations still suffer from directivity, the residual ghost and the residual bubble effect, and further processing methods need to be investigated.

Recently, Amundsen and Zhou [2013] presented a low frequency filter for a single monopole source, which typically dehosts the data up to half of the second frequency notch.

In this report I generalise the method of Amundsen and Zhou [2013] to any marine source array. I suggest two practical approximations of the designature operator of a marine array: 1) at low frequency and zero take-off angle and 2) at all frequencies and small take-off angles. The second approximation can be used to design a spatially compact source designature filter based on the far field signature of the source array, which handles directivity, directional source ghost effects and the residual bubble to some extent.

This filter is then applied on synthetic and field data to the particular Delta 3 source configuration, a synchronised multi-depth source array, and its limitations are analysed. The real data results are finally compared to a standard data processing best practice, the Calibrated Marine Source (CMS) method, and the present filter is used in conjunction with CMS to account for directivity effects.

In terms of organisation of this report, the present introductory chapter is followed by three main parts: theory, synthetic data and real data. Each of them is divided in several chapters.

In this first chapter, the basic concepts of source ghost and bubble effect are briefly discussed, the Delta 3 source is then presented and, finally, the CMS method is explained.

Next, the theoretical part of this report is divided in two chapters: whereas chapter 2 discusses each contribution to the far field of a marine source array and their effects in the wavelet and its spectra, the small take-off angle filter and its limitations are derived and discussed in chapter 3. Additionally, an alternative low frequency filter applicable to any source array is developed, but it is not further tested on synthetic and real data in this report.

In the second part (chapter 4), the more promising small take-off angle filter is tested on synthetic data and the results are compared with the theoretical limits of the present approach.

The last part of the report is again separated in two chapters: first, the real data (from the Bruce field) is presented in chapter 5 and the strategy to implement the present method together with the flow used are discussed. The results obtained are then shown, discussed and compared with CMS results in chapter 6.

Finally, the main conclusions of this report are summarised in chapter 7 and future lines of investigation are suggested.
1-1 The bubble effect and the source ghost

Conventionally, airguns are widely used as sources in marine seismic acquisition. The basic principle behind consists of the injection of highly-compressed air into the water, creating a bubble and causing the propagation of a (high-intensity) pressure wave that propagates from the source to the subsurface with a distinct shape in time, also named the notional signature (or notional).

The physics of the near field signature is a non-linear fluid theory actively discussed in some papers, such as in [Ziolkowski et al., 1982] and [Safar et al., 1983]. Intuitively, the bubble initially expands until the hydrostatic pressure is greater than the internal pressure. Then, the expansion is slowed down and the bubble loses some energy due to heat dissipation. It then collapses until the pressure inside equates the hydrostatic pressure. At this point, the bubble starts expanding again and the same procedure is repeated many times, with energy being dissipated through time [Drijkoningen, 2003].

Thus, the signature of an airgun shows periodic explosion/implosions of the bubbles and the natural decay due to the energy dissipation. This is the so-called "bubble effect".

According to Drijkoningen [2003], the bubble period - which describes the source signature of an airgun - scales as the 1/3 power of the volume of the chamber of the airgun (modified Rayleigh-Willis formula). Hence, larger sources (in volume) produce more usable seismic energy than smaller ones (larger periods) [Dragoset, 1990], but the deployment costs are indeed bigger. Usual volumes go from 10 to $\sim 200 \text{ in}^3$.

Although the source signature of an airgun is inconveniently long (around $200 \text{ ms}$ in an airgun array), oscillatory and its spectrum is multi-peaked and not minimum phase, the airgun is widely used due to its reliability and signature repeatability [Ziolkowski et al., 1982].

Ideally, the pressure field generated by an airgun travels downwards towards the sea-bottom and it is transmitted, refracted and reflected in the different interfaces of the subsurface. The modified wavefield is then recorded at the receivers. This is indicated, schematically, by the green arrow in figure 1-1 for the sea-bottom reflection.

But the pressure field is emitted in all directions. Thus, there is an up-going component that travels upwards from the source, reaches the sea-surface and it is reflected downwards due to the (negative) sea surface reflection coefficient (given the impedance contrast between the air and the sea water, the latter is close to $-1$), as indicated by the yellow and red arrows in this picture.

This is the so-called source ghost, which has opposite polarity with respect to the emitted wavefield and interferes with the down-going direct field, reducing the useful bandwidth by introducing notches at certain frequencies (see section 2-2-2) and, ultimately, affecting the resolution of the recorded data. Source ghosts are classified as short-path multiples [Stoker et al., 1997] and normally considered as coherent noise\footnote{A coherent noise is normally defined as the undesirable seismic energy that shows consistent phase from trace to trace, according to [Schlumberger, 2014b]. Thus, an incoherent noise does not show a consistent phase from trace to trace.}. Analogously, a receiver ghost is generated from the reflection of the up-going field at the sea surface close to a receiver.
Interestingly, the bubble effect of an airgun is also reduced due to the source ghost given that the latter is delayed with respect to the primary down-going field and has opposite polarity (as depicted in figure 1-1 for vertical take-off angle). The far field of an array will be further investigated in chapter 2.

1-2 The Delta 3 V-shaped source array: a notchless configuration

Airgun arrays rather than single airguns are used to tune the final output and reduce the bubble effect, but directivity is introduced such that the wavefield is not equal in all directions.

Normally, airgun arrays present source ghost notches at frequencies that depend on the source depths, the water velocity and the departure angle from the array.

The volumes and the distribution of the sources in a Delta 3 source array are designed such that there are no source ghost notches below 150 Hz within a cone of 20° (for all directions) while leading to a reduced bubble effect.

This array consists of two sub-arrays towed at 6 m depth and separated 12 m in the cross-line direction and a single sub-array towed at 9 m depth in between the last two, such that it is 6 m apart from each of them in the cross-line direction [WesternGeco-Isometrix, 2012].

Each sub-array contains 8 sources of different volumes, in which the first four - closer to the boat - are grouped in pairs (clusters) leading to a non-symmetric directivity pattern.

The source positions and the system of coordinates used throughout this report are depicted in figure 1-2, in which source take-off angles are positive towards the streamer and negative...
towards the boat, comprised between $-90^\circ \leq \theta^s \leq 90^\circ$, and source azimuths are defined such that $-90^\circ \leq \varphi^s \leq 90^\circ$.

The firing of the deeper sub-array is delayed by 2 ms, which results in a reinforcement of the down-going field and attenuation of the source ghost. This is observed when the amplitude spectrum at zero take-off angle of the Delta 3 source array is compared to that of a standard single depth array towed at 6 m depth and with 8 m sub-array separation, as shown in figure 1-3. The Delta 3 source gives a slightly larger low frequency output ($\sim 1.5 dB$) and has no notches below 150 Hz for all directions within a 20$^\circ$ cone, whereas the single depth array has a notch at 125 Hz.

The contributions to the far field of the Delta 3 source array are further investigated in section 2-2 and the small take-off angle source designature filter derived in this report is applied on synthetic and real data for this configuration in the following chapters.
1-3 Calibrated Marine Source

The objective of the Calibrated Marine Source (CMS) process is to remove the shot to shot variations due to minor perturbations in wave height, gun pressure, array geometry, etc. An operator is designed for each shot that designatures the different traces and shapes them to a target wavelet [Schlumberger, 2014a].

First, a near field hydrophone measures the pressure field of each gun, affected by the neighbouring guns. Then, the the far field notional signature is estimated following the method described by Ziolkowski et al. [1982], where the notionals can be thought as the signature of a single gun as if it was isolated. The notionals of the different guns in an array are finally combined to generate a far field signature of the array at a certain angle for a single shot.

Ideally, the far field signature obtained from CMS is the actual far field, but there are shot-to-shot variations that are corrected for by designing an operator different for each shot, which shapes each trace to a target wavelet. This is designed to be an edited average far field signature such that it is a broad-band zero phase wavelet in which the residual bubble has been smoothed out from the spectrum.

Currently, directivity effects are not included from a production point of view. In this report, CMS is applied on the real data in chapters 5 and 6 and compared with the result of applying the small take-off angle filter. Under certain practical simplifications, a correction with directivity to the CMS method is found when applying the small take-off angle filter with a target wavelet equal to the one obtained by the CMS technique.

For further information on the CMS process, the reader is referred to [Schlumberger, 2014a].
Part I

Theory
Far field of a marine source array: application to the Delta 3 source

The first step towards source deghosting the seismic data acquired with a marine source array is understanding what are the different contributions to the pressure signal due to its sources and their spatial distribution.

Unlike a single source, a marine source array is composed by many sources and not only the signature of each source has a contribution in the radiated pressure field, but also the source positions and depths have a relevant effect. In this chapter, the emitted pressure far field of a marine source array is discussed and further analysed for the particular case of a Delta 3 source.

Based on the understanding of the different contributions to the so-called far field, chapter 3 presents a practical and general designature filter able to remove the source ghosts, compensate for the directivity pattern due to an array and remove the residual bubble up to a certain extent.

2-1 Far field of a marine source array

Each of the sources in a marine source array has a contribution to the total emitted pressure field. Typically, the volumes and positions of each of the guns (depth, inline and crossline positions) are designed such that the resulting field has little bubble effect in comparison to a single gun, i.e. the peak-to-bubble ratio is increased. Such procedure is named tuning of a marine source array.

Nevertheless, each of the guns in an array generates a source ghost, which has an impact on the recorded data by introducing notches in the spectrum. By adjusting the source locations, several source arrays have been deployed that not only minimize the bubble...
Far field of a marine source array: application to the Delta 3 source

The effect of the different airguns but that also have a notchless far field spectrum by synchronising the sources and positioning them at different depths. Yet, directivity (or antenna) effects are introduced. In other words, the radiated field depends on the direction of emission.

All these contributions are taken into account to obtain an expression for the pressure field due to a marine source array in an infinite homogeneous and isotropic medium, derived as a generalisation of the approach taken by Ziolkowski [1985] to \( N \) sources.

The resulting expression is valid in the far field, in other words, far enough away from the array that small changes in the measurement position do not significantly affect the signature [Dragoset, 1990]. That is, at a distance \( r \gg \lambda \), where \( \lambda \) is the dominant wavelength.

According to Hopperstad et al. [2013] and Halliday [2013], the far field of a marine source array in the frequency domain is written as follows:

\[
W = e^{-ik_z^s z_{\text{ref}}} \sum_{n=1}^{N} S_n(\omega) e^{i(k_x^s x_n + k_y^s y_n)} \left( e^{ik_z^s z_n} - |r_0| e^{-ik_z^s z_n} \right) e^{-i\omega \tau_n}
\]

\[ (2-1) \]

Here \( N \) is the total number of guns in the array, \( k_x^s, k_y^s \) and \( k_z^s \) are the components of the wavenumber vector, \( x_n, y_n \) and \( z_n \) are the source positions, \( z_{\text{ref}} \) is a reference depth (at which the phase is minimised), \( \tau_n \) is a firing time delay (to synchronise the sources such that the vertical down-going fields are aligned), \( S_n(\omega) \) is the source signature of the \( n^{th} \) gun, \( |r_0| \) is the absolute value of the sea surface reflection coefficient \(( -1 \leq r_0 < 0 )\), \( \omega \) is the angular frequency, \( c \) the velocity of acoustic waves in the sea-water and \( i = \sqrt{-1} \).

The sign convention used for the temporal and spatial Fourier transforms throughout this report is such that the Fourier transform of a function that depends on time and the spatial variables, \( v(t,x,y,z) \), is:

\[
V(\omega, k_x, k_y, k_z) = \int_{\mathbb{R}^3} \int_{-\infty}^{\infty} v(t,x,y,z) e^{-i\omega t} e^{i(k_x x + k_y y + k_z z)} dt dx dy dz
\]

\[ (2-2) \]

Where capital letters denote that a function is in the frequency and/or wavenumber domain and the sign of the exponentials is the opposite for the inverse Fourier transforms.

Table 2-1 summarises the different contributions to the total far field in equation (2-1), their dependencies and the basic physical principles and effects on the resulting far field. Each of these contributions is further developed in the following sections.

A list of the variables used and their expressions is presented in table 2-2.

Now, the different contributions to the far field are separately studied for the Delta 3 source array with the ultimate objective to improve the bandwidth by deconvolving, to some extent, some of them. Analogously, this analysis can be extended to any marine source array by considering the far field expression given in equation (2-1), which is a general expression.
Table 2-1: Contributions to the total far field of a marine source array, their dependency on the different variables and some comments.

<table>
<thead>
<tr>
<th>Contribution of:</th>
<th>Expression</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source signature</td>
<td>$S_n(\omega)$</td>
<td>• Pressure signal of each gun.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Introduces the bubble effect at low frequencies ($\approx 5 - 10 , Hz$).</td>
</tr>
<tr>
<td>Source ghost</td>
<td>$G_n(k_s^z, z_n) = e^{ik_s^z z_n} -</td>
<td>_{\text{Primary}}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Introduces notches in the spectra reducing the frequency bandwidth (seismic resolution).</td>
</tr>
<tr>
<td>Synchronisation</td>
<td>$D_n(k, z_n) = e^{-i\omega \tau_n}$</td>
<td>• Source synchronisation to align the vertical down-going field.</td>
</tr>
<tr>
<td>Antenna effect</td>
<td>$A_n(k_s^x, x_n, k_s^y, y_n) = e^{i(k_s^x x_n + k_s^y y_n)}$</td>
<td>• Directivity due to the different source positions.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Introduces notches in the spectra reducing the frequency bandwidth (seismic resolution).</td>
</tr>
<tr>
<td>Reference depth</td>
<td>$R(k_s^z, z_{ref}) = e^{-ik_s^z z_{ref}}$</td>
<td>• Angle dependent phase shift towards the phase center of the array that minimises the phase error for a range of take-off angles.</td>
</tr>
</tbody>
</table>

Table 2-2: Variable list, nomenclature used and their expressions, when necessary.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>Frequency</td>
<td>$f$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular frequency</td>
<td>$\omega = 2\pi f$</td>
</tr>
<tr>
<td>$c$</td>
<td>Velocity acoustic waves in sea-water</td>
<td>$c = \lambda f$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength</td>
<td>$k = \frac{\omega}{c} = \sqrt{(k_s^x)^2 + (k_s^y)^2 + (k_s^z)^2}$</td>
</tr>
<tr>
<td>$\theta^s$</td>
<td>Departure take-off angle from the source</td>
<td>$k_s^x = k \sin \theta^s \cos \phi^s$</td>
</tr>
<tr>
<td>$\phi^s$</td>
<td>Departure azimuth from the source</td>
<td>$k_s^y = k \sin \theta^s \sin \phi^s$</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of sources of the array</td>
<td>$k_s^z = k \cos \theta^s$</td>
</tr>
<tr>
<td>$x_n, y_n, z_n$</td>
<td>$n^{th}$ source coordinates</td>
<td>$n^{th}$ source coordinates</td>
</tr>
<tr>
<td>$\tau_n$</td>
<td>Firing delay of the $n^{th}$ source</td>
<td>$\tau_n = \frac{z_n - z_{\text{min}}}{c}$</td>
</tr>
<tr>
<td>$z_{\text{ref}}, z_{\text{min}}$</td>
<td>Reference and minimum depth of the arrays</td>
<td></td>
</tr>
</tbody>
</table>
2-2 Far field of the Delta 3 source array: analysis of its contributions

By inspection of equation (2-1), several effects modify the far field response of a marine source array.

Below these lines, the amplitude and phase spectra of the total far field for a Delta 3 source are first introduced and its time response as a function of take-off angle is shown. Then, each contribution to the total far field is separately analysed. These are the (modelled) source signatures, the seismic ghosts, the source synchronisation, the antenna effect and the phase center.

In section 1-2, the Delta 3 source array was presented. The distribution of sources, as shown in figure 1-2, is such that the peak-to-bubble ratio (PB) of the total far field is increased and the source array is notchless. This is observed in the amplitude and phase spectra plotted in figure 2-1 for the Delta 3 source considering 24 sources and their source positions, obtained by the appropriate substitutions in equation (2-1) ($\phi^s = 0^\circ$ is taken).

According to the system of coordinates in figure 1-2, take-off angles are positive towards the streamers.

The top row in figure 2-1 is computed considering impulsive monopoles (this would be the ideal case), i.e. $S_n(\omega) = 1\mu Pa \cdot m$ in the frequency domain. Yet, in practice the guns are not impulsive and this is considered in the bottom plots of this figure, where (modelled) source signatures have been used. This is further discussed in the next section.

In both situations, the reference depth is set at $z_{ref} = 6 \text{ m}$, as it is later on justified in the last section of this chapter, and a sea reflection coefficient of $r_0 = -1$ is chosen.

Several relevant observations can be made from these plots:

- The modelled source signatures introduce a residual bubble at low frequencies (around $5 - 10 \text{ Hz}$), seen as the rapid change from blue to yellow and from yellow to red at the top part of the amplitude spectrum for the modelled notionals. This is not observed in the amplitude due to impulsive monopoles.

- The Delta 3 source array is notchless (amplitude notches represented in dark blue colour) for $f < 150 \text{ Hz}$ except at $0 \text{ Hz}$ (green arrow). This is achieved by synchronising the sources.

- Yet, a directivity notch that roughly starts at $90 \text{ Hz}$ and $\theta^s = \pm 90^\circ$ all the way down to $150 \text{ Hz}$ and $\theta^s = \pm 25^\circ$ is visible (purple arrow). A polarity change for these frequencies in the phase spectra is observed.

- Lack of symmetry of the spectra due to the non-symmetric source distribution (clusters in figure 1-2). This effect is more pronounced when the modelled source signatures are considered (bottom row).

- The phase variation with take-off angle between $\theta^s = 0^\circ$ and $\theta^s \sim 30^\circ$ is minimised. This is due to the choice of the reference depth, $z_{ref} = 6 \text{ m}$. Beyond these angles, the phase drastically varies due to the antenna response.

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These effects leave an imprint in the time domain far field wavelets, as shown in figure 2-2 for different take-off angles and for both impulsive and modelled source signatures (although in practice, they are difficult to be separately identified on shot gathers). The amplitude and phase spectra of each of these wavelets correspond to a trace at the same angle in figure 2-1.

The following is observed from figure 2-2:

- When modelled notionals are used (bottom row), the bubble effect is observed as a secondary low-frequency peak after the direct arrival and the source ghosts.

- After the direct arrival, the reflection of the emitted field at the sea surface reaches the receiver with opposite amplitude. These are the source ghosts due to the sources at 6 m, which arrive first, and the sources at 9 m, which arrive some 2 ms later.

- The separation between each direct arrival and its ghost decreases with take-off angle (more easily seen for impulsive guns, top row). This is due to the source ghost dependency with take-off angle.

- The amplitude of the direct arrival and the source ghosts decreases with take-off angle. Moreover, when the notionals are considered, the PB ratio decreases with take-off angle.
Figure 2-2: Far field wavelets for different take-off angles considering impulsive sources (top) and the modelled source signatures (bottom).

(as indicated in the figure) due to directivity effects [Van der Schans and Ziolkowski, 1983].

- The separation in time amongst the direct arrivals from each source increases with take-off angle, which is a directivity effect and will be later on analysed. This is more easily observed for impulsive guns. Note one single arrival is observed at $\theta = 0^\circ$, given that all arrivals occur at the same time, whereas several direct arrivals with smaller amplitudes are visible for larger take-off angles because each source is at a different inline position. Analogously, a similar effect is observed for the source ghosts. The ripples are due to a reduced bandwidth.

All these contributions are now separately analysed.

2-2-1 Source signature

As explained in the introductory chapter (see chapter 1), the bubble generated by an airgun oscillates and generates pulses of smaller amplitude than the first one, which introduce source-generated noise. This is minimised by deploying an array of guns at different positions. Yet, a residual bubble remains in the data, which shows up at low frequencies (around $\approx 5 - 10$ Hz for the Delta 3 source).
The guns in a Delta 3 source configuration have been tuned such that the residual bubble is minimised, leading to a large PB ratio. In the previous section, either impulsive or modelled source signatures were used.

The latter have been modelled by Mark Ramsay (personal communication, 7 March, 2014) using Gundalf™ (the reader is referred to [Oakwood, 2012]), a well-known 3rd party marine source array modelling software. The volumes and positions of each of the sources in a Delta 3 source configuration have been taken into account to generate them. From now onwards, they will be referred as the modelled notionals or just the notionals. The latter will be shown and compared to the notionals used in the real data example in chapter 5.

The far field pressure signal at zero take-off angle in figure 2-3 shows the benefit of using the Delta 3 configuration to reduce the bubble effect. This is compared with a single gun (number \( n = 5 \) in figure 1-2). The PB ratio of the zero take-off angle wavelet for the Delta 3 source array is roughly 6 times larger than the one for the 5th source alone.

Yet, a low-frequency residual bubble still remains in the data, which can also be appreciated in figure 2-1 for frequencies between 5 and 10 Hz.

![Figure 2-3: Far field wavelets for the Delta 3 source considering all the sources and their ghosts (left) and only one of their sources (the 5th) and its ghost (right). \( r_0 = -1 \) and \( z_{ref} = 0 \) m have been considered but an additional time shift of 6 ms has been applied for display reasons.](image)

### 2-2-2 Source ghost notches

In the marine case, sources (and receivers) are normally placed close to the sea surface for the following reasons:

- For towing and handling reasons.

---

*August 8, 2014*
There is no practical gain by placing them deeper.

A shallow source gives a source ghost notch at higher frequencies than a deeper one (except at $0 \, Hz$), hopefully outside the useful bandwidth.

The latter is now examined.

Let us consider a monopole source (impulsive source) at a certain distance $z$ from the sea surface, as depicted in figure 2-4, that emits plane waves (blue arrows). The up-going wave emitted by the monopole that impinges at the sea surface is reflected downwards with opposite polarity given that the sea surface reflection coefficient is negative and close to $r_0 = -1$.

This is represented by a mirror source ($-z$) with the same strength and opposite phase as the real source (oscillating in anti-phase), as if it was within the sea-water emitting plane waves simultaneously with the real source. For simplicity, $r_0 = -1$ is considered.

After firing the source, the pressure field is radiated in all directions. A receiver at location $Q$ records the far field signal (again, this means that $r \gg \lambda$).

According to equation (2-1), table 2-1 and the geometry in figure 2-4, the far field recorded at $Q$ due to a single monopole and its ghost for $\varphi^* = 0^\circ$ is given by:

$$W = G(\omega, \theta^*, z) = e^{i\omega z \cos \theta^*/c} - e^{-i\omega z \cos \theta^*/c} = 2i \sin \left( \frac{\omega z \cos \theta^*}{c} \right)$$
The amplitude of the far field is then $|W| = |G(\omega, \theta^s, z)| = |2 \sin (\omega z \cos \theta^s/c)|$, which has notches for frequencies:

$$f_{n,sg} = \frac{nc}{2 \cos \theta_z}, \quad n = 0, 1, 2, \ldots$$  \hspace{1cm} (2-4)

Where the subscript of the frequency indicates this is a notch due to a source ghost.

This frequency is take-off angle dependent and several notches appear within the spectrum for the different multiples of the fundamental frequency ($n = 1$ in the previous equation), as depicted in figure 2-5.

Furthermore, shallow guns have a notch at higher frequencies than deeper guns for $n > 0$ according to this expression.

![Figure 2-5: Modulation of the amplitude due to the source ghost. Source: modified from Drijkoningen, 2003.](image)

In a Delta 3 source configuration, the sources are placed at depths of 6 m and 9 m. After substitution in the previous equation for an approximate water velocity of 1500 m/s, source ghost notches occur for the following frequencies:

$$f_{n,sg}(z = 6 \text{ m}) = \frac{125n}{\cos \theta^s} \text{ Hz}, \quad n = 0, 1, 2, \ldots$$  \hspace{1cm} (2-5)

and,

$$f_{n,sg}(z = 9 \text{ m}) \approx \frac{83.3n}{\cos \theta^s} \text{ Hz}, \quad n = 0, 1, 2, \ldots$$  \hspace{1cm} (2-6)

Therefore, both the sources at 6 m and 9 m present amplitude notches at different frequencies ($125 \text{ Hz}$ and $83.3 \text{ Hz}$ for the sources at 6 m and 9 m and zero incidence angle, respectively) and common notches for frequencies of $250n/\cos \theta \text{ Hz}$. Figure 2-6 illustrates this for two impulsive sources at 6 m and one source at 9 m in a Delta 3 source configuration, as indicated in the figure. Amplitude and phase spectra are plotted as a function of frequency and take-off angle considering zero azimuth, $\varphi^s = 0^\circ$, and $z_{ref} = 0 \text{ m}$. In this figure, dark blue colours represent notches.

Moreover, a rapid phase change is observed at a source notch, which can be explained by analysing the phase of the far field in equation (2-3). The real part of the far field is always zero, whereas the imaginary part takes the form of a sinusoid, $\Im{G} = 2 \sin (\omega z \cos \theta^s/c)$. When the latter is zero, the phase is undetermined and it is represented by zero in the plots (zero phase not visible at 83.3 Hz or 125 Hz due to the frequency sampling of 2 Hz used in
Far field of a marine source array: application to the Delta 3 source

Figure 2-6: Amplitude and phase plots for two impulsive sources at 6 m (top row) and a single impulsive source at 9 m (bottom row), typical in a Delta 3 source array configuration. Notches in amplitude take dark blue colors whereas a change of polarity is observed after crossing a notch in the phase plots.

When the imaginary part is non-zero, the phase is $\pm \pi/2$, where the sign is given by the sign of the imaginary part.

In the time domain and for zero take-off angle, the source ghost for an impulsive gun arrives after the primary arrival, where the delay is determined by the depth of the source. But this delay changes with take-off angle, as shown in figure 2-7 for the two situations drawn in figure 2-6.

The advance and the delay of the direct arrival and the ghost, respectively, can be obtained by performing an inverse Fourier transform of equation (2-3). The direct arrival (positive exponential in this equation) is an advanced Dirac delta in the time domain, with an advance of $t_d^0 = \frac{z \cos \theta_s}{c}$, whereas the ghost is a delayed spike with a delay of $t_g^0 = -\frac{z \cos \theta_s}{c}$. Thus, the separation in time between the two is $t_d^0 - t_g^0 = \frac{2z \cos \theta_s}{c}$, which is larger for deeper sources and it decreases with take-off angle.

For the two impulsive monopoles at 6 m, this difference is 8 ms at zero take-off angle and it decreases to $\sim 6.9$ ms at $\theta_s = 30^\circ$. This difference is larger for the source at 9 m, with a value of 12 ms for $\theta_s = 0^\circ$ and $\sim 10.4$ ms at $\theta_s = 30^\circ$. August 8, 2014
**Figure 2-7:** Ghost function in the time domain for two monopoles at 6 m and a monopole at 9 m as shown in figure 2-6 as a function of take-off angle. \( z_{\text{ref}} = 0 \text{ m} \) here but, for display reasons, and additional time shift of 9 ms has been applied.

This time separation and the difference in polarity between the direct arrival and the ghost cause the spectra to have notches, as in figure 2-6. Moreover, it is observed for the total field in figure 2-2 that the distances between the direct arrivals and their corresponding ghosts are shortened with increasing take-off angle.

Finally, the amplitude of the spikes in figure 2-7 is nearly unchanged with time as the sea reflection coefficient is set to \( r_0 = -1 \) and the amplitude of the direct arrival and the ghost are always proportional to the number of impulsive sources considered. Yet, there is a slight decrease of amplitude due to the loss of high-frequencies when ripples are introduced, as in practice a band-limited spike is computed.

### 2-2-3 Source synchronisation: the notchless source

In a Delta 3 source configuration, the sources at 9 m are fired with a delay of \( \tau_9 = 2 \text{ ms} \) with respect to the sources at 6 m, which is key to have a notchless source.

To intuitively justify this, figure 2-8 illustrates the path of the emitted field at normal take-off angle for different time frames in which, for simplicity, three impulsive monopoles at 6–9–6 m depth are considered, respectively, which emit plane waves. The firing of the source at 9 m is delayed by \( \tau_9 = 2 \text{ ms} \) and the wave field is recorded at 12 m depth (in fact, it is recorded in the far field and time shifted at this level).

At zero time, the sources at 6 m are fired (green dots in the drawing) and the pressure field is radiated in all directions. Here, only zero take-off angle is considered.

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After 2 ms, the down-going pressure waves reach the 9 m level and the source at this level is then fired. The latter reinforces the down-going direct field and from this point onwards the vertical down-going fields are aligned. But an up-going wave is emitted as well, which in turn will generate an additional ghost. Two milliseconds later, the first up-going waves reach the sea surface and they are reflected downwards with opposite polarity (assuming $r_0 = -1$) generating the first source ghost, whilst the field travelling upwards from the deepest source has not yet touched the sea surface. Moreover, the down-going field from the three sources impinges at the receiver at 12 m at the same time and their amplitudes are added in time. This is the direct arrival shown in the time graph at the bottom of figure 2-8, with a positive amplitude equal to +3 (amplitude radiated by the source is here considered positive).

From here onwards, the down-going waves continue propagating downwards and the up-going waves are reflected at the sea surface. First, the up-going field due to the sources at 6 m is reflected at 6 ms, generating the source ghost, and at 10 ms the up-going field due to the source at 9 m is reflected.

The first ghosts reach the receivers after 12 ms, with an amplitude of −2 due the two sources at 6 m and the negative sea surface reflection coefficient.

Finally, the remaining ghost reaches the receiver 4 ms later with an amplitude of a single source and negative sign, i.e. −1.

The amplitude and phase spectra corresponding to these three monopoles for different take-off angles and with (bottom row) and without (top row) source synchronisation are shown in figure 2-9.
2-2 Far field of the Delta 3 source array: analysis of its contributions

Figure 2-9: Phase and amplitude spectra for three monopoles with the 6 – 9 – 6 m configuration when the deepest source is fired with no delay (top row) or with a delay of 2 ms (bottom row). Due to the synchronisation, the amplitude spectrum at the bottom row has no notches.

By combining the sources at 6 m with the one at 9 m without synchronisation (top row in figure 2-9), the resulting spectra does not have a notch at $f_{n,sg}(z = 9 m) \approx 83.3$ Hz from the source at 9 m or at $f_{n,sg}(z = 6 m) = 125$ Hz due to the sources at 6 m, but it has a notch in between, around 108 Hz for zero take-off angle.

The corresponding wavelet in the time domain is also shown in this image, where $z_{ref} = 0 m$ has been chosen and the wavelet has been shifted by 9 ms for display reasons. Due to the sources not being synchronised, the direct arrival from the monopole at 6 m - with strength +1 - arrives first at the receiver followed by the direct arrivals of the other two sources, with strength of +2. Nevertheless, the ghosts from the shallow guns arrive first at the receivers as their distance from the guns to the sea surface and back to the receiver is smaller than for the deeper source.

On the other hand, the amplitude spectrum when the sources are synchronised (bottom row of figure 2-9) has no notch due to the source ghost for frequencies $f < 150$ Hz except at 0 Hz within a large range of take-off angles. Yet, not a constant amplitude is observed throughout the spectra (note red colour dominates at frequencies from 20 Hz to 90 Hz, whereas it turns into orange from there onwards) due to the fact that the spectra of the two sources at 6 m is superposed with the spectrum of the single gun at 9 m, which has half the amplitude and a different spectrum.

In the time domain, the direct arrival from the three monopoles is synchronised and the
amplitudes are added, as seen in the third figure at the bottom of figure 2-9. Due to the different depth of the sources, the source ghosts arrive at different times.

When 24 non-impulsive and non-monochromatic airguns are considered, this principle still holds and the amplitude spectrum is notchless for \( f < 150 \text{ Hz} \) except at 0 Hz. In fact, this holds for a cone of source angles between \( \pm 20^\circ \), according to [WesternGeco-IsoMetrix, 2012]. Beyond these angles, directivity notches are introduced, as observed in figure 2-1. This is now discussed.

### 2-2-4 Directivity notches

As previously mentioned, an array of guns can reduce the bubble effect but introduces directivity (or antenna or array effects), i.e. the far field depends on the direction of emission.

Now, let us consider two monopoles with the geometry described by figure 2-10 that emit plane waves (blue arrows). The two guns are located at opposite inline positions with respect to the center of the array, \( \pm x \). For simplicity, the reflection at the sea surface is now neglected and zero azimuth is again considered, \( \varphi^s = 0^\circ \).

![Figure 2-10](image)

**Figure 2-10:** Two monopoles separated a certain distance from the center of the array are used to study the antenna effect.

Both sources fire simultaneously in all take-off angles. A receiver at position Q records the far field at a certain take-off angle \( \theta^s \).

Given this geometry, equation (2-1) and table 2-1, the far field measured by the receiver is:

\[
W = A(\omega, \theta^s, x) = e^{i\omega x \sin \theta^s / c} + e^{-i\omega x \sin \theta^s / c} = 2 \cos \left( \frac{\omega x \sin \theta^s}{c} \right) \quad (2-7)
\]
Thus, the amplitude of the far field \( |W| = |2 \cos (\omega x \sin \theta s / c)| \) also has notches at frequencies:

\[
|W| = 0 \iff f_{n,a} = \frac{(2n + 1)c}{4x \sin \theta s}, \quad n = 0, 1, 2, \ldots
\] (2-8)

Here, the subscript indicates these notches are due to antenna effects.

Hence, the larger the separation from the sources to the center of the array is, the lower the frequency at which the notches start to appear.

Regarding the phase, it varies between \( 0^\circ \) if the real part is positive and \( 180^\circ \) if the real part is negative, as the imaginary part of the far field is zero. Figure 2-11 illustrates this for two different pairs of impulsive sources in a Delta 3 source configuration, as indicated in this figure, for \( \varphi = 0^\circ \) and \( z_{ref} = 0 \text{ m} \).

In each of these examples, the pair of sources are at opposite inline positions, \( \pm x \). In the first case (top row of this figure) the guns are at \( \pm x = 7.5 \text{ m} \), whereas in the second example (bottom row) they are at \( \pm x = 4.5 \text{ m} \). The corresponding notch frequencies can be found by.
substitution in equation (2-8) as a function of take-off angle.

It is important to highlight that the antenna notches (see equation (2-8)) do not exist at $\theta^a = 0^\circ$, whereas source ghost notches are present at vertical angle (see equation (2-4) and compare the notches in the amplitude spectra of figures 2-6 and 2-11, both shown in dark blue colour).

In addition, single monopoles do not introduce antenna notches if only the inline component is considered (only one exponential in the $A_n$ term of table 2-1), but they may introduce them if its cross-line position or other monopoles at different locations are also considered (see equation (2-1)).

When many non-impulsive sources are considered, the underlying principle is the same and the shape of the antenna notches can be differentiated from the source ghost notches because the former do not introduce notches at zero take-off angle because its spectrum is always a (generally complicated) function of cosine functions. On the other hand, due to the negative sea surface reflection coefficient, the source ghost notches are usually related to sines (strictly, only if $r_0 = -1$).

In the time domain, an impulsive monopole gives a spike. For zero take-off angle, the pressure fields due to two sources separated $\pm x$ from the center of the array arrive at the same time and the amplitudes are added. For other take-off angles, the travel time of the emitted field from each source is different, being it longer for the furthest source from the receiver.

According to the system of coordinates for the Delta 3 source presented in section 1-2, i.e. $\theta^s > 0$ towards the streamers, and equation (2-7), the far field wavelet due to the source at position $x$ is an advanced spike with an advance of $t^a_0 = \frac{x \sin \theta^s}{c}$, whereas the source at $-x$ gives a spike delayed by $t^{-a}_0 = -\frac{x \sin \theta^s}{c}$. Thus, the time separation between the two is $t^a_0 - t^{-a}_0 = \frac{2x \sin \theta^s}{c}$, which takes a zero value for zero take-off angle and increases with take-off angle.

Figure 2-12 shows this behaviour for the examples depicted in figure 2-11. Here, $z_{ref} = 0$ m has been chosen and an additional 9 ms delay has been applied for display reson.

A similar trend is observed in the total far field due to impulsive sources in figure 2-2 (top row), where the distance amongst the different primary arrivals is zero for zero take-off angle and the amplitudes are added, but they split for larger take-off angles. The same is valid amongst the source ghost arrivals. Moreover, the source ghost shortens the separation between a direct arrival and its ghost with take-off angle. This is more difficult to observe when the notionals are considered.

2-2-5 Acoustic center of the Delta 3 source array

According to Hopperstad et al. [2008a], the (acoustic) center of a marine source array is the point from which the signal appears to have radiated, which can vary with frequency. In particular, the phase center is the one that minimises the phase variation with take-off angle.
In equation (2-1), $z_{ref}$ shifts the reference point from the origin of the system of coordinates - i.e. the sea surface in the system of coordinates used, see figure 1-2 - to a reference depth at which the phase is minimised: the phase center.

Hopperstad et al. [2013] describe a method to determine the phase center of a marine source array. I follow here the same procedure to determine the phase center of the Delta 3 source.

First, the amplitude and phase spectra for $\theta^s = 0°$ and $\theta^s = \theta^s_{max}$ are plotted in figure 2-13 (top left and top right, respectively, which are the extracted amplitude and phase spectra from the bottom row in figure 2-9) considering impulsive sources and that the phase center is at the sea surface, i.e. $z_{ref} = 0 m$.

By design, the Delta 3 source has been optimised to operate within $\theta^s = \pm 20°$ [WesternGeco-IsoMetrix, 2012]. Thus, $\theta^s_{max} = 20°$. Consistently with the coordinate system depicted in figure 1-2, take-off angles are positive towards the streamers and negative towards the boat. Hence, it is important to minimise the phase difference towards positive angles (phase not
minimised between $\theta^s = 0^\circ$ and $\theta^s_{\text{min}} = -20^\circ$).

The amplitude and phase differences between these two take-off angles as a function of frequency are then computed (the bottom two graphs).

Then, this phase difference is computed for the same take-off angle range but for reference depths that vary from $z_{\text{ref}} = 0 \, \text{m}$ to the maximum depth in the array, being $z_{\text{ref}} = z_{\text{max}} = 9 \, \text{m}$ for the Delta 3 source.

In fact, $z_{\text{ref}}$ is not strictly limited to the maximum depth of the array and it can be extended further. Nonetheless, the optimum value tends to be in between the sea surface and the maximum depth.

This difference is shown in figure 2-14 for reference depths from $z_{\text{ref}} = 0 \, \text{m}$ to $z_{\text{ref}} = 10 \, \text{m}$, slightly above the maximum depth of the array, and assuming that $\theta^s_{\text{max}} = 20^\circ$.

![Figure 2-13](image)

**Figure 2-13:** Top: Amplitude (left) and phase (right) spectra for the Delta 3 source considering impulsive sources for $\theta^s = 0^\circ$ and $\theta^s_{\text{max}} = 20^\circ$. Bottom: amplitude difference (left) and phase difference (right) of the curves shown in the top row. $z_{\text{ref}} = 0 \, \text{m}$ is taken for all the plots.

An optimum choice for $z_{\text{ref}}$ is the one that minimises this difference across the bandwidth.

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Figure 2-14: Phase difference for impulsive sources in a Delta 3 source configuration between take-off angles of $\theta^s = 0^\circ$ and $\theta^s_{\text{max}} = 20^\circ$ for various depth references and a zoomed in image of the first 100 Hz (right).

Clearly, the maximum phase difference is minimised with a reference depth larger than $z_{\text{ref}} = 0\, \text{m}$. Either $z_{\text{ref}} = 8\, \text{m}$ or $z_{\text{ref}} = 10\, \text{m}$ minimise the phase and, thus, one could take its average as a compromise, i.e. $z_{\text{ref}} = z_{\text{max}} = 9\, \text{m}$.

The same procedure is now repeated taking into account modelled source signatures instead of impulsive sources.

Figures 2-15 and 2-16 present the amplitude and phase spectra, their differences and the phase difference for different reference depths between $\theta^s = 0^\circ$ and $\theta^s_{\text{max}} = 20^\circ$ for the modelled notionals in a Delta 3 source configuration.

The same conclusion can be drawn from these images, as either $z_{\text{ref}} = 8\, \text{m}$ or $z_{\text{ref}} = 10\, \text{m}$ minimise the phase.

Yet, the Delta 3 source can operate for larger take-off angles. If the maximum value is relaxed to $\theta^s_{\text{max}} = 30^\circ$, the phase error is then minimised if $z_{\text{ref}} = 6\, \text{m}$, as shown in figure 2-17 for both impulsive sources and the modelled source signatures.

Overall, it is important to emphasise that taking $z_{\text{ref}} = 0\, \text{m}$ does not minimise the phase variation within the normal bandwidth.

Considering that the range of take-off angles at which the Delta 3 source operates (not necessarily in an optimum way) is larger than $20^\circ$, $z_{\text{ref}} = 6\, \text{m}$ is taken as a compromise from now onwards unless specified ($z_{\text{ref}} = 0\, \text{m}$ is taken in the synthetics part for simplicity). Nevertheless, further research should be carried out in order to exactly determine the phase center, which seems to be located between $6\, \text{m}$ and $10\, \text{m}$.
Figure 2-15: Top: Amplitude (left) and phase (right) spectra for the Delta 3 source considering the modelled notionals for $\theta_s = 0^\circ$ and $\theta_{smax} = 20^\circ$. Bottom: amplitude difference (left) and phase difference (right) of the curves shown in the top row. $z_{ref} = 0\, m$ is taken for all the plots.

Figure 2-16: Phase difference considering the modelled source signatures of the guns in a Delta 3 source configuration between take-off angles of $\theta_s = 0^\circ$ and $\theta_{smax} = 20^\circ$ for various depth references and a zoomed in image of the first 100 Hz (right).
Figure 2-17: Phase difference for impulsive sources (left) and modelled notionals (right) in a Delta 3 source configuration between take-off angles of $\theta_s = 0^\circ$ and $\theta_{s,\text{max}} = 30^\circ$ for various depth references for the first 100 Hz.
A practical method to partially deghost the data acquired with any marine source array is presented in this chapter. Its application is not only valid for normal incidence but it also works for a certain range of take-off angles which varies as a function of frequency. Furthermore, the presented compact filter also improves the bandwidth by dealing with directivity effects and the residual bubble.

In terms of structure of the chapter, the reflectivity convolution model is first discussed, which is the basis to deghost the data. The ideal source designature operator is then presented, which from a practical point of view is not useful due to the lack of angular information and the coarse shot sampling.

Hence, the next section discusses a small take-off angle approach which, under certain approximations, can be applied on real data regardless of any angle information. Amplitude and phase plots are used to estimate the accuracy of this method.

The last section briefly presents an alternative method valid at low frequencies, which is not further investigated.

### 3-1 Far field deconvolution: convolutional model and approximations

The seismic data recorded by a receiver is usually expressed as a convolution of a seismic wavelet and the earth’s impulse response under certain assumptions. Yilmaz [1987] refers to this as the convolutional model of the recorded seismogram:

\[
m(t) = w(t) * e(t) + n(t)
\]

Here, \( m(t) \) is the measured seismogram, \( w(t) \) is the seismic wavelet, \( e(t) \) is the earth’s impulse response, \( n(t) \) represents random noise and \( * \) denotes convolution. This model assumes normal
incidence plane wave propagation across a 1D horizontal subsurface and does not take into account spherical spreading, inelastic absorption, transmission losses or multiples. In practice, two additional assumptions are made: the noise component is zero and the seismic wavelet is known\(^1\).

As usually done in geophysics, the earth’s response is considered as a linear system, where the principle of superposition holds.

Hence, the seismic wavelet can be described as a superposition of several contributions. According to equation (2-1), the signature of the complete source array is a superposition of the far field source signatures, the directivity effect of the array, the source ghosts, the synchronisation of the sources and a phase shift towards the phase center of the array.

As a consequence, the seismic wavelet is time and angle dependent. In the frequency domain, convolution is expressed as a multiplication. Here, the far field depends on the position of each source, \((x_n, y_n, z_n)\), but also on the horizontal wavenumbers from the source, \((k_{sx}, k_{sy})\), and the wavenumber modulus, \(k\), such that \(W = \sum_{n=1}^{N} W_n(k, k_{sx}, k_{sy}, x_n, y_n, z_n)\). The superscript \(s\) denotes that the angle associated with this variable is the departing angle from the source.

Alternatively, one can express this in terms of source take-off angle and azimuth. Then, the far field can be written as \(W = \sum_{n=1}^{N} W_n(\omega, \theta^s, \phi^s, x_n, y_n, z_n)\), where \(\theta^s\) and \(\phi^s\) are defined as the take-off angle and the azimuth from the center of the array to a receiver and, therefore, being the same for each source within an array for a single receiver. Equivalently, I refer to them as the source take-off angle and the source azimuth, respectively, or just the take-off and the azimuth angles.

As stated by Van der Schans and Ziolkowski [1983], angular-dependent deconvolution is required to account for the directivity of a source array, which modifies the source signature as a function of take-off angle such that the peak-to-bubble ratio decreases with increasing take-off angle. The antenna term in the approach described by equation (2-1) accounts for this.

Theoretically, the angular-dependent deconvolution of the far field from the measured data can be performed on a trace-by-trace basis in the \(\tau - p_{sx}, p_{sy}\) or in the \(f - k_{sx}, k_{sy}\) domains by decomposing the measured data into its plane-wave components. According to equation (2-1), in the \(f - k_{sx}, k_{sy}\) domain (recall \(\omega = 2\pi f\)) the filter takes the following expression:

\[
F \equiv \frac{1}{W} = \frac{1}{e^{-ik_{sz}z_{ref}} \sum_{n=1}^{N} S_n(\omega) e^{i(k_{sx}x_n + k_{sy}y_n)} (e^{ik_{sz}z_n} - |r_0| e^{-ik_{sz}z_n}) e^{-i\omega \tau_n}} \tag{3-2}
\]

This is the designature operator and it can be multiplied (convolved in the time domain) with a desired target wavelet. Here, a spike in the time domain is chosen, i.e. 1 in the frequency domain (nominator of the filter).

From a practical point of view, the filter needs to be applied in the frequency-wavenumber or tau-p domains of data sorted in CRGs, in which the shot spacing is too coarse.

\(^1\)For instance, CMS is a method to estimate the seismic wavelet.
Thus, several approximations are made for a practical and simple implementation of this filter:

1. Azimuthal dependency of the far field is neglected, i.e. $p_y^s \approx 0$ and $k_y^s \approx 0$. Equivalently, $\varphi^s \approx 0^\circ$.

2. By neglecting azimuthal dependency, $F$ can be applied in the $\tau - p_x^s$ or $f - k_x^s$ domains. Data must be sorted in common receiver gathers (CRG), where each trace corresponds to one source take-off angle. Inline interpolation is required as shot spatial sampling is generally coarse. Alternatively, $F$ in equation (3-2) could be transformed to $f - k_{x}^r$ and be applied in CSG.

3. A 1D horizontally layered Earth is considered: the departure take-off angle from the source is equal to the incidence angle at the receiver according to Snell’s law, $\theta^s = \theta^r$. Hence, $p_x^s = p_x^r \equiv p_x$ and $k_x^s = k_x^r \equiv k_x$ and $F$ can be applied trace-by-trace in common shot gathers (CSG). Inline interpolation is now not required as receiver spacing is generally much smaller than shot spacing.

4. The source take-off angle is small, which is usually valid at the target level but it is not so much on shallow events, where arrivals have larger angles (such as for the seabottom). An approach based on this is presented in the next section, applicable on data in the $f - x^r$ or $t - x^r$ domains (data sorted in CSG).

Under these approximations, the so-called small take-off angle approach is further investigated as a practical source deghosting and designature filter, as it is directly applied in CSG without the need for any spatial Fourier transform of the data. Even more, it compensates for directivity effects up to a certain extent.

A filter under the first and second (perhaps the third) approximations is expected to work better on more complicated geological structures as $\theta^s \neq \theta^r$, but it requires trace interpolation and sorting of the data in CRG. Yet, this filter is unstable at those frequencies at which the far field, $W$, has notches, where the filter infinitely boosts the signal and it is limited by the noise level as the filter is applied to both the data and the noise. These limitations also hold for the small take-off angle filter obtained after the last four approximations.

Two possible solutions are suggested to deal with the instability of the filters. The first one is to add a small, positive and real-valued stabilisation parameter $\epsilon$ to the complex conjugate of the far field times the far field such that the filter is stable for all frequencies:

$$F \equiv \frac{1}{W} \approx \frac{W^*}{W^*W + \epsilon}, \quad \epsilon \in \mathbb{R}^+$$

(3-3)

In which the superscript $*$ denotes conjugation. But this presents two problems: 1) it is difficult to quantify how much $F$ is modified, 2) a criteria for determining $\epsilon$ must be used.

A suggestion is to define $\epsilon$ as a percentage of the maximum value of the modulus of the far field, $\epsilon = (\xi/100) \max (W^*W)$, where $\xi$ represents this percentage and it must be chosen with some criteria.
Compact source deghosting filters

An alternative solution to overcome the instability of this filter is to apply a band-pass filter that removes those frequencies at which it is unstable, i.e. at which the denominator is zero.

In particular for the Delta 3 source, strong source ghost notches (stronger than the antenna notches, as seen in figure 2-1) are present at frequencies of \( 0 \, Hz \) and \( 250 \, Hz \), which are infinitely boosted by the small take-off angle filter.

Next, the small take-off angle approach is discussed.

3-2 Small take-off angle source deghosting filter

Neglecting azimuthal dependency, this section introduces a small take-off angle approximation of the designature operator in equation (3-2), able to remove the source ghosts, directivity and bubble effects from seismic data up to a certain extent.

To find the deghosted data sorted in CRG, \( M^{DG}(\omega, x^s) \), the inverse of the far field (or designature operator), \( F \), is applied to the measured data in the same domain, \( M(\omega, x^s) \), where \( x^s \) denotes the horizontal spatial variable in a receiver gather.

The inverse of the far field in equation (3-2) is first expanded as a Taylor series around zero source take-off and azimuth angles, such that 

\[
F \approx \sum_{n=1}^{N} F_n(k, \theta^s, (\theta^s)^2, \varphi^s, (\varphi^s)^2, x_n, y_n, z_n) \quad (A-5)
\]

Then, azimuthal dependency is neglected and the latter takes the following expression:

\[
F \approx F(\theta^s = \varphi^s = 0^\circ) + \theta^s \frac{\partial F}{\partial \theta^s} \bigg|_{\theta^s=\varphi^s=0^\circ} + \frac{(\theta^s)^2}{2} \frac{\partial^2 F}{\partial (\theta^s)^2} \bigg|_{\theta^s=\varphi^s=0^\circ} \equiv A + B^' \theta^s + C'(\theta^s)^2 \quad (3-4)
\]

The Fourier sign convention used here is the same as in section 2-1. Appendix A presents the derivation of the previous equation and the expressions for the coefficients including azimuthal dependency. The zero azimuth assumption is briefly evaluated in the next section for the Delta 3 source.

After a 2D spatial inverse Fourier transform, the latter is expressed in terms of spatial derivatives, 

\[
F \approx \sum_{n=1}^{N} F_n(k, \partial^s_x, (\theta^s)^2, x_n, z_n), \quad \text{where } \partial^s_x \equiv \partial_x \text{ and } (\partial^s_x)^2 \equiv \partial_x^2 \text{ are the first and second spatial derivatives in the common receiver domain.}
\]

However, this approach also suffers from coarse sampling of CRGs. In this domain, the spatial sampling between traces corresponds to the shot sampling (\( \sim 25 \, m \)), which is usually much larger than the receiver sampling (\( \sim 3 \, m \)). Therefore, in order for the spatial derivatives to give an accurate result, inline interpolation of traces is required.

For simplicity, an additional approximation is made. A 1D horizontal earth is assumed, such that the offset controls the propagation (Snell’s law applies) and the departure take-off angle from the source is nearly the same as the incident angle in the receiver (third approximation in the previous section). In other words, \( \theta^s \approx \theta^r \) and the small take-off angle filter can either
be applied in CRGs or in CSGs. Interpolation is expected to improve the result but it is not required given that the spatial sampling in CSGs corresponds to the sampling between receivers, which is usually small (≈ 3 m).

Under this approximation, one can write

\[ F \approx \sum_{n=1}^{N} F_n(k, \theta_x^x, (\partial_x^x)^2, x_n) \equiv \sum_{n=1}^{N} F_n(k, \theta_x^x, \partial_x^2, x_n, z_n), \]

in which frequency and take-off angle operators are decoupled. According to the derivation in appendix A:

\[ F \approx A + B \partial_x^x + C \partial_x^2 \]  

(3-5)

Here, \( A, B \) and \( C \) are angle independent coefficients that depend on the frequency and the vertical and horizontal (inline) positions of the sources as well as the source signatures in the following fashion:

\[
A = \left[ \sum_{n=1}^{N} S_n(\omega) \left( e^{ikc_n^+} - |r_0| e^{ikc_n^-} \right) \right]^{-1} \equiv [W(k, \theta^s = 0^\circ)]^{-1}
\]

\[
B = A^2 \sum_{n=1}^{N} S_n(\omega) x_n \left( e^{ikc_n^+} - |r_0| e^{ikc_n^-} \right)
\]

\[
C = \frac{B^2}{A} + \frac{iA^2}{2k} \sum_{n=1}^{N} S_n(\omega) \left[ \left( z_{\text{ref}} - z_n + ikx_n^2 \right) e^{ikc_n^+} - |r_0| \left( z_{\text{ref}} + z_n + ikx_n^2 \right) e^{ikc_n^-} \right]
\]

Where \( c_n^\pm = \pm z_n - z_{\text{ref}} - \tau_n c \) and \( \tau_n = (z_n - z_{\text{min}})/c \). These coefficients are related to \( B' \) and \( C' \) such that \( B = B'/(-ik) \) and \( C = C'/(-ik)^2 \).

Equation (3-5) is a generalisation of Amundsen and Zhou [2013, equation 7] to \( N \) non-impulsive sources while neglecting azimuthal dependency \( (k_y^s \approx 0) \). Thus, the latter is valid for any marine source array.

The first coefficient of the expansion, \( A \), corresponds to the inverse of the far field at zero take-off angle. If only this is considered, a zero order small take-off angle approach can be applied to the data which does not require the computation of spatial derivatives.

Similarly to the exact expression for \( F \) presented in the previous section, coefficient \( A \) requires stabilisation at those frequencies in which the denominator cancels, i.e. where the source ghosts of the sources at different depths have a common notch. This will be partly compensated for when applied to synthetic and real data with the approaches suggested in the previous section.

The second coefficient, \( B \), introduces a weighted correction to the filter in terms of take-off angle, where the weights are the inline position of the sources, \( x_n \). Thus, by including the second term an improved correction due to directivity and source ghost effects with respect to take-off angle is expected. In fact, \( B \) compensates for the asymmetry of the source array.

Finally, the third term is also a weighted correction that includes larger take-off angle variations. The weights take now a more complicated expression as a function of the frequency and vertical and horizontal (inline) positions of the sources.

\footnote{Note that an opposite sign convention for the Fourier transforms is used.}
It is worth mentioning that both $B$ and $C$ may require additional stabilisation at low frequencies as these are amplified by the factors $1/k$ and $1/k^2$ included in the latter. A suggestion is to apply a steep band-pass filter at low frequencies on these two coefficients in order not to excessively amplify signal and noise at low frequencies, as discussed in the following chapters.

In the next subsection, the zero azimuth assumption is evaluated for the Delta 3 source array using the modelled notionals.

Then, the theoretical benefits of either using only $A$, $A$ and $B$ or $A$, $B$ and $C$ as well as the coefficients’ behaviour with frequency are shown below these lines.

In the following chapters, these different approximations are also tested on synthetic and real data and the advantages and drawbacks of including each term are analysed.

3-2-1  The zero azimuth assumption for the Delta 3 source

The small angle filter presented in equation (A-9) with coefficients given by equation (A-8) in the appendices have been particularised for zero source azimuth. This approximation is made given the lack of azimuthal information in seismic data, but the term $B_2 \partial_y x$ in equation (A-9) can be included if one wants to take that into account, provided cross-line data is available. Moreover, take-off angle variations are usually more important due to the symmetry of marine source arrays and the location of the streamers. Thus, azimuthal dependency is neglected.

To illustrate this point, the far field signature due to a Delta 3 source array considering the modelled notionals is computed as a function of frequency and take-off angle for different azimuths. The resulting amplitude and phase spectra are shown in figure 3-1.

From these plots it is observed that the phase varies more with azimuth than the amplitude. Moreover, both amplitude and phase spectra are symmetric with azimuth but they are not with take-off angle. This is expected as the array in figure 1-2 is symmetric with respect to azimuth but it is not with take-off angle due to the presence of the clusters.

An important observation is that variations in the spectra for angles within a cone of ±20°, at which the Delta 3 source is designed to optimally operate, are more important with take-off angle than what they are with azimuth. This is observed at high frequencies, where directivity effects are important. For instance, an amplitude trace at a fixed frequency of 50 Hz or 120 Hz has more variation with take-off angle than with azimuth.

Furthermore, streamers are usually aligned with the source arrays in the inline direction (x axis in figure 1-2) and their length in this direction is much longer than in the cross-line direction. Thus, a large variation in take-off angles is recorded - as it is the angle between the vertical and the inline direction -, whereas the range of azimuths measured is limited. However, this is not true for near offset measurements of the outer receiver lines, where azimuthal dependency is important and needs to be taken into account.

Although a small angle approach around $(k_x s)^2 + (k_y s)^2$ can be obtained in a similar fashion, the small take-off angle filter represents a practical approach in which small inline receiver
Figure 3-1: Far field amplitude (top row) and phase spectra (bottom row) for a Delta 3 source configuration for azimuths between $\varphi_s = -60^\circ$ and $\varphi_s = 60^\circ$ (see system of coordinates in figure 1-2). The modelled notionals have been used.

3-2 Coefficients A, B and C for the Delta 3 source

A, B and C in equation (3-6) are the general expression for the coefficients of the small take-off angle approximation of the designature operator, valid for any source array. Now, these are particularised for the Delta 3 source array configuration and their spectral and time response are computed.

The small take-off angle coefficients in the frequency domain are found by substitution of the source positions in figure 1-2 and the notionals in equation (3-6). Moreover, not only coefficients $A$, $B$ and $C$ are studied but also $B' = (-ik)B$ and $C' = (-ik)^2C$, as the latter are of the same order of magnitude and have the same dimensions ($\theta^s$ is in radians). But the former are of practical use (alternatively, one could apply $A$, $B'$ and $C'$ in the $\tau - p_x^s$ domain).

Here, $z_{ref} = 6\ m$ (see section 2-2-5), $r_0 = -1$ and two different situations are considered: impulsive sources, in which $S_n(\omega) = 1\ \mu Pa\cdot m$, and the modelled notionals for the Delta 3 source.

The amplitude spectra of all the coefficients are shown in figure 3-2.
Figure 3-2: Amplitude spectra of coefficients $A$, $B'$ and $C'$ for both impulsive and modelled notionals (top) and amplitude spectra of coefficients $B$ and $C$ for the same situations. The dashed lines represent the original spectra, whereas the continuous lines are the spectra after applying a band-pass filter (a cosine window) in order for the coefficients to be stable.

In this figure, the original amplitude spectra of the coefficients is depicted by dashed lines. As expected, the $A$ coefficient is unstable at zero frequency and at $250\, Hz$, where the sources at $6\, m$ and at $9\, m$ have a common notch. A band-pass filter is applied in order to make this coefficient stable and not infinitely boost very low and very high frequencies.

A standard power cosine window is used as a band-pass filter [De Levie, 2004, p. 245-255]. The exponent of the cosine window has been set to 2 ($s = 1$, von Hann window) for coefficients $A$, $B'$ and $C'$ with a low cut frequency of $f_{\text{low}}' = 4\, Hz$ and a high cut frequency of $f_{\text{high}}' = 200\, Hz$. Their spectra after filtering corresponds to the continuous lines plotted on the top row of figure 3-2.

On the other hand, a power cosine window with exponent 4 ($s = 2$, alternative Blackman) has been applied on $B$ and $C$, given that a steeper filter is needed as they present a faster variation at low frequencies (due to the $1/k$ and $1/k^2$ factors multiplying them with respect to $B'$ and $C'$ that boost the low frequencies even more). A low cut frequency of $f_{\text{low}} = 5\, Hz$ and a high cut frequency of $f_{\text{high}} = 200\, Hz$ have been selected. Their spectra after filtering corresponds to the continuous lines of the two graphs at the bottom row of figure 3-2.
The following is observed from figure 3-2:

- All the coefficients boost the very low frequencies and the high frequencies to compensate for the notches at 0 Hz and at 250 Hz from the sources at 6 m and 9 m.
- In order not to boost too much the frequencies around the notches at 0 Hz and 250 Hz, a cosine window band-pass filter is applied to the coefficients.
- Coefficients A and B’ are of the same order of magnitude, whereas C’ is larger specially around 90 Hz, where it rises drastically in comparison to A and B’. This will be further analysed in the following section.
- When considering the notionals, the filters include the low frequency variations due to the bubble effect. Hence, they are expected to remove the residual bubble from the data.
- The difference between the amplitude spectra with the notionals and with impulsive sources is not constant as the amplitude spectra of the notionals varies with frequency. Still, they follow the same trend.

The band-pass filtered coefficients are then transformed to the time domain via a 1D inverse Fourier transform, as presented in figures 3-3 and 3-4 for the modelled notionals and for impulsive sources, respectively.

Coefficients A, B’ and B present a similar shape, being a high frequency signal (note the fast variation) and having a similar amplitude. But both A and B present a low frequency tail before and after zero time.

Coefficients C’ in figures 3-3 and 3-4 (top right) have also a similar shape compared to the previous coefficients, but their amplitude is roughly 10 times larger. This is explained by the fact that their amplitude spectrum at frequencies higher than 90 Hz shown in figure 3-2 increases over A and B’.

Finally, coefficients C have larger magnitude than C’ for either impulsive or modelled notionals, as expected according to their amplitude spectra in figure 3-2 (bottom right).

Furthermore, C has a much slower variation than the other coefficients and within a larger time range. The main explanation for this is that C has a higher content in low frequencies, which been amplified by the factor $1/k^2$. In this case, the band-pass filtering is essential not to boost too much the low frequencies and this will be taken into account when applying this approach to synthetic and real data.

It is worth mentioning that the small take-off angle coefficients for impulsive sources in a Delta 3 source array configuration are the same independently of the survey, whereas they vary when the notionals are considered as the source signatures are not the same for each shot in a survey (due to several reasons such as the change in hydrostatic pressure, the actual towing depth, etc.).

In the real data case, the data is filtered in time by convolving it with these coefficients as in equation (3-5). Hence, the coefficients in time have a finite length, which is chosen as short as possible, and similar filters as in figures 3-3 and 3-4 are truncated. As a consequence, their spectra is slightly modified and this will be later on considered in the real data chapter.
Figure 3-3: Coefficients for the small take-off angle approximation of the inverse of the far field in the time domain for the modelled notionals of a Delta 3 source array.

3-2-3 Error plots for the Delta 3 source

The theoretical benefits of using the small take-off angle approach rather than a zero take-off angle approach are now analysed. Amplitude and phase plots are used in order to estimate the validity of the small take-off angle approach and its accuracy.

The small take-off angle approach in equation (3-4) is compared with the exact expression for the inverse of the far field, presented in equation (3-2). Different terms of the Taylor series expansion can be considered separately in order to investigate the benefits of using higher orders in this expansion.

Thus, the amplitude and phase spectra of the designature operator (denoted as $|F|$ and $\phi(F)$, respectively, being $F$ the filter in equation (3-2)) are compared with the corresponding spectra of the first term in equation (3-4), i.e. $A$, the spectra of the first and the second terms in this equation, $A + B'\theta^s$, and the first three terms, $A + B'\theta^s + C'(\theta^s)^2$.

The first column in figures 3-5 and 3-6 shows the amplitude spectra of the exact filter and the small take-off angle approximation including each of the terms in the Taylor series expansion,
Figure 3-4: Coefficients for the small take-off angle approximation of the inverse of the far field in the time domain for impulsive sources in a Delta 3 source array configuration.

progressively, as a function of frequency and take-off angle. Whereas the plots in figure 3-5 have been generated using the modelled notionals, impulsive sources have been used in figure 3-6 in a Delta 3 source configuration. Note that the amplitude and the phase of both exact filters (first and last plots of the first row in these figures) are not exactly symmetric due to the clusters.

The second column in these plots contains the difference in amplitudes between the approximations and the exact amplitude.

In the next column, the amplitude of the difference between the approximations and the exact filter is plotted, which also takes into account the phase of the filters. Here, dark blue colour means the amplitude of the difference is small.

Finally, the last column in figures 3-5 and 3-6 shows the phase for each filter.

To generate these plots, the reference depth and the sea surface reflection coefficient have been set to $z_{ref} = 6\, m$ and $r_0 = -1$, whereas the source positions are the ones for the Delta 3 source.

In addition, the same plots are made considering the low frequency filter suggested by Amundsen and Zhou [2013, equation 10] (with $z = z_{avg} = 7\, m$) for comparison, as shown at the
bottom row of figure 3-6. The previous filters for impulsive sources are thus multiplied by 24 so that they can be compared with the latter, which only accounts for a single source.

Figure 3-5: Error plots for the exact inverse of the far field, $A$, $A + B'\theta^a$ and $A + B'\theta^a + C'(\theta^a)^2$ considering the modelled notionals.

The amplitude and phase of the exact filter in the first row of these plots are such that they compensate for the effects of the directivity, the source ghosts and the bubble effect (the
latter only when the notionals are considered, figure 3-5) that have an impact on the far field, as explained in chapter 2 and as shown in figure 2-1. Thus, notches in the far field amplitude spectrum in figure 2-1 are amplified (shown in red colours) whereas the amplitude in other areas is decreased in order to obtain a flat spectrum, corresponding to a spike in the time domain. Again, the amplitude and phase spectra are not symmetric due to the clusters and amplitudes are larger in figure 3-5 because the notionals have bigger amplitudes than the impulsive sources. In terms of the phase, the impulsive sources lead to a slightly different phase response than the notionals.

In contrast, the following is observed when comparing the different Taylor approximations to the exact filter either with notionals or with impulsive sources:

1. In terms of amplitude (first column in both plots), $A$ alone (second row) represents perfectly the trace at the center in the first plot on the right (first column) as it is the inverse of the far field at zero take-off angle, but it is unable to compensate for the directivity notches. By considering $B'$ (third row), the amplitude starts being asymmetric and slightly changed from the former as it models some directivity effects. Moreover, when $C'$ is included (fourth row) directivity notches are compensated for up to a certain extent.

2. The difference in amplitudes with respect to the exact (second column), does not consider the phase. It is noticeable that by including each term in the Taylor expansion, the difference with respect to the exact is smaller for a larger range of take-off angles as each terms models the directivity effects and the source ghost variation with take-off angle more accurately. This is a common feature of filters approximated by Taylor series, which are accurate in their domain of validity but they differ outside this domain. Nevertheless, amplitudes are over amplified when including $C'$ for frequencies larger than $90 \, \text{Hz}$ and take-off angles from $\pm 90^\circ$ to $\pm 25^\circ$, where the approximation breaks.

3. The third column in both plots shows the accuracy of the small take-off angle approach taking into account amplitude and phase. From this, it is visible that the accuracy of the approximation (blue and dark blue colours show little difference, and hence high accuracy) is improved by including the different terms in the series expansion. Whereas considering $A$ alone gives a very limited accuracy around zero take-off angle, this range is significantly improved when $B'$ is included (from $\sim \pm 5^\circ$ degree accuracy from 0 to $90 \, \text{Hz}$ with only $A$ to $\approx \pm 12^\circ$ considering $B'$). This is further improved (up to $\sim \pm 30^\circ$ for these frequencies) when including $C'$. For higher frequencies, the full (including all terms) approximated filter is accurate up to $\sim 20^\circ$.

4. The phase (last column in both plots) is improved by including more terms in the Taylor expansion (compare the fourth figure in the second column with either the fourth picture in the third row or the fourth row in both plots) and the rapid change in polarity due to the antenna notches cannot be modelled by any of them, which is a limitation of the approach.

5. Regarding the behaviour in terms of amplitude and phase of the filter proposed by Amundsen and Zhou [2013] (last row of figure 3-6), these are only accurate at low frequencies (in comparison to the first picture in the first column of the same figure),
Compact source deghosting filters

up to $\approx 55 \text{ Hz}$, and over the same range of take-off angles as only considering $A$ with impulsive sources.

Hence, by including the different terms in the small take-off angle approximation presented in equation (3-4), the accuracy is improved as the approximated filter is able to model directivity effects and source ghost variations with take-off angles, up to a certain extent. One could suggest to add more terms in the Taylor series, but this becomes less practical and practical issues need to be discussed (for instance, an additional coefficient, $D$, might amplify even more than $C$ high frequencies at large take-off angles).

Thus, the suggested method is expected not only to deghost the data but also to compensate for directivity effects. What is more, this is a general approach that can be extended to any source array by appropriately changing the parameters in equation (3-6).

Next, the small take-off angle approach is tested on synthetic and real data in the following chapters and the different advantages and drawbacks are discussed.

The inaccuracy introduced by the spatial derivatives when considering equation (3-5) is later on analysed on synthetic data. This depends on how well the data is sampled in space and also on the finite difference operator used to compute the derivatives.

Before testing the validity of this approach on synthetic data, an alternative low frequency approach is presented. Yet, this has not been tested on synthetic or real data and requires further research.

3-3 An alternative approach

At the beginning of this chapter, a small take-off angle approximation of equation (3-2) was presented. Now, a low frequency approximation of equation (3-2) is derived, which can be specialised to zero take-off angle.

To derive a low frequency approximation of the designature operator in equation (3-2), it is assumed that the far field in equation (2-1) can be approximated at low frequencies by $W \approx W'_1 \omega + W'_2 \omega^2 + W'_3 \omega^3$, where $W'_1$, $W'_2$ and $W'_3$ are the coefficients of the series expansion. The inverse of the far field can then be written in the following fashion:

$$F \approx \frac{A_{-1}}{i\omega} + A_0 + i\omega A_1$$

(3-7)

Here, coefficients $A_{-1}$, $A_0$ and $A_1$ depend on the take-off angle but not on the frequency.

Yet, this is impractical due to the lack of angular information on real data and these coefficients are specialised to $\theta^* = 0^\circ$ such that:
An alternative approach

\[ A_{-1} = \frac{ie}{W_1} = \frac{c}{2i} \left[ \sum_{n=1}^{N} S_n(\omega)z_n \right]_{\omega=0} \]

\[ A_0 = -\frac{W_2}{i\epsilon W_1^2} = \left[ \frac{i \sum_{n=1}^{N} \frac{\partial S_n(\omega)}{\partial \omega} z_n + \sum_{n=1}^{N} S_n(\omega)z_n(z_{ref} + \tau_n c)}{2 \sum_{n=1}^{N} S_n(\omega)z_n} \right]_{\omega=0} \]

\[ A_1 = \frac{W_2^2}{i\epsilon W_1^2} - \frac{W_3}{i\epsilon W_1^2} = \left[ \frac{\sum_{n=1}^{N} \frac{\partial^2 S_n(\omega)}{\partial \omega^2} z_n + 2i \sum_{n=1}^{N} \frac{\partial S_n(\omega)}{\partial \omega} z_n(z_{ref} + \tau_n c) - \frac{i}{2} S_n(\omega)z_n(z_n^2 + 3z_n^2 + 6z_n^2 + 12z_{ref} + 3\tau_n c^2)}{4 \sum_{n=1}^{N} S_n(\omega)z_n^2} \right]_{\omega=0} \]

(3-8)

Which are further simplified for impulsive sources (see equation (B-20) and figure B-1).

The current approach is a generalisation of the low frequency approach presented in Amundsen and Zhou [2013, equation 10] to \( N \) non-impulsive sources. Indeed, when one source is considered the present approach converges to the low frequency filter suggested by Amundsen and Zhou [2013] (see equation (B-21)). For further details, the reader is referred to the full derivation in appendix B.

The main advantage of this approach is that, under the zero take-off angle assumption and in the time domain, it can be directly applied to the data in a trace-by-trace basis as a sum of an integral of the data, a time derivative of the data and the data itself conveniently convolved with the previous coefficients in the time domain in the following fashion:

\[ m_{dg}(t) = a_{-1}(t) * \int_0^t m(t')dt' + a_0 * m(t) + a_1(t) * \partial_t m(t) \quad (3-9) \]

Where \( m(t) \) is the measured data in time for one trace, \( m_{dg}(t) \) is the corresponding deghosted data at low frequencies and \( a_{-1} \), \( a_0 \) and \( a_1 \) are the time domain coefficients of the low frequency approximation for zero take-off angle. However, the integral term is unstable at low frequencies and a low-cut filter is required. Again, equation (3-9) is a generalisation to \( N \) sources of Amundsen and Zhou [2013, equation 10] in the time domain.

It should be emphasised that the expression for the coefficients in equation (3-8) are general for any source array and zero take-off angle, but their validity extends over a certain frequency band and a range of take-off angles (its validity for the Delta 3 source is shown in figure B-1, which compares it to the exact filter for impulsive sources and it shows that it is valid up to a certain frequency and a range of take-off angles).

The accuracy of this approach with respect to \( F \) is presented in figure B-1. This shows that there is an improvement with respect to the approach suggested by Amundsen and Zhou [2013], specially as the phase is improved when the depths of the different sources are considered. Nevertheless, the improvement is not dramatic as \( \theta^s = 0^\circ \) is considered for practical reasons.

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Yet, further research on its application on synthetic and real data is required, as well as the assessment of its benefits and drawbacks with respect to the small take-off angle approach.

Nevertheless, the small take-off angle approach is expected to give better results as it includes all the frequencies and take-off angles different than zero, whereas the latter is a approximated at low frequencies and its application according to equation (3-9) requires zero take-off angle.

Hence, only the small take-off angle approach is now tested on synthetic and real data.
Figure 3-6: Error plots for the exact inverse of the far field, $A$, $A + B\theta^*$, $A + B\theta^* + C^*(\theta^*)^2$ and the low-frequency filter as per Amundsen and Zhou [2013] for impulsive sources.
Part II

Synthetic data
Chapter 4

Small take-off angle approach: tests on synthetic data

In the previous chapter, the small take-off angle approximation of the inverse of the far field was presented. This is now applied on a simple synthetic example to test its validity and accuracy.

First, the geometry used to generate the synthetic data is depicted and the different parameters are listed. Then, the input data is filtered using the small take-off angle approach in the form described by equation (3-4), which considers that the angle information is known. In addition, the small take-off angle filter in terms of spatial derivatives presented in equation (3-5) is tested, which has a practical implementation on real data.

The inaccuracy introduced by the derivatives in the latter and the benefits of using the different variations of the small take-off angle approach are discussed with the help of frequency-space amplitude spectra of the resulting data.

This is first done considering the modelled notionals and later on considering impulsive sources, both in a Delta 3 source array configuration.

All the previous results are then further discussed in the last section of this chapter.

4-1 Geometry and modelling parameters

To test the small take-off angle approach, synthetic data with the geometry depicted in figure 4-1 has been generated considering a $6 - 9 - 6 \text{ m}$ depth Delta 3 source array. The far field pressure signal is recorded by an array of receivers at $500 \text{ m}$ depth regularly distributed over $4000 \text{ m}$ offset, as illustrated in figure 4-1. The latter are situated along the $x$ axis such that there is no azimuthal dependency ($\varphi^s = 0^\circ$) and travel times are computed from the sea surface ($z_{ref} = 0 \text{ m}$).

Receiver spacing is initially set to a constant value of $5 \text{ m}$ and this is later on changed to $2.5 \text{ m}$ and $1.25 \text{ m}$ to investigate the accuracy of the finite difference operator that computes
the spatial derivatives in equation (3-5). In consequence, the source take-off angle as depicted in figure 4-1 is not regularly distributed amongst the receivers. In other words, the source take-off angle associated to one shot (the same angle for all the sources) does not vary linearly with offset.

Figure 4-1: Illustration of the geometry used to generate the synthetic data. Distances are not in a 1:1 scale.

Given that the receiver spacing is varied, the number of receivers differs in each case but the range of source take-off angles is the same for the same offset as the Delta 3 source array and the line of receivers remain at the same positions.

Figure 4-1 illustrates a CSG, although really a CRG is needed. Nevertheless, as reciprocity holds here the Delta 3 source can be replace with a single receiver and the array of receivers be replaced with different Delta 3 source arrays at each location. Thus, both are equivalent here.

Once the geometry is defined, synthetic data is generated by adding the contribution to the far field of each source to each receiver as if the former were monopoles. According to Wapenaar and Berkhout [1989, equation I-18a], the wave field for a single causal monopole in a homogeneous fluid at a distance $r_n = \sqrt{(x - x_n)^2 + (y - y_n)^2 + (z - z_n)^2}$ from the source, being $(x, y, z)$ the coordinates of the receiver and $(x_n, y_n, z_n)$ the coordinates of the monopole, is given by:

$$p_n(r_n, t) = \frac{s_n(t - r_n/c)}{r_n}$$

(4-1)

Where the source signature, $s_n(t)$, is expressed in $bar \cdot m$ and it includes the term $1/4\pi$ in Wapenaar and Berkhout [1989, equation I-18a].

The far field has the same units as the source signature and it is defined as $w_n(r_n, t) = s_n(t - r_n/c)$. According to the principle of superposition and taking into account the additional delay due to the synchronisation of the sources, $\tau_n$, the far field is computed as the sum of the far field due to each source in a Delta 3 source array and their ghosts:
\[ w(r, t) = \sum_{n=1}^{N} \left[ s_n(t - r_n/c) \ast \delta(t - \tau_n) - |r_0|s_n(t - r'_n/c) \ast \delta(t - \tau_n) \right] \] (4-2)

Being \( r'_n \) the distance from each mirror source (that generates the source ghost far field) to the receiver, i.e. \( r'_n = \sqrt{(x - x_n)^2 + (y - y_n)^2 + (z - (-z_n))^2} \). The source signatures used here are the modelled notionals for the Delta 3 source, as in the previous chapter, which already include the synchronisation of the sources.

The time sampling is \( \Delta t = 0.5 \text{ ms} \) and, thus, the Nyquist frequency is \( f_N \equiv 1/(2\Delta t) = 1000 \text{ Hz} \). Commonly, the time sampling in seismic data is around 2 ms, with a corresponding Nyquist frequency of 250 Hz, and the maximum observed frequency is roughly between 150 Hz and 180 Hz. Hence, in order to make the synthetic example more realistic, a zero phase 11th order Butterworth filter\(^1\) with a high cut frequency of \( f_{cut} = 150 \text{ Hz} \) is applied to the synthetic data before applying the small take-off angle filters. Alternatively, the notionals could have been resampled at 2 ms.

Synthetic data is then generated and filtered with the small take-off angle filters presented in the previous chapter. Only for display reasons, the normal moveout is undone after filtering with the small take-off angle filters such that the single event due to the direct arrival and the ghost from the Delta 3 source array can be seen as a flat event, in which comparisons are more easily made. To achieve that, each trace - corresponding to the same shot but a different receiver position - is shifted in time by \( t_0 = -r/c \), being \( r = \sqrt{x^2 + y^2 + z^2} \) the distance from the origin of the system of coordinates to a receiver (travel times are computed from \( z_{ref} = 0 \text{ m} \)). It is essential that this is performed after filtering the data, because spatial derivatives are computed from a weighted sum of data with moveout.

Furthermore, an additional time shift is applied such that the direct arrival sits at 200 ms. This way, waveforms are more easily identified.

The resulting filtered data with the different terms of the Taylor series expansion in equations (3-4) and (3-5) are compared in the following sections.

As stated in chapter 3, coefficient \( A \) is unstable at very low (around 0 Hz) and very high (around 250 Hz) frequencies, where the signal is boosted. Here, the denominator in \( A \) is stabilised similarly to equation (3-3) for frequencies \( f \leq 4 \text{ Hz} \) and \( f \geq 150 \text{ Hz} \) with a stabilisation order of \( \xi = 5 \), i.e. 5% of the maximum of the absolute value of denominator is added to it. Coefficients \( B, C, B' \) and \( C' \), which depend on \( A \), are not stabilised here as it is enough to do it for the latter.

In order to apply the small take-off angle filter as in equation (3-5), the spatial derivatives are computed with a nine point finite difference stencil, which introduces an additional error. This will be later on investigated with the help of amplitude plots in the frequency-space domain.

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\(^1\)This is achieved in MATLAB® by designing the desired Butterworth filter (specifying the cut frequency and the order of the filter), obtaining the coefficients of the filter and using them to filter the data with the command `filtfilt`. 

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An additional test is then performed: the small take-off angle filters presented in equations (3-4) and (3-5) are multiplied (convolved in the time domain) with another desired target wavelet. Let this be the far field signature at zero take-off angle, which is the inverse of $A$ as defined in equation (3-6). Thus, one can write:

$$
\frac{F}{A} = F \cdot W(k, \theta^s = 0^\circ, z_n) \approx 1 + \frac{B'}{A} \theta^s + \frac{C'}{A} (\theta^s)^2
$$

(4-3)

or, under the same assumptions that lead to equation (3-5):

$$
\frac{F}{A} = F \cdot W(k, \theta^s = 0^\circ, z_n) \approx 1 + \frac{B}{A} \theta^s + \frac{C}{A} \theta^s^2
$$

(4-4)

If either of these is applied to the input data, the result is the same input data (first term in the previous equations) plus corrections with take-off angle to first order (terms $B'/A$ or $B/A$) and second order (terms $C'/A$ or $C/A$). Thus, the latter are not expected to deghost the data because the desired wavelet includes the source ghost at zero take-off angle, but they represent a correction due to directivity effects and source ghost variation with take-off angle.

It should be emphasised that the latter is again applicable to any marine source array.

The remaining parameters used to generate the synthetic data are summarised in table 4-1.

<table>
<thead>
<tr>
<th>Modelling parameter</th>
<th>Nomenclature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water velocity</td>
<td>$c$</td>
<td>1500 m/s</td>
</tr>
<tr>
<td>Sea surface reflection coefficient</td>
<td>$r_0$</td>
<td>$-0.95$</td>
</tr>
<tr>
<td>Source array type</td>
<td></td>
<td>Delta 3 source</td>
</tr>
<tr>
<td>Number of sources per array</td>
<td>$N$</td>
<td>24</td>
</tr>
<tr>
<td>Source signatures</td>
<td>$s(t)$</td>
<td>Modelled (Gundalf$^\text{™}$)</td>
</tr>
<tr>
<td>Receiver spacing</td>
<td>$\Delta x$</td>
<td>2.5 m, 5 m or 10 m</td>
</tr>
<tr>
<td>Receiver depth</td>
<td></td>
<td>500 m</td>
</tr>
<tr>
<td>Receiver cross-line positions</td>
<td></td>
<td>0 m</td>
</tr>
<tr>
<td>Maximum and minimum receiver offsets</td>
<td></td>
<td>$\pm 2000$ m</td>
</tr>
<tr>
<td>Time sampling</td>
<td>$\Delta t$</td>
<td>0.5 ms</td>
</tr>
<tr>
<td>Nyquist frequency</td>
<td>$f_N$</td>
<td>1000 Hz</td>
</tr>
<tr>
<td>Reference depth</td>
<td>$z_{\text{ref}}$</td>
<td>0 m</td>
</tr>
<tr>
<td>High-cut Butterworth filter applied on the data: cut frequency</td>
<td>$f_{\text{cut}}$</td>
<td>150 Hz</td>
</tr>
<tr>
<td>High-cut Butterworth filter applied on the data: filter order</td>
<td>$p$</td>
<td>11</td>
</tr>
<tr>
<td>Stabilisation of $A$: frequencies</td>
<td>$f \leq 4$ Hz and $f \geq 150$ Hz</td>
<td></td>
</tr>
<tr>
<td>Stabilisation of $A$: order</td>
<td>$\xi$</td>
<td>5</td>
</tr>
</tbody>
</table>

| Table 4-1: Parameters used to compute the synthetic data with the geometry depicted in figure 4-1, their nomenclature and their value, when appropriate. |
4-2 Filtered data: modelled notionals

The obtained synthetic data according to the geometry depicted in figure 4-1 and the modelling parameters in table 4-1 is now presented and then filtered by using the small take-off angle filter. In this section, the source signatures of the filters used are the modelled notionals for a Delta 3 source.

After removing the normal moveout, the left panel in figure 4-2 shows the input data. In this and the following pictures, the vertical axis is the time axis (in seconds) and the trace number and the take-off angles are both indicated at the bottom axis (the latter is additionally indicated at the top one).

The second panel in this figure shows the direct down-going wave from the array to the receivers, as if there were no source ghosts. Finally, the third panel shows the result of applying the exact filter as shown in equation (3-2) to the input data.

Several things are observed from these images. For the input data:

1. It contains the direct arrival (positive event around 196 ms) and the source ghost (negative event around 204 ms).

2. The time separation between the direct arrival and the source ghost decreases with
offset, which is an effect of the source ghost variation with take-off angle, as described in section 2-2-2.

3. The time separation between the different arrivals increases with take-off angle, which is more visible around $\theta_s \sim 50^\circ$ (note the direct arrival seems to split). This is due to directivity effects, as described in section 2-2-5. For negative angles the effect is not so visible due to the asymmetry of the array. This points out that directivity effects are smaller on the cluster side, i.e. the far field does not vary so significantly with direction on this side.

4. The residual bubble leaves a low frequency imprint on the input data, which is observed as the small peak and troughs after the ghost and deeper down between 280 ms and 240 ms and also in the region between 360 − 400 ms. These correspond to the residual bubble peaks and troughs shown in figure 2-3 (left).

The bubble effect is more pronounced when the down-going field alone is considered (second panel), where low frequency bands can be appreciated after the direct arrival. The combination with the source ghosts reduces the bubble effect. Note the deflection of the down-going field with offset due to its variation with take-off angle. The same splitting of the direct arrival with take-off angle due to directivity effects is observed in this picture for $\theta_s \sim 50^\circ$. The aim of the small take-off angle filter is not only to obtain the direct wave shown in the second panel but also to compensate for directivity and the bubble.

When the input data is filtered with the designature operator in its exact form (see equation (3-2)), which is considered as the reference result, the following is observed (right panel in figure 4-2):

1. The source ghost has been removed and only the direct arrival remains, which has a higher temporal resolution than the input data (the width of the peak is smaller than in the input data) and presents a constant waveform with take-off angle.

2. The direct arrival after filtering is delayed with respect to the direct arrival in the input data (compare the first and the third panels). This happens because the origin in time has been chosen to be computed from the sea surface according to how the far field is derived. With this time origin, the direct arrival appears to have departed from the sea surface at $-6 \text{ m}/1500 \text{ m/s} = -4 \text{ ms}$ (before zero time), whereas the ghost arrives at $6 \text{ m}/1500 \text{ m/s} = 4 \text{ ms}$. When the far field is deconvolved from the data, the result is a spike at zero time, i.e. at the sea surface, which is in between the direct arrival and the source ghost. This is always observed when the desired wavelet is a spike as $z_{ref} = 0 \text{ m}$ (sea surface), independently whether notionals or impulsive sources are used, and it is not further mentioned if a spike is taken as the desired wavelet.

3. The direct arrival does not bend with take-off angle, as the variation of the source ghost and the direct arrival with take-off angle has been compensated for.

4. There is no splitting of the direct arrival with take-off angle, as directivity effects have been taken into account. Moreover, the amplitude of the direct arrival is constant with take-off angle as directivity has been accounted for.
5. The residual bubble has been removed by considering the modelled notionals. Furthermore, amplitudes are smaller than the input data as the notionals have been removed.

6. Nevertheless, we see peaks and troughs around the direct arrival, which have smaller amplitudes than the latter. This is due to the choice of the desired wavelet, i.e. the nominator of equation (3-2). Here, 1 has been used. Ideally, this represents a spike in the time domain but, given that the bandwidth is limited, the resulting wavelet is a zero phase sinc function. Thus, these ripples correspond to the side-lobes of a sinc and the wider the bandwidth the narrower the main peak of the sinc and the smaller the ripples [Lee, 2006, p. 86]. If desired, a minimum phase sinc function or a Butterworth filter with a cut frequency equal to the largest frequency allowed in the bandwidth can be used to obtain a minimum phase wavelet as a desired wavelet.

Despite the many advantages of using this filter, the latter is inapplicable on real data due to the lack of take-off angle information. As mentioned in the previous chapter, the small take-off angle approach with spatial derivatives represents a practical solution. First, figure 4-3 shows the input data filtered with the different terms of the the small take-off angle series expansion presented in equation (3-4), provided the source take-off angle is known.

![Figure 4-3: From left to right: input data filtered with the zero order term in equation (3-4), up to the first order and up to the second order in take-off angle in this equation.](image)

The following observations are made:
1. In all the plots, the source ghost has been removed similarly to the right panel in figure 4-2. Resolution is again improved with respect to the input data (left panel in figure 4-2).

2. In a certain range of take-off angles that varies in each case, the distance between the direct arrival and the residual ghost is constant and the direct arrival does not bend, given that source ghost variations with take-off angle have been partially compensated for. This happens for angles between $-25^\circ$ and $20^\circ$ when only $A$ is considered, it then extends from $-27^\circ$ to $25^\circ$ including $B'$ and from $-56^\circ$ to $27^\circ$ considering $C'$ as well. The non-symmetry is again due to the clusters.

3. Directivity effects are compensated for in a range of take-off angles different in each case: from $-25^\circ$ to $15^\circ$ in the first panel, from $-27^\circ$ to $20^\circ$ in the second panel and from $-56^\circ$ to $25^\circ$ in the third, approximately. Within this range, the amplitude of the traces resembles the amplitude of the trace at zero take-off angle. Beyond these angles, the amplitude of the direct arrival fades away and it splits in the different arrivals, specially for positive angles due to the non-symmetry introduced by the clusters.

4. In all the three situations, the residual bubble is removed and the amplitudes are smaller than the input data because the notionals have been removed.

5. Before and after the direct arrival, ripples are observed due to the target wavelet being a sinc and not a spike.

6. When including $C'$, the high frequencies of the direct arrival and the ripples of the sinc are amplified for large take-off angles. This is an artefact due to $C'$, as it can be appreciated in figure 3-5 (second column, fourth row).

Hence, the small take-off angle approach removes the source ghost from the data, the residual bubble and it compensates for directivity effects and source ghost variation with take-off angle up to a certain extent. This is expected to widen if more terms of the small take-off angle approximation are included.

Next, the far field signature at zero take-off angle is used as the target wavelet, as described by equation (4-3). In figure 4-4, the input data is compared with the correction to first and second order suggested by the small take-off angle approach with take-off angle.

Here, only directivity effects and take-off angle variations have been compensated for. Thus, the residual bubble and the source ghost remain in the filtered data.

Two main observations are made here: the first one is that the direct arrival after the corrections is flattened (correcting the bending due to the angle dependency of the direct arrival) and, secondly, the amplitude of the direct arrivals resembles the one at zero take-off angle for a wider range of take-off angles, which increases as more terms in the series expansion are incorporated. Although not easily visible for positive angles, this results in a higher temporal resolution (for instance, see the sharpness of the trace at $-27^\circ$ in the right-hand-side panel compared to the other two). Note the improved consistency of the bubble in the third panel.

Additionally, there is no time shift with the direct arrival of the input data because the desired wavelet is again the far field signature at zero take-off angle, in which the direct
Figure 4-4: From left to right: input data and small take-off angle filter suggested in equation (4-3) applied to the input data to first and second order in take-off angle, respectively. Here, the target wavelet is the far field signature at zero take-off angle.

arrival happens at $-4 \text{ ms}$, as in the input data. Thus, independently of the source signatures used, no time shift is observed when using the far field signature at zero take-off angle as a target wavelet.

This approach will be later on analysed in more detail.

Now, the input data is filtered with the small take-off angle filter using spatial derivatives (see equation (3-5)), which can be practically implemented on real data. A nine point stencil finite differences scheme is used to compute the derivatives before undoing the normal moveout from the input data. Figure 4-5 shows the results of applying this filter to the input data considering three different receiver spacings: 2.5 $m$, 5 $m$ and 10 $m$ from left to right in this figure. Note that data is not spatially aliased for the two first spacings (in which the aliasing frequencies are $c/2\Delta x = 1500 \text{ m/s}/(2 \cdot 2.5m) = 300 \text{ Hz}$ and $c/2\Delta x = 1500 \text{ m/s}/(2 \cdot 2.5m) = 150 \text{ Hz}$, the latter equal to the maximum allowed frequency as the synthetic data has been filtered with a Butterworth with this cut frequency) but it is aliased when the spacing is 10 $m$, as the aliasing frequency is then $75 \text{ Hz}$.

When compared to the third panel in figure 4-3, little differences are appreciated for receiver spacings of 2.5 $m$ and 5 $m$. What is more, only a few small differences can be appreciated amongst the first two panels in figure 4-5, meaning that the inaccuracy due to the finite
differences scheme is small for these spacings. But larger inaccuracies are introduced when the receiver spacing is 10 m (third panel in figure 4-5), observed as smaller amplitudes and less resolved wavelets.

On real data, the small take-off angle approach will be tested in CSGs with a receiver spacing of 3.125 m and, thus, the inaccuracy introduced by the spatial derivatives should be similar to the first panel in figure 4-5.

Finally, a hybrid filter is suggested to compensate for the fact that amplitudes at high frequencies and at large take-off angles are over amplified when \( C \) and \( C' \) are included - which is a limitation of the Taylor series -, as discussed in figure 4-3 and as shown by the fourth plot in the second column of figure 3-5.

The hybrid filter is achieved by using the small take-off angle filter as in equations (3-4) and (3-5) up to 90 Hz and only coefficients \( A \) and \( B' \) or \( B \) from this frequency onwards, respectively, which do not boost too much these amplitudes (see third plot of the second column in figure 3-5). Figure 4-6 compares the filtered data with the small take-off angle filter as a function of \( \theta_s \), shown in the third panel of figure 4-3, with the suggested hybrid filters.

The high frequency amplitudes for large take-off angles (\( \sim \pm 35^\circ \)) are not amplified so much by using the hybrid filters and, still, an improvement is seen when compared to only using the zero order term in the series expansion, i.e. \( A \) (see first panel in figure 4-3).
4-2 Filtered data: modelled notionals

Figure 4-6: From left to right: input data filtered with the small take-off angle approach as in equation (3-4) and with hybrid small take-off angle filters of those presented in equations (3-4) and (3-5).

4-2-1 Frequency-space spectra

In order to evaluate the benefits and drawbacks of using the small take-off approach with the modelled notionals, amplitude spectra are obtained from the previous plots in the frequency-space (FX) domain, as shown by figures 4-7 and 4-8. As the source take-off angle associated with each trace is known in synthetic data, the latter are not only represented in terms of frequency and offset (or trace number), but also in terms of frequency and source take-off angle. The resulting plots are comparable to those in figure 3-5.

The following is observed from these plots:

1. The input data is similar to the first plot in the second row of figure 2-1, corresponding to the modelled far field signature for a Delta 3 source array. Hence, the different contributions to the far field as discussed in chapter 2 are observed (i.e. the residual bubble at low frequencies, the antenna notches and no notches due to source ghosts for frequencies smaller tan $150 \, \text{Hz}$ except at $0 \, \text{Hz}$).

2. The amplitude decreases at $f_{\text{cut}} = 150 \, \text{Hz}$ due to the Butterworth filter applied to the data. This is sharper when the small take-off angle filters are applied to the data as the stabilisation of coefficient $A$ for frequencies $\leq 4 \, \text{Hz}$ and $\geq 150 \, \text{Hz}$ (see table 4-1) is similar to an additional band-pass filter within these frequencies.

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3. When $F$ in its exact form is applied (first plot in the second row of figure 4-7), the amplitude spectra is flat as all the effects that modify the spectra as discussed in chapter 2 have been removed.

4. The remaining plots in the second row of this figure show the improvement of using each term of the small take-off angle filters with respect to $A$ alone. When only $A$ is used, the data is deghosted and debubbled within the region indicated with red-orange
colour, which is not symmetric due to the impact of the clusters and the gun volume distribution in the array. The next figure shows the improvement by including $B'$. The spectra is flattened for a larger range of take-off angles, which is almost symmetric around $\theta_s = 0^\circ$. This area is extended when $C'$ is included, but high-frequencies with large take-off angles are over amplified. These pictures are comparable to the extent of the green area in the second column of figure 3-5.

5. The third row in figure 4-7 shows the correction with take-off angle with respect to the input data shown in figure 4-4. The first order and the second order corrections cause the amplitudes to be more symmetric around $\theta_s = 0^\circ$ and they correct for the aperture notches up to a certain extent. Thus, the third picture in this row shows that the amplitudes for traces between $\theta_s \sim \pm 56^\circ$ resemble to the trace at zero take-off angle, and that antenna notches have been attenuated within this angles. Again, high frequencies at large take-off angles are boosted too much.

6. The inaccuracy due to the spatial derivatives in the small take-off angle approach can be analysed in the first row of figure 4-8 by comparing the last three plots to the first one in this row. Although the dark orange area is a bit reduced due to the derivatives for spacings of 2.5 m and 5 m, the shape is very similar to the result of applying the filter by knowing $\theta_s$ (first plot in this row). Thus, a smaller improvement in amplitude...
with take-off angle is due to the inaccuracy of the spatial derivatives, but this is not very important when using a nine point stencil and a receiver spacing between 2.5 m and 5 m. When this is 10 m (last plot in this row), the differences are larger from 75 Hz onwards due to the coarser sampling and also because the data is aliased.

7. By using the hybrid filters (last row in figure 4-8), amplitudes with high frequencies at large take-off angles are not amplified too much while keeping the benefits of using the full small take-off angle approach for frequencies \( \leq 90 \text{ Hz} \).

These results are further discussed in the last section of this chapter.

4-3 Filtered data: impulsive sources

The same input data is again filtered with the small take-off angle filters but now considering impulsive sources of equal strength in a Delta 3 source configuration. Later in this section, the corresponding FX plots are shown.

Throughout this section, the different filters are multiplied with a factor \( N = 24 \) in order to later on compare it with the result obtained by applying the low-frequency filter suggested by Amundsen and Zhou [2013, equation 10], which considers a single source. Furthermore, the time delay due to the synchronisation of the sources must be now included in the filters (recall this was included in the notionals in the previous section).

The first test is to filter the generated synthetic data with the designature operator as in equation (3-2), but this time the source signatures correspond to impulsive sources, i.e. \( S_n(\omega) = 1 \mu \text{Pa} \cdot \text{m} \) for each gun. Figure 4-9 compares the input data and the filtered data with the modelled notionals with the input filtered considering impulsive guns.

As expected, by using impulsive sources the bubble is not removed and this also affects the temporal resolution of the direct arrival, which is also delayed in comparison to the direct arrival in the input data because the travel times have been computed from the sea surface. In practice, one could apply a debubble filter on beforehand.

The ripples due to the sinc function are also visible around the direct arrival. Yet, the source ghost has been removed.

As before, this filter has no practical use as it requires take-off angle information and the small take-off angle filters are used. The input data is then filtered with the small take-off angle filter in equation (3-4) and the results are shown in figure 4-10. This is also impractical for the same reasons but it has some potential when using the spatial derivatives as in equation (3-5).

The output is comparable to the one considering the notionals, figure 4-3, except for the impact of the bubble that has not been removed. It is worth comparing the first panel in this figure with the input down-going field in the second panel of figure 4-2. Note the result is similar except that now the direct wave does not bend downwards and it is more resolved for large take-off angles. This is because the impact of the ghosts for the 24 sources has been removed as if all the traces corresponded to \( \theta^* = 0^\circ \) and the asymmetry due to the clusters has been partially compensated for.
Similarly as for the notionals, the input data is deghosted and the temporal resolution of the direct arrival is improved by including the first order correction and the second order correction in take-off angle as directivity and angle dependencies have been removed up to a certain extent. Yet, amplitudes with high frequencies at large take-off angles are boosted by including $C'$, which is observed in the third panel in this figure for $|\theta_s| \geq 40^\circ$. Once again, a possible solution is to use an hybrid approach and combine the full small take-off angle approach with only the correction to first order.

The inaccuracy introduced by the derivatives when considering impulsive monopoles is comparable to the one for modelled notionals and, hence, a receiver spacing between 5 m and 2.5 m leads to a result which is close to the third panel in figure 4-10. Thus, these results are not shown here.

Next, the far field signature at zero take-off angle is again chosen as a target wavelet according to equation (4-3), where the source signatures are now ones in the frequency domain. The resulting filtered synthetic data is presented in figure 4-11.

In terms of units and order of magnitude, this is equivalent to applying the same filter with notionals (see figure 4-4) because the source signatures of the different sources are both in the nominator and denominator in the corrections of the filter in equation (4-3). Once again, directivity and other variations with take-off angle are compensated for by including the first order and then the second order corrections. The amplitudes of the

![Figure 4-9](image)

Figure 4-9: From left to right: input data and input data filtered with $F$ as in equation (3-2) considering the modelled notionals and considering impulsive sources, respectively.
66 Small take-off angle approach: tests on synthetic data

Figure 4-10: From left to right: input data filtered with the zero order term in equation (3-4), up to the first order and up to the second order in take-off angle in this equation considering impulsive sources.

direct arrival are better aligned after the corrections and the waveforms resemble the one at $\theta^o = 0^o$ within a range of take of angles between $\sim -30^o$ and $\sim 25^o$ when only the first correction is considered and from $\sim -50^o$ to $\sim 30^o$ when including the second order correction.

Finally, the low frequency filter presented in Amundsen and Zhou [2013, equation 10], which only considers a single impulsive source, is applied to the input data and compared with the result after applying the zero order term of the small take-off angle approach considering impulsive sources, where the different depths of the sources in a Delta 3 source array have been used. For the former, a single source depth of 7 m - average of the source depths in a Delta 3 source configuration - is used. The result is shown in figure 4-12.

Several observations are made from this figure:

1. Neither of the two filters removes the bubble as both filters use impulsive sources, but the modelled notionals can be incorporated in the small take-off angle approach (see, for instance, left panel in figure 4-3) or, alternatively, a debubble filter can be applied on beforehand.

2. The source ghost has been attenuated in both approaches, but $F \approx A$ considers the source ghosts due to all the different depths of the sources and it is expected to work
4-3 Filtered data: impulsive sources

Figure 4-11: From left to right: input data and small take-off angle filter suggested in equation (4-3) applied to the input data to first and second order in take-off angle, respectively, for impulsive sources of equal strength. Here, the target wavelet is the far field signature at zero take-off angle.

better, specially in terms of phase (see figure 3-6). Yet, this is difficult to appreciate due to the low frequencies introduced by the residual bubble.

3. Both filters suffer from directivity effects (note the separation of the direct arrival in its multiple arrivals) and variations with take-off angle of the source ghost (downward trend of the direct arrival with $\theta^s$). Yet, this can be corrected by incorporating the first and second order terms of the small take-off angle approximation.

4. The width of the direct arrival - and hence the temporal resolution - is sharper by using $A$ for impulsive sources given that all frequencies and depths of the sources in a Delta 3 source configuration have been considered.

5. A time shift exists between the direct arrivals in the second and third panel. This is due to the fact that travel times are computed from the sea surface in the former but from the chosen average depth of 7 m in the latter. This can be changed in the small take-off angle approach by finding the appropriate $z_{ref}$, as discussed in chapter 2.

Thus, even the zero order term of the small take-off angle approach seems more appropriate than the low frequency (and zero take-off angle) filter presented by Amundsen and Zhou

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Small take-off angle approach: tests on synthetic data

Figure 4-12: From left to right: input data, input data filtered with the zero order term of the small take-off angle filters with impulsive sources in a Delta 3 source configurations and input data filtered with the low frequency filter of equation 10 in Amundsen and Zhou [2013], where \( z = 7 \) m is used.

[2013] when dealing with data acquired (or generated, if synthetic data) with a marine source array rather than a single source. Even for one source, the former should give better results as it includes all frequencies and, hence, the filter presented by Amundsen and Zhou [2013] is not tested on real data.

4-3-1 Frequency-space spectra

Once again, FX spectra are used to further study the implications of using the previous filters, in which impulsive sources were considered. These are presented in figures 4-13 and 4-14 and similar conclusions to those in section 4-2-1 are made.

The next observations are now made:

1. Taking into account the different scale between the second and third plots in the first row - which is due to the fact that \( F \) with the modelled notionals removes the notionals but \( F \) for impulsive sources does not - it is observed that antenna notches are removed in both situations and the spectra is flattened. Yet, in the latter the residual bubble is not removed (note the ripples between \( 5 - 10 \) Hz).
2. Again, the second row compares the different terms of the small take-off angle approach. The more terms are included, the more directivity is handled and the flatter the spectra is. Nevertheless, high frequency amplitudes at large take-off angles are over amplified by including $C'$ ($f > 90 \text{ Hz}$ and $|\theta^o| \geq 45^o$).

3. When using the zero take-off angle far field signature as a target wavelet (third row), amplitude is improved in terms of symmetry by including $B'/A$, and even more when including $C'/A$. Directivity notches are also handled to some extent with the latter and, thus, the bandwidth is improved.

4. The improvement in the bandwidth by using $F \approx A$ with impulsive sources rather than the low frequency filter suggested by Amundsen and Zhou [2013] can be seen in the fourth row of this figure. This is clearly visible for frequencies $\geq 80 \text{ Hz}$.

These results and those in the previous sections of this chapter will now be further analysed.
4-4 Comparison and discussion of the results

The small take-off angle approach has been tested on synthetic data - generated with an array of sources in a Delta 3 source configuration - by first including the modelled notionals and then considering impulsive sources of equal strength. By using impulsive sources, the residual bubble remains in the data and an additional filter to debubble the data is required.

Moreover, two different target wavelets have been tested: a spike in the time domain as originally derived in equations (3-4) and (3-5), which in practice means a sinc function due to the limited bandwidth, and the far field signature at zero take-off angle.

The former is able to deghost the data and remove directivity effects up to a wide range of take-off angles if all the coefficients in the series expansion are included. When the notionals are included, the residual bubble is also removed, leading to a higher resolution. Nevertheless, care needs to be taken into account when considering $C'$ or $C$ as amplitudes with high frequencies at large take-off angles are boosted too much, and noise at these frequencies on real data could be over amplified. An hybrid solution is suggested to overcome this problem.

This method has a high potential as it can be applied on real data acquired with any marine source array. Then, the approach with the spatial derivatives needs to be used.
(see equation (3-5)) as angular information is not directly available. Although it should be
performed in the common receiver domain, where the shot spacing is coarse and interpolation
between traces is required, the latter can be applied in CSGs under the 1D earth assumption.
Figure 4-5 has shown that a receiver spacing between 2.5 m and 5 m is enough to get accurate
results with a nine point finite differences stencil.

But a very interesting result is obtained when using the far field signature at zero take-off angle
as a target wavelet (see figures 4-4 and 4-11). Although the input data is not deghosted or
debubbled by doing this, the directivity effects and variations with take-off angle are certainly
reduced such that the waveforms remain fairly constant across many traces around $\theta = 0^\circ$.

To further illustrate this point, the root-mean-square (RMS) amplitude between 160 ms and
260 ms of the different panels in figures 4-4 and 4-11 and their analogous considering the
spatial derivatives (i.e. by multiplying equation (3-5) with $1/A$) are computed and presented
in figure 4-15. This window includes the direct arrival, the source ghost and part of the
residual bubble.

When including the notionals (left column in figure 4-15), the asymmetry in amplitude around
$\theta = 0^\circ$ of the input data (blue curve) is made symmetric by including the first correction in
take-off angle (green line), as the impact of the clusters and the volumes of the guns across
the array are taken into account by this correction, which also includes the inline positions
of the sources (see equation (4-3)). Once the amplitudes are symmetric, the second order
term (red line) compensates for directivity effects, such that the amplitude stays the same
between $\pm 35^\circ$ and slightly decreases up to $\pm 50^\circ$. From here onwards, the small take-off angle
approximation breaks and the amplitudes change. If additional terms of this series expansion
were included, the constant amplitudes would expand further but it would probably be over
complicated and impractical. The hybrid approach (cyan) is an intermediate solution that
does not suffer so much from the artefacts generated by $C$ or $C'$ at large take-off angles and
high frequencies.

On the other hand, when the same is performed for impulsive sources of equal strength (right
column in figure 4-15), the directivity effects are also compensated for but the result is not as
symmetric as previously obtained with notionals (compare the red lines in these two graphs).
The main reason for this is that 24 impulsive sources of equal strength have been considered,
$S_n(\omega) = 1 \mu Pa \cdot m$, and this does not hold for a Delta 3 source array, in which the volumes
of each gun have been adjusted to obtain a large PB ratio.

Thus, a suggestion is to still use impulsive sources (i.e. with a constant strength with fre-
quency) but with different strengths adjusted to the cubic root of each gun’s volume, according
to Dragoset [1990, table 1].

Overall, note that the corrections with take-off angle are small for this target wavelet (for
instance, compare the amplitude difference between the red and green curves in the first plot
in figure 4-15 between $\theta = 0^\circ$ and $\theta = 30^\circ$), as it only represents a correction in terms
of directivity effects. Thus, small improvements are expected on real data when using this
target wavelet.

Finally, when using spatial derivatives (second row in figure 4-15) the amplitudes follow
the same trend as when using the known take-off angles (compare it with the top row in
this figure) and differences are more visible at large take-off angles, specially for $C$ and $C'$
Figure 4-15: RMS amplitude of the input data (blue), the first order corrections with take-off angle (green), the second order correction (red) and the hybrid result (cyan) considering the modelled notionals (left column) and impulsive sources (right column) computed from figures 4-4 and 4-11 (top row) and their analogous with spatial derivatives (bottom row) for 2.5 m receiver spacing. The window used to compute the RMS amplitude goes from 160 ms to 260 ms.

coefficients due to the artifacts generated by them at those angles and high frequencies. Again, the hybrid filters attenuate these effects.

Hence, the small take-off angle approach has potential to deghost the data but also to deal with directivity effects and remove the residual bubble for any source array configuration. Depending on the chosen target wavelet, only some of the previous effects - such as directivity - are compensated for.

This approach is now tested on a real data set and different practical aspects are assessed.
Part III

Real data
Chapter 5

Bruce field data and practical aspects

The potential of the small take-off angle filters in removing the source ghost, the bubble and directivity effects has so far been theoretically demonstrated and proved on synthetic data in the previous chapters. Now, this approach is tested on real data and its practical implementation is discussed.

The seismic dataset used here was acquired over the Bruce field, a condensate gas field located 340 km north east of Aberdeen\(^1\) in the UK sector of the Northern North Sea, as illustrated in figure 5-1.

\[\text{Figure 5-1: Location of the Bruce field, mainly operated by BP. Source: slides from [Cunnell et al., 2014].}\]

\(^{1}\)Blocks 9/8a, 9/9a, 9/9b (Licence Nos P.209, P.090, P.276)
This field is situated in a historically prolific area of the North Sea, the Brent Group, containing the most significant Middle Jurassic hydrocarbon fields in the North Sea. Close to the Shetland and Beryl basins, water depths are between 100 m and 120 m. In fact, condensate - a light crude that exists as gas under reservoir conditions - was first discovered in 1974 at 4000 m depth below the seabed in a water depth of 121 m.

The Bruce field is geologically complex, as depicted and described by Johnson et al. [2005], both throughout the overburden as well as at the reservoir level, and it is formed by three reservoirs: the Turonian limestone (gas condensate), the Bruce sandstone (oil and gas condensate) and the Statfjord sandstone (oil and gas condensate). These conditions create different geophysical imaging challenges. For instance, the hard and flat seabed interface generates short-period multiple contamination throughout the section, including at the reservoir level.

For further information, Cunnell et al. [2014] and Johnson et al. [2005] discuss in detail the geological structure of this field, whereas more technical aspects are found in BP [2014] and TOTAL [2013].

The data used here was acquired along a sailing line with a 3D multi-sensor streamer towed at ~ 18 m depth (thus, the receiver ghost at vertical incidence is at 2 · 18 m/1500 m/s = 24 ms two-way traveltime). The pressure, the vertical and the cross-line components of particle acceleration were recorded but, in this report, only the data from an inner cable is used (azimuthal dependency should be accounted for when including more cables not aligned with the boat). The Delta 3 source array was used while the sea was calm during the acquisition (swell height of 1 m).

Moreover, basic pre-processing has been applied on the data (navigation merge, coherent noise attenuation and standard on board processing sequence).

The small take-off angle filter - discussed in chapter 3 and applied on synthetic data in chapter 4 - is now applied on this dataset.

First, different practical aspects are discussed in the following section and the flow used to apply the small take-off angle filter is presented. The results are then shown and discussed in chapter 6.

5-1 Practical aspects of the source deghosting/designature workflow

In order to apply the small take-off angle method described in chapter 3 on the real dataset from the Bruce field, the input pressure data is processed with the flow depicted in figure 5-2.

The (basic) pre-processed pressure data is first receiver deghosted by combination of multi-component streamer pressure and vertical particle acceleration, as described by Caprioli et al. [2012].

Next, low frequency dipping noise is attenuated by building up a noise model with a controlled range of dips and frequencies and then subtracting it from the input data in a controlled manner. In this case, the low frequency noise attenuation (LFNA) is performed for frequencies
Figure 5-2: Suggested processing flow to deghost and designature the data.

≤ 8 Hz and only applied for t ≥ 1 s. The range of dips attenuated are between 1200 m/s and 2800 m/s.

This step is optional but it is applied here to reduce the low frequency noise before source deghosting, as receiver deghosting not only amplifies low frequency signal but also low frequency noise. Indeed, some of this noise already exists in the input pressure and vertical acceleration data.

The data is then source deghosted by using the small take-off angle source deghosting and designature approach (STASD) described in chapter 3. Again, to mitigate the low frequency noise amplified by the source deghosting approach, LFNA is applied once more with the same parameters as before (optional).

Additionally, the data after each step is stacked without further processing - only normal moveout and top mute are applied and nominal geometry is used. I refer to this as a brute stack.

The STASD job in the previous flow is depicted in figure 5-3. The latter is the processing sequence suggested to remove the source ghost, designature the data and compensate for directivity effects for any source array as discussed in chapter 3, which also accounts for shot to shot variations.

The different steps depicted in this workflow are the following:

1. The first trace (i = 1) in the first CRG (j = 1) is selected as the input data, $M_{ji} = M_{11}$.
   Each trace in a CRG corresponds to one shot and, thus, to different notionals and different small take-off angle coefficients, $A_i$, $B_i$ and $C_i$.

2. Then, the small take-off angle filter in equation (3-5) is applied on this trace. Simultaneously, $A_1$, $B_1 \partial_x$ and $C_1 \partial^2_x$ are convolved (in time) with the input data, $M_{11}$. The spatial derivatives are computed with an n-point finite difference stencil (7-point stencil is used here, i.e. 15 points in total: the current trace and 7 points before and 7 points after the current trace).

3. After separately applying the zero, first and second order terms of the small take-off angle filter to the data, the outcome is convolved in the time domain with the desired target wavelet. This is done at this stage to compare the different terms of the approximation, but it could be done once at the end of the flow.

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4. The output by using $A_1$ alone is saved together with the contributions of $B_1$ and $C_1$, respectively, which are summed to the first one.

5. Additionally, the hybrid filter is achieved by filtering the outcome of $C_1$ with a high-cut filter in which the high-cut frequency is determined by computing the error plots, as discussed later in this section and shown in appendix C. The outcome is then saved.

6. At this point, the next trace in the first CRG is selected and the process is repeated by updating the input data to $M_{12}$ and the coefficients to next shot, i.e. $A_2$, $B_2$ and $C_2$. This is repeated until the last trace in the first CRG, $M_{1N_{shots}}$. Then, if there are further CRGs to process the flow starts again with the next CRG. This process is repeated until the last trace of the last CRG ($i = N_{shots}$, $j = N_{CRG}$). But coefficients $A_i$, $B_i$ and $C_i$ only need to be computed once for each trace in the first CRG, as the same trace across different CRGs corresponds to the same shot and, therefore, the notionals and the small take-off angle coefficients are the same.

This workflow is general for any marine source array, where coefficients $A_i$, $B_i$ and $C_i$...
need to be conveniently adapted to the layout (source positions) of each array according to equation (3-6). Moreover, it accounts for removing shot to shot variations and corrects for directivity effects, removing the source ghost and reducing the residual bubble.

Two additional simplifications are made here as a first test. The first one is to consider a 1D horizontally layered earth, in which data sorted in CSGs is equivalent to data sorted in CRGs (Snell’s law, which is a reasonable assumption for this dataset according to the picked velocities, not shown here). Thus, CSGs are processed, in which interpolation is not required for the spatial derivatives.

Secondly, the notionals for each gun in a Delta 3 configuration across all shots are averaged. Hence, coefficients $A_i$, $B_i$ and $C_i$ defined by equation (3-6), which vary from shot to shot as the notionals of each source slightly change, are the same for all the shots.

To further justify this approximation, the measured notionals for each gun of the Delta 3 source array across all the shots are plotted in the middle panel of figure 5-4 (the colour lines indicate the source positions, as denoted in the caption) together with the stacked notionals (right panel) and the modelled notionals used in the previous section for this array (left panel).

It is worth mentioning that the measured far field notionals have been first recorded with near field hydrophones (NFH) and then obtained in the far field in a similar manner as described by Ziolkowski et al. [1982].

First, note that the modelled notionals resemble the measured notionals. Yet, the clusters are counted as two different sources in the former and as one in the latter, which explains the different amplitudes of the first trace for each sub-array between the modelled notionals and the stacked notionals. The polarity of the modelled notionals has been reversed to account for the fact that the measured notionals start with a negative peak (a negative impulse response).

Second, some variations are observed within the different shots (middle panel in figure 5-4) and, thus, using average notionals seems reasonable for a first test as it simplifies the processing flow. These variations can be included by updating $A_i$, $B_i$ and $C_i$ for each shot, i.e. for each CSG or, alternatively, by using the calibrated marine source (CMS) process.

Regarding the choice of the target wavelet in figure 5-3, two different tests are performed. The first one is to consider that the target wavelet is a negative spike in the time domain, corresponding to an impulsive gun with negative strength. The negative sign is to compare with the input data, as the notionals start with a negative peak.

Moreover, the latter is multiplied by 24 to achieve amplitudes similar to the input data, as the small take-off angle as defined in equation (3-5) removes the notionals due to the $N$ sources of the array. Thus, $T_w = -24$ is used to test the small take-off angle approach. With this target wavelet, the source ghost, the residual bubble and directivity effects are attenuated. In this case, LFNA is applied before and after source deghosting, as both this filter and the receiver deghosting flow (see figure 5-2) boost low frequency noise.

Secondly, in order to compare the small take-off angle approach with the outcome after applying the CMS sequence, the desired (and edited) target wavelet obtained from the latter is used as a target wavelet in the flow in figure 5-3, i.e. $T_w = T_{CMS}$. 

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Figure 5-4: From left to right: modelled notionals for the Delta 3 source array used in previous chapters (multiplied by \(-1\)), notionals for \(N = 18\) sources for all the shots in the studied seismic line in the Bruce field and the stacked far field notionals across all shots. The coordinate positions of the sources for each shot according to the system of coordinates in figure 1-2 are indicated in green (inline), orange (cross-line) and black (depth) colours in the middle panel which, for display reasons, have been scaled by a factor 10 and their zero is set at 0.7 s (red arrow indicates positive values).

By doing this, the zero order term with averaged notionals, \(T_{CMSA} = T_{CMS} / \langle W_i(k, \theta_s = 0^\circ) \rangle\), is similar to the matching filter of CMS, \(MF = T_{CMS} / W_i(k, \theta_s = 0^\circ)\), with the difference that the matching filter from CMS takes into account the shot to shot variations of the notionals in the far field signal at zero take-off angle (again, this can be included in the small take-off angle approach by updating the coefficients for each shot gather).

Here, LFNA is only applied once after source deghosting in order to compare with CMS, as low frequency noise has not been attenuated before processing for CMS, only after.

Thus, a small correction due to directivity is expected when including \(B\) and \(C\) with this target wavelet, similar as obtained on synthetic data when \(T_w = 1/A\) was considered (see figure 4-15). The difference now is that \(T_w = T_{CMS}\) is the average of the far field signature at zero take-off angle, zero phased and where the residual bubble has been smoothed out from the amplitude spectra, whereas \(T_w = 1/A\) was the far field signature at zero take-off angle (for the stacked notionals, in this case) still containing the bubble.

Hence, the aim of using this target wavelet is to estimate whether there is an improvement by including the small take-off angle corrections, which is expected to correct for directivity effects with this choice but not deghost the data, as \(T_{CMS}\) includes the (average) source ghost at \(\theta_s = 0^\circ\). A small correction is expected in this case (for instance, note the small corrections
seen in the synthetic RMS plots in figure 4-15 when the far field signature at vertical angle was used).

Figure 5-5 shows the target wavelet from CMS (left) and its amplitude spectrum (right). Additionally, the amplitude spectrum before debubble is overlayed.

![Figure 5-5: CMS target wavelet for the Bruce line studied here (left), its amplitude spectrum (right, red line) and the corresponding spectrum before debubble (right, blue line).](image)

To compute the small take-off angle coefficients, the acoustic center of the array is again investigated following the same procedure as described in section 2-2-5, in which stacked notionals have been used. The phase difference is shown in appendix C (figure C-1), which leads to the conclusion that $z_{ref} = 6 \text{ m}$.

The coordinate frames for sources and receivers should be consistent, and the spatial derivatives computed accordingly. Here, the system of coordinates in figure 1-2 has been used. According to this figure (streamers are at $\theta^s > 0$), error plots are again computed for the exact far field in equation (3-4) with the average notionals for zero source azimuth and also for the different terms of the small take-off angle filter - as in section 3-2-3 -. The results are shown in appendix C (figure C-2). Additionally, error plots are computed considering that the desired wavelet is the one from CMS (see figure C-3 in appendix C, in which it is observed the improvement with respect to CMS by using the small take-off angle filter with this target wavelet).

In these plots the theoretical improvement in terms of amplitude and phase obtained for this sailing line by adding the first and second terms of the small take-off angle approximation is observed, considering the stacked notionals.

By inspection of the second and third plots in the fourth column in figures C-2 and C-3, a high-cut filter with frequency equal to 90 Hz is applied on $C$ to limit the boosting of amplitudes above these frequencies. This is the so-called hybrid filter.

The remaining parameters from the real data (receiver spacing, time sampling, etc.) and those used to generate coefficients $A$, $B$ and $C$ with the stacked notionals are summarised in tables C-1 and C-2 of appendix C. It is important to mention that receiver spacing is 3.125 m and the shot spacing is 25 m. Thus, inaccuracies due to the spatial derivatives are much smaller in CSGs and interpolation is not required as discussed in the previous chapter,
where receiver spacings between 2.5 m and 5 m resulted in similar results as those obtained knowing the angles.

With these parameters, the amplitude spectra of the coefficients and their wavelet are shown in figure 5-6. A cosine window filter has been used to stabilise the coefficients and they have been then tapered in time to a finite number of samples (see table C-2).

![Figure 5-6](image)

**Figure 5-6:** Top: Amplitude spectra of coefficients A, B and C obtained with the stacked notionals (brown), their amplitude spectra after filtering the coefficients to make them stable (green) and after filtering and tapering (black, blue and red for A, B and C, respectively). Bottom: filtered and tapered coefficients in the time domain (time sampling is 2 ms).

In order to avoid artefacts due to the length of the coefficients (such as wrapping around or mirror effects), the number of samples for each coefficient is chosen as short as possible, trying to include the main variations of the original wavelet.

For coefficients A and B, the length of the filter is of 501 samples, with the zero time at sample 251, whereas more samples are required to include the full behaviour of C (1001 samples with zero time at sample 501, which does not produce any visible artefact). Note the relative global amplification between A, B and C and the different behaviour of C, also observed in chapter 4.

These tapered wavelets are then convolved with the input data.

The following chapter presents the results obtained after applying the small take-off angle filter with stacked notionals, as described in this chapter, to the real data from the Bruce field, both with a negative impulsive target wavelet and the edited target wavelet from CMS. Then, the benefits and drawbacks of this approach are discussed and compared with the current CMS approach.
Chapter 6

Results and discussion

In this chapter, the results obtained by applying the source deghosting and designature small take-off angle filter - as discussed in the previous chapter - to the receiver deghosted data (which is the input data being filtered) are shown and compared with the pressure data (not receiver deghosted) and the input data after CMS.

First, pre-stack data results are shown for single shots in the next section and all the shots are then (brute) stacked and shown in the second section. The outcome is further discussed in the last section of this chapter.

For display purposes, a $t^2$ gain function is applied to all the results [Claerbout, 1985, p. 233-234]. Low frequency noise attenuation (LFNA) is applied on the input, as receiver deghosting amplifies low frequency signal and noise. Additionally, LFNA is applied again when the source deghosting and designature small take-off angle filter is applied to the input, independently of the target wavelet used, as the latter also acts on the low frequency signal and noise in a similar manner. However, LFNA is not applied when only displaying (not receiver deghosted) pressure data.

6-1 Pre-stacked data

Tests of the small take-off angle filter are first performed on pre-stack data. Unless otherwise mentioned, the results are shown for shot 1071 as this has less noisy or anomalous traces. Nevertheless, all the shots are stacked in the next section.

Figure 6-1 compares the following pre-stack data for this shot: (a) (not receiver deghosted) pressure, (b) input (pressure data receiver deghosted), (c) input filtered with $(A + B\partial_x + C\partial^2_x)T_w$ hybrid and $T_w = -24$, (d) input after CMS, (e) input data filtered with $(A + B\partial_x + \partial^2_x)T_w$ hybrid with $T_w = T_{CMS}$ and (f) difference between (e) and (d).

Very briefly, the direct arrival is first observed as a function of time (straight line starting at $\sim 50\ ms$) followed by the sea-bottom reflection (SBR) at $\sim 150\ ms$, its first multiple at
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Figure 6-1: Pre-stacked data: (a) pressure, (b) input or pressure receiver deghosted (with PZ combination), (c) input filtered with \((A + B\partial_x + C\partial_x^2)T_w\) hybrid and \(T_w = -24\), (d) input after CMS, (e) input data filtered with \((A + B\partial_x + \partial_x^2)T_w\) hybrid with \(T_w = T_{CMS}\) and (f) difference between (e) and (d) $\sim 300\ ms$ and other events. Normally, larger take-off angles and frequencies are expected at early times, which is only strictly true if the Earth is 1D. The following is observed when comparing the different results:

1. The receiver ghost has been attenuated in the input data (compare b and a) and, thus, the latter contains less events than in the pressure data (for instance, the receiver ghost of the SBR at around 170 $ms$ and the receiver ghost of the SBR ghost following it.
have been attenuated). Low frequencies are present in both due to the residual bubble (note the low frequency negative amplitude between the SBR at $\sim 150 \text{ ms}$ and its first multiple, at $\sim 300 \text{ ms}$). This is not further analysed in the following results, either pre- or after-stack, as receiver deghosting is out of the scope of this report.

2. When applying the small take-off angle filter with $T_w = -24$ (figure c), the residual bubble is again removed and events are now much sharper than in the input data and CMS applied to the input (for instance, compare the time width of the events in figures d, e and b between 500 ms and 550 ms). But after the SBR several high frequency events (of around $\sim 107 \text{ Hz}$) between $\sim 155 \text{ ms}$ and $\sim 200 \text{ ms}$ appear. Its origin will be investigated later. It is worth mentioning that the amplitudes of (d) are different with respect to the other plots as with this target wavelet the notionals are removed from the data, but the colour scale used is comparable to the rest of the plots. This will not be further mentioned in the following results.

3. CMS has removed the low frequencies due to the residual bubble while converting the wavelets into the designed target wavelet, such that they are zero-phase but in a 1D sense. Fewer and sharper events are now observed when compared to the input (compare d and b, for instance the positive event at $\sim 440 \text{ ms}$).

4. Some differences are observed when using the hybrid small take-off angle filter with $T_w = T_{CMS}$ (figure e) compared with the CMS result (figure d). Although both results are similar as the target wavelet is the same, directivity effects have been corrected for in (e). Thus, differences are not so visible in terms of resolution (but this is also improved, as it is observed in the negative event starting at $\sim 575 \text{ ms}$) but more in terms of equalisation of the amplitude with respect to offset (angle). Hence, amplitudes are slightly higher and events are more continuous in (e). Figure (f) shows the differences between (e) and (d). Note these are not an amplitude scaling but they vary with offset (take-off angle) and, hence, the small take-off angle approach amplifies the amplitudes differently with offset. For instance, the SBR (starting at 150 ms) is a negative event and the amplitudes are larger (more negative) in (e) than in (d), such that the difference is still negative in (f). On the other hand, its first multiple ($\sim 300 \text{ ms}$) is a positive event, stronger in (e) than in (d), and the difference is still positive in (f) and the amplitude varies with offset.

5. Although hardly visible, edge effects are observed for the first traces in figures (c) and (e) due to the finite length of the spatial derivatives.

Next, normalised amplitude spectra of the data shown in figure 6-1 are computed and displayed in figure 6-2 for two windows: one containing mainly signal and some noise and the second one containing only noise, as specified in the caption of figure 6-2. The amplitude behaviour for each dataset is similar in both windows, as the same filters are applied to both signal and noise.

The small take-off angle filter with $T_w = T_{CMS}$ (black curve) shows a similar behaviour to the input processed with CMS (green curve) at low frequencies, and it is above the latter from 90 Hz onwards as it corrects for directivity effects (see figure C-2, first row). The shape of the CMS spectra is similar to that of the far field signature at zero take-off angle, but with other effects. Little variations at low frequencies are due to the fact that CMS accounts for
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Figure 6-2: Normalised amplitude spectra of the pre-stacked input and processed data for two windows: from $0.474$ s to $1.779$ s and from trace 1 to 74 (top, mainly signal and also some noise) and from $0.002$ s to $0.868$ s and from trace 518 to 894 (bottom, mainly noise). In all graphs, blue is the input or pressure deghosted data, green is the input processed with standard CMS, black is the input filtered with $(A + B\partial_x + C\partial_x^2)T_{CMS}$ hybrid and red is the input filtered with $(A + B\partial_x + \partial_x^2)(-24)$.

Shot to shot variations whereas averaged notionals have been used for the small take-off angle filter as a first test. Shot to shot variations could be included by using the workflow depicted in figure 5-3.

When using $T_w = -24$ (red curve), the amplitude shows a similar behaviour at intermediate frequencies than CMS and the small take-off angle with $T_w = T_{CMS}$, but it is above the previous two at low frequencies and also at frequencies $> 90 \text{ Hz}$.

At low frequencies a low-cut filter is applied to the target wavelet from CMS (see figure 5-5), whereas by using $T_w = -24$ low frequencies are also amplified. On the other hand, the bump in amplitude at $f > 90 \text{ Hz}$ is because the latter target wavelet tries to flatten the spectra (but it’s not compensating for other effects, such as spreading) and, thus, amplifies the amplitudes that are smaller in the spectra of the Delta 3 source array (see figure 1-1, pink curve). Due to the improvement of the useful bandwidth, the resolution is improved. Unfortunately, noise with these frequencies is also amplified.

Now, the data in figure 6-1 is filtered with a low-cut filter with a cut frequency of $f_{cut} = 105 \text{ Hz}$ in order to investigate the origin of the high frequency events that appear below the SBR when using $T_w = -24$ (see plot d in figure 6-1). The result is shown in figure 6-3, ordered.

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according to the caption in this figure.

Figure 6-3: Low-cut filter with $f_{\text{cut}} = 105\,\text{Hz}$ (i.e., pass-band from 105 Hz onwards) and a slope of $18\,\text{dB/octave}$ applied to the following pre-stacked data: (a) pressure, (b) input or pressure receiver deghosted, (c) input processed with standard CMS, (d) input filtered with $(A + B\partial_x + C\partial_x^2)T_w$ hybrid and $T_w = T_{\text{CMS}}$ and (e) input filtered with $(A + B\partial_x + \partial_x^2)T_w$ hybrid with $T_w = -24$.

By inspection of the different filter panels, it is possible that these events are already in the input data (figure b), as they happen at the same time as when applying the small take-off angle approach with $T_w = -24$ (figure e). However, these are of higher amplitude and they increase with angle in the latter as frequencies greater than 90 Hz are amplified according to the previous spectra for this target wavelet. Indeed, the frequency of these events is $\sim 107\,\text{Hz}$, which falls in the area of the spectra in which amplitudes are amplified according to the error plots in appendix C.

Nevertheless, by band-pass filtering the data - and hence reducing the bandwidth - we might...
generate ripples as well. Additionally, their origin can also be due to the limitations of the small take-off angle filter not only in terms of amplitude but also in terms of phase (see figures C-2 and C-3, third column). At early times, large take-off angles are expected in a 1D earth (Snell’s law), such that the filter falls out of its validity range and, thus, not only the amplitude but also the phase are incorrectly modelled, which can generate ripples. Thus, most likely these events are an artefact due to the limitations (specially in phase) of the approximated filter. Further research should be done to confirm this argument. Additionally, they may also be caused by the fact that the notionals have been measured in the near field and then computed in the far field, which may contain errors, or also because average notionals have been used.

Figure 6-4 compares the results of filtering the data with the small take-off angle filter and \( T_w = -24 \) using: (a) only \( A \), (b) \( A \) and \( B \) and (c) \( A \), \( B \) and \( C \). Differences between these plots are shown in plots (d) to (f), as indicated in the caption.

The main differences between these plots is that directivity effects are corrected for when including more terms in the Taylor series, observed as higher and equalised amplitudes as a function of offset when compared to the zero take-off angle result. Additionally, when using the three coefficients the outcome is loaded with low and high frequency noise (compare figure f with d for traces 1 to 101). This noise has a frequency of roughly 10 Hz and it seems to be contained in the input data (see figure 6-1, b) and amplified by \( C \), but it does not seem related to the residual bubble. LFNA does not attenuate this noise because it happens for \( t < 1 \text{s} \) and frequencies lower than 8 Hz.

Amplitude spectra of figures (a) to (c) are shown in figure 6-5 for two different windows, as in figure 6-2: from 0.474 s to 1.779 s and from trace 1 to 74 (top, mainly signal and also some noise) and from 0.002 s to 0.868 s and from trace 518 to 894 (bottom, mainly noise).

Differences are important again from 90 Hz onwards, were directivity effects are more important and are thus corrected by including \( B \) (recall that \( C \) is not included from 90 Hz onwards in the hybrid approach and, thus, the blue and the green lines in the previous graph overlay). As observed in figure 6-4, larger low frequency amplitude content is present when including all the terms, which is also observed in the spectra. It is easier to observe this in the noise window given that the latter is shorter.

Finally, the improvement in equalisation of the amplitude and continuity of the events when filtering the data with the small take-off angle approach with \( T_w = T_{CMS} \) is compared with the input processed with CMS in figure 6-6 for shot 1001. With this aim, two different normal moveouts (NMOs) are applied to the input data, the input data processed with CMS and the input filtered with the different coefficients of the small take-off angle approach and \( T_w = T_{CMS} \): 1) with constant water velocity and 2) with the primary picked velocities. Figures (a) to (c) show the result of constant velocity NMO on the input, the input data processed with CMS and the input filtered with the hybrid small take-off angle filter with \( T_w = T_{CMS} \), whereas (d) shows the result after NMO with the primary velocities only for the input data.

In order to quantify the improvement of including any of the coefficients of the small take-off angle filter, RMS amplitudes for the SBR are computed when constant water velocity is used in the window displayed in (a) and the outcome is presented in (e). The same is done for
the data processed with a NMO using the primary velocities, where the window in which the RMS amplitudes are computed is indicated in (d), which is at the target level, and the resulting RMS amplitudes are shown in (f) for all the processed datasets.

First, note the expected dephasing in (b) and (c) due to the target wavelet. Moreover, a continuity improvement is observed when using the small take-off angle approach with respect to CMS, for instance at the first SBR multiple (around \( \sim 300 \text{ ms} \)). The RMS graph in (e) shows the amplitude correction due to directivity effects when including \( B \) and \( C \) as a function of trace number and take-off angle. The latter has been estimated assuming a flat sea-floor and taking a water depth of \( z_{wd} = 120 \text{ m} \), such that \( \theta_s = \tan^{-1}(\frac{x_s}{2z_{wd}}) \), where \( x_s \) is the offset.

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Figure 6-5: Amplitude spectra (not normalised) of the pre-stack input data processed with \(A\) (red), \(A\) and \(B\) (blue) and \(A\), \(B\) and \(C\) (green) for two windows: from 0.447 s to 2.663 s and from trace 4 to 96 (left, mainly signal and also some noise) and from 0.002 s to 0.925 s and from trace 527 to 886 (right, mainly noise), as in figure 6-2.

In the region of validity of the approximated filters (i.e., below 25° for \(B\) and below 35° when including \(C\), but it is frequency dependent as observed in figure C-2), amplitude is not only amplified but also corrected, as it is not a constant amplitude shift (see the different slope of the blue and orange curves with respect to the blue curve from trace 11 onwards). This is linked to what it was observed when applying the present method to synthetic data (see figure 4-15). Note the correction is small, as expected with this target wavelet, and the difference between using the different terms is comparable to the difference in amplitude in figure 6-5.

Beyond the region of validity, the hybrid filter or the filter with only \(A\) and \(B\) present the same trend as CMS but with higher amplitudes, as observed in the synthetic example. The edge effect due to the second derivatives is also observed at the first traces for the hybrid filter. Note that amplitudes between CMS data and the input filtered with \(AT_{CMS}\) are very similar, as expected (green and black curves, respectively).

At the target level, the small take-off angle filter also corrects the amplitudes, although the effect is smaller as the angles are expected to be lower as well (here the RMS is required to estimate the source take-off angles). Whereas in some areas the amplitudes are similar (from traces 150 to 350), they differ for larger offsets (from trace 450 onwards), as the angle is expected to increase with offset. It is worth mentioning that the RMS amplitudes have not been computed from the first trace as there are a few anomalous traces in the input data (actually, in the vertical velocity data used for receiver deghosting), which are amplified by the \(C\) coefficient. Nevertheless, the behaviour of the RMS amplitude with offset is similar to that observed in the synthetic example (see figure 4-15).

6-2 Brute stacked data

A brute stack of the pre-stacked data is now performed and the results are shown in figures 6-7 and 6-8 for the top part of the stack and at the target level (the latter between 2.4 s and 2.6 s), respectively. The following stacks are shown: (a) pressure data, (b) input (pressure deghosted...
Figure 6-6: Top: NMO panels with constant water velocity of 1500 m/s for (a) the input or pressure deghosted data, (b) the input processed with standard CMS and (c) input filtered with \((A + B\partial_x + \partial_x^2)Tw\) hybrid with \(Tw = TCMS\) and NMO panel with the primary picked velocities (d) for the input. Bottom: RMS amplitudes of the input (red), the input processed with CMS (green), the input filtered with \((A + B\partial_x + \partial_x^2)Tw\) hybrid and \(Tw = TCMS\) (blue) and the input filtered with \((A + B\partial_x + \partial_x^2)Tw\) hybrid with \(Tw = -24\) (orange) for (e), a window around the SBR as indicated in (a) for the NMO panels with water velocity, and (f), a window at the target level, as indicated in (d) for the NMO panels with primary picked velocities.

Similar observations as in the pre-stacked data in figure 6-1 are made:

1. The receiver ghost is removed in (b), such that there are less events, but the residual bubble is still present (low frequencies between 200 ms and 400 ms).

2. CMS removes the residual bubble (events that were masked by the low frequencies of the residual bubble are unravelled, such at the very top or also at the target level, in figure 6-8) and turns the wavelets into zero-phase wavelets, as observed in (c).

3. Events are much sharper in (d) as the bandwidth has been improved by using \(Tw = -24\),
in which the notionals (and hence the residual bubble) have been removed, directivity effects have been corrected for and the spectrum without including other effects such as spreading has been flattened. However, high frequency noise has been amplified. This is not the case in figure 6-8 due to the lower content in high frequencies and the low-cut filter applied on the coefficients, which is not as steep as for $T_{CMS}$, as will be later observed in the spectra.

4. Directivity effects are corrected for in (e) with respect to CMS in figure (c), observed as
slightly higher amplitudes, more similar in amplitude throughout different traces and improved continuity, and the residual bubble is also removed. For instance, compare the negative event starting at $\sim 550 \text{ms}$ or the positive event at $\sim 625 \text{ms}$ in both images. These corrections are small with this target wavelet. And still, the shot to shot variations have been neglected in (e) whereas they are taken into account in (c) as a first attempt. Figure (f) shows the differences between these two.

5. At the target level in figure 6-8, the differences between (c), (d) and (e) are not so
important given that low frequency directivity effects are not so important for a wide range of take-off angles (see figure C-2), but there is still a correction when using the small take-off angle approach.

For completion, the amplitude spectrum at two different windows is computed and displayed in figure 6-9 for the previous images: the first mainly containing signal and the second one with mainly noise, defined in the caption of figure 6-9.

![Figure 6-9](image.png)

**Figure 6-9:** Normalised amplitude spectra of the stack input and processed data for two windows: from 0.268 s to 1.714 s and from CMP 11595 to 13066 (top, mainly signal and some noise) and from 4.485 s to 6.943 s and from CMP 11425 to 12877 (bottom, mainly noise). In all graphs, blue is the input or pressure deghosted data, green is the input processed with standard CMS, black is the input data filtered with \((A + B\frac{\partial}{\partial x} + C\frac{\partial^2}{\partial x^2})T_{CMS}\) hybrid and and red is the input filtered with \((A + B\frac{\partial}{\partial x} + \frac{\partial^2}{2})(-24)\) hybrid.

Again, the amplitude behaviour is similar to the one of the pre-stacked data (see figure 6-2), such that an amplitude correction is seen for frequencies > 90 Hz when using the small take-off angle filter with \(T_w = T_{CMS}\) and this is even larger when \(T_w = -24\), as the latter tries to flatten the spectra of the input data in this area, so that amplitudes are amplified in this area. Yet, the present filter is not able to fully model the phase behaviour present in the data due to the far field from the array and it has limitations in terms of the phase above 90 Hz (see figure C-2).
6-3 Discussion of the results

Several benefits have been observed after the application of the small take-off angle filter to receiver deghosted pressure data in the previous sections, such as an improvement in the continuity of the events and equalisation of the amplitudes due to correction of directivity effects with respect to CMS and a better time resolution, the latter specially when a spike is used as a target wavelet in which the data is also source deghosted. Furthermore, the residual bubble is removed and shot to shot variations can be included, although for simplicity, averaged notionals have been used to compute the coefficients of the filters.

The results obtained for the real data are comparable to those obtained in the synthetic data (chapter 4), although other effects complicate its understanding (spherical spreading, amplitude attenuation, etc.).

Nevertheless, this filter has some limitations and the following issues need to be considered in each case:

1. The filter derived here is an approximate take-off angle filter and the range of validity varies with frequency, as shown in the error plots, usually decreasing with frequency as directivity effects become important and the filter is unable to model sudden changes in amplitude and phase with take-off angle (this happens for this particular array and it may be different for other arrays). Beyond this range, not only the amplitude is incorrectly modelled but also the phase, which might generate artefacts at shallow events containing high frequencies (ripples). Most likely, additional terms in the series expansion might improve both the amplitude and phase of the approximate filter reducing these effects, but it might become impractical.

2. When the filter is applied on the data, it is applied in both the signal and noise. Hence, it can be useful to apply noise attenuation before applying the present source deghosting filter. Alternatively, the filter can be applied to raw data before receiver deghosting, which also contains noise but it has not been amplified at low or high frequencies by the receiver deghosting method. Theoretically, both operations should be commutative but source deghosting should then also be applied to particle acceleration data.

3. The choice of the target wavelet depends on the objective of the application of the filter, i.e. either source designature or directivity correction, but also on the noise level in the data and the far field spectrum at zero take-off angle of the array. For instance, when choosing $T_w = -24$ for the Delta 3 source the filter tries to flatten the spectra (without considering spreading and other effects) and, thus, for frequencies in which the amplitude of the input decreases (above $90 \, \text{Hz}$ in this case, see figure 1-3) the filter amplifies signal but also noise in this range. On the other hand, when using $T_w = T_{CMS}$ the target wavelet has the spectrum of the far field signature at zero take-off angle, as shown in figure 5-5, in which the signal and noise at the same frequencies are not amplified. Yet, by choosing the latter the source ghost is not removed from the data but directivity effects are corrected with the small take-off angle filter. This is a small correction.

4. If shot to shot variations are to be included, the small take-off angle coefficients need to be updated for each shot according to the workflow in figure 5-3.
5. The first and second spatial derivatives generate small edge effects at the initial traces. To overcome this, a few traces can be extrapolated at the near offset before the application of the filter and later on removed.

6. If the filter is applied in CRGs as in figure 5-3, in which the 1D earth approximation is not required, interpolation is normally necessary as shot spacing is coarser than receiver spacing.

Hence, before the application of the small take-off angle approach these issues need to be assessed for each particular dataset, but assessing the improvements and drawbacks of source deghosting is not an easy task.

Computation of amplitude and phase error plots are recommended to investigate possible artefacts of the $A$, $B$ and $C$ coefficients of the small take-off angle approach due to their limitations in terms of phase and amplitude for large take-off angles.

Overall it has been observed that the source signature corrections are small for the Delta 3 source and, hence, quality control is difficult. Thus, data acquired with this array is ready for processing and only little corrections due the source signature need to be taken into account, which can be included with the present approach.
In this report a general pre-stack source designature solution has been investigated, valid for any marine source array.

First of all, the different contributions to the far field spectrum of a marine source array have been separately studied, in particular for the Delta 3 source. It has been observed that the synchronisation of the sources in this configuration is key to have a notchless spectrum below 150 Hz (except at 0 Hz) within a 20° take-off angle cone. Beyond these angles, the spectrum becomes more take-off angle dependent, specially at high frequencies (over 90 Hz) as the wavelength becomes comparable to the size of the array and directivity effects are more significant.

Deconvolution of the data from the far field signature has then been studied, which requires the filter to be applied in the frequency-wavenumber or tau-p domains and the data to be sorted in CRGs. Thus, two practical approximations of the source designature operator have been derived as a generalisation of the approach followed by Amundsen and Zhou [2013] to any source array, applied on a trace-by-trace basis: 1) at low frequency and zero take-off angle and 2) at all frequencies and small source take-off angles (azimuthal dependency can potentially be included).

These filters only require the knowledge of the characteristics of the airguns (their localisation, eventual firing source synchronisation and the notionals) and they can be customised to different scenarios (for instance, impulsive sources) while symmetries of the array may simplify the expressions. Moreover, the source signature is replaced by a user defined target wavelet.

The first approximation is suggested given that low frequencies are important in seismic data, specially at the reservoir level. Yet, the more promising small take-off angle approximation has been further investigated, as it is a compact designature filter that handles to some extent directivity, angle-dependant ghost effects and the residual bubble for all frequencies while limiting smearing of shot to shot perturbations due to its short length.

The latter is a practical and fast alternative to a frequency-wavenumber CRG global approach. This filter requires the computation of spatial derivatives of CRGs and interpolation is normally required due to the coarse shot sampling.
Next, the present method has been tested on the synchronised multi-depth Delta 3 source. First, it has been successfully applied on synthetic data, in which its benefits and its limitations have been analysed (for this configuration, the range of validity of the approximation is from take-off angles between $\sim \pm 35^\circ$ and frequencies smaller than $\sim 90\ Hz$, and it narrows down to $\sim \pm 20^\circ$ for higher frequencies). A small but visible correction in directivity effects of the input data has been observed.

Furthermore, the synthetic data tests have been useful to obtain a better insight in the source array deconvolution operator.

As a compromise between the theory and its practical application, the present approach has then been tested on field data under two additional approximations: 1) the Earth is 1D and 2) the notional for the different shots have been averaged. Under the first approximation, the approach has been applied on CSGs (Snell’s law), in which the trace spacing corresponds to the receiver sampling and interpolation is not required, which seems reasonable for this dataset (the Bruce field). Under the second approximation, shot to shot variations have not been accounted for, but they could be included.

The real data results show the benefits of the present approach at the target level, such as the improved continuity of the events, the equalisation of the amplitudes and the sharpening of the wavelets. Nevertheless, the present approach has also its limitations: 1) it is an approximate solution and, thus, might lead to artefacts outside the validity range, 2) the filter design needs to be carefully done, 3) data needs to be prepared in CRGs or CSGs and 4) quality control is difficult as the improvement is small (RMS plots, amplitude spectra and NMO have been used to assess the results).

Nevertheless, in this report I have focused on relatively shallow water data (the Bruce field, $\sim 120\ m$ water depth), and the small take-off angle approximation will be better suited for data acquired over a larger water layer.

Furthermore, the present method has been compared and used in conjunction with a current data processing best practice that only considers vertical take-off angle, the CMS method, and an expected small improvement in directivity effects has been observed.

Further work might focus on alternative approaches to the designature deconvolution operator (i.e. frequency-wavenumber windowing) for designing compact filters, but also higher order terms could be included in the present approach to improve the validity range of the approximation, although the practical aspects should then be assessed. It is also suggested to compare the present results with the global frequency-wavenumber approach of the designature operator in order to further analyse the benefits and drawbacks of the present method.

Additionally, the approach should be tested in interpolated CRGs, where the 1D approximation is not necessary, and the impact of noise with respect to the chosen target wavelet could be further investigated.


Appendix A

Small take-off angle source deghosting filter: derivation

The inverse of the far field presented in equation (2-1) is the basis of the source deghosting filters developed in this report. But in the exact form, as shown in equation (3-2), the filter does not have a practical implementation.

In this chapter, the small take-off angle approximation of the latter is derived. Under certain approximations, the resulting filter can be used in practice.

Let us consider the far field expression presented in equation (2-1), in the $f - k_s^x, k_s^y$ domain, which can be written in a more compact form as:

$$W(\omega, \theta^s, \varphi^s) = \sum_{n=1}^{N} S_n(\omega) \left( e^{ikb^+_n,s} - |r_0|e^{ikb^-_n,s} \right)$$  \hspace{1cm} (A-1)

Here, the Fourier sign convention is the same as specified in section 2-1 and coefficients $b^\pm_{n,s}$ include the dependency of the far field on the $n^{th}$ source positions and the departure take-off angle and azimuth from the source, indicated with the subscript $s$ such that:

$$b^+_n,s \equiv \sin \theta^s (\cos \varphi^s x_n + \sin \varphi^s y_n) + \cos \theta^s (\pm z_n - z_{ref}) - \tau_n c$$ \hspace{1cm} (A-2)

Therefore, $b^+_n,s = b^-_{n,s} + 2z_n \cos \theta^s$. Thus, the inverse of the far field in equation (3-2) is written as:

$$F \equiv \frac{1}{W} \approx \frac{1}{\sum_{n=1}^{N} S_n(\omega) \left( e^{ikb^+_n,s} - |r_0|e^{ikb^-_n,s} \right)}$$ \hspace{1cm} (A-3)
This can be decomposed in an infinite Taylor series around zero take-off angle and zero azimuth. Yet, the expansion is truncated at second order as its application shows some benefits with respect to only considering the first term of the series. If this is done, one can write:

\[
F \approx F(\theta^s = 0^\circ, \varphi^s = 0^\circ) + \theta^s \frac{\partial F}{\partial \theta^s} |_{\theta^s = \varphi^s = 0^\circ} + \varphi^s \frac{\partial F}{\partial \varphi^s} |_{\theta^s = \varphi^s = 0^\circ} + \frac{1}{2} \left[ (\theta^s)^2 \frac{\partial^2 F}{\partial (\theta^s)^2} |_{\theta^s = \varphi^s = 0^\circ} + (\varphi^s)^2 \frac{\partial^2 F}{\partial (\varphi^s)^2} |_{\theta^s = \varphi^s = 0^\circ} + 2 \theta^s \varphi^s \frac{\partial^2 F}{\partial \theta^s \partial \varphi^s} |_{\theta^s = \varphi^s = 0^\circ} \right]
\]

(A-4)

After some algebra, the latter is written as:

\[
F \approx A + B' \theta^s + B'' \varphi^s + C' (\theta^s)^2
\]

(A-5)

Where the previous coefficients take the following expressions:

\[
A = \left[ \sum_{n=1}^{N} S_n(\omega) \left( e^{i k c_n^+} - |r_0| e^{i k c_n^-} \right) \right]^{-1} \equiv [W(k, \theta = 0^\circ, \varphi^s = 0^\circ)]^{-1}
\]

\[
B' = -A^2 k \sum_{n=1}^{N} S_n(\omega) x_n \left( e^{i k c_n^+} - |r_0| e^{i k c_n^-} \right)
\]

\[
B'' = -A^2 k \sum_{n=1}^{N} S_n(\omega) y_n \left( e^{i k c_n^+} - |r_0| e^{i k c_n^-} \right)
\]

\[
C' = \frac{(B')^2}{A} - i k A^2 \sum_{n=1}^{N} S_n(\omega) \left[ (z_{ref} - z_n + i k x_n^2) e^{i k c_n^+} - |r_0| (z_{ref} + z_n + i k x_n^2) e^{i k c_n^-} \right]
\]

(A-6)

with \( c_n^\pm = \pm z_n - z_{ref} - \tau_{nc} \).

Furthermore, for small source azimuth and small source take-off angle, \( k^s_x = k \sin \theta^s \cos \varphi^s \approx k \theta^s \) and \( k^s_y = k \sin \theta^s \sin \varphi^s \approx k \theta^s \varphi^s \) at second order. According to the sign convention used, \((i k^s_x)^m \) and \((i k^s_y)^m \) in the wavenumber domain are respectively transformed to \((-1)^m \partial_x^m \) and \((-1)^m \partial_y^m \) in the space domain, where \( m \) is the exponent of the wavenumber.

Thus, equation (A-5) at small angles in the frequency-space domain is as written as follows:

\[
F \approx \sum_{n=1}^{N} F_n(k, \partial_x^s, (\partial_x^s)^2, x_n, \partial_y^s, y_n, z_n) = A + B_1 \partial_x^s + B_2 \partial_y^s + C \partial_x^2
\]

(A-7)

With:

\[
F \approx A + B' \theta^s + B'' \varphi^s + C' (\theta^s)^2
\]

(A-5)
Here, coefficients $B_1$, $B_2$ and $C$ are related with $B'$, $B''$ and $C'$ in equation (A-8) such that $B_1 = B'/(-ik)$, $B_2 = B''/(-ik)$ and $C = C'/(−ik)^2$, respectively.

Now, an additional assumption is made: azimuthal dependency is neglected, such that $B_2 \partial_y$ is not considered in equation (A-9). Then:

$$F \approx \sum_{n=1}^{N} F_n(k, \partial_{x'}(\partial_{x'})^2, x_n, \partial_{y'} = 0, y_n, z_n) = A + B \partial_{x'} + C \partial_{x'}^2 \tag{A-9}$$

Where $B \equiv B_1$. The previous equation is the so-called small take-off angle approximation of the inverse of the far field. The latter leads to the result presented in Amundsen and Zhou [2013, equation 7] neglecting azimuthal dependency if a single monopole source is considered.
An alternative approach to the small take-off angle filter to deghost the data presented in section 3-2 is to perform a low frequency approximation of the far field in equation (2-1) and then a second low frequency approximation of the inverse of the latter. This is the so-called low frequency approximation in chapter 3-3 and it is derived below these lines.

Let us consider the far field expression presented in equation (2-1), in the \( f - k^s_x, k^s_y \) domain. Under the approximation that the far field is independent of the azimuth (i.e. \( \varphi^s \approx 0^\circ \) and hence, \( k^s_y \approx 0 \)), the latter is written in a more compact form as:

\[
W(\omega, \theta^s, \varphi^s = 0^\circ) = \sum_{n=1}^{N} S_n(\omega) \left( e^{ikb^+_n,s} - |r_0| e^{ikb^-_n,s} \right) \tag{B-1}
\]

Here, the Fourier sign convention is the same as specified in section 2-1 and coefficients \( b^+_n,s \) include the dependency of the far field on the \( n^{th} \) source positions and the departure take-off angle from the source, indicated with the subscript \( s \), such that:

\[
b^+_n,s \equiv \sin \theta^s x_n + \cos \theta^s (\pm z_n - z_{ref}) - \tau_n c \tag{B-2}
\]

Therefore, \( b^+_n,s = b^-_n,s + 2z_n \cos \theta^s \). The far field is then expanded as a Taylor series around zero frequency as follows:

\[
W \approx W(\omega = 0) + \omega \frac{\partial W}{\partial \omega} \bigg|_{\omega=0} + \frac{\omega^2}{2} \frac{\partial^2 W}{\partial \omega^2} \bigg|_{\omega=0} + \frac{\omega^3}{6} \frac{\partial^3 W}{\partial \omega^3} \bigg|_{\omega=0} = W_0 + W_1 k + W_2 k^2 + W_3 k^3 \tag{B-3}
\]

With \( k = \omega/c \) and \( W_0, W_1, W_2 \) and \( W_3 \) angle dependent coefficients that take the following expressions:
In marine surveys, \( r_0 \approx -1 \). If \( r_0 = -1 \) is considered, the previous coefficients simplify to:

\[
W_0 = 0
\]

\[
W_1 = \left[ \sum_{n=1}^{N} \left( (1 + r_0) \frac{\partial S_n(\omega)}{\partial \omega} + i \sum_{n=1}^{N} S_n(\omega) \left( b_{n,s}^+ + r_0 b_{n,s}^- \right) \right) \right]_{\omega=0}
\]

\[
W_2 = \left[ \sum_{n=1}^{N} \left( (1 + r_0) \frac{c^2 \partial^2 S_n(\omega)}{\partial \omega^2} + i c \frac{\partial S_n(\omega)}{\partial \omega} (b_{n,s}^+ + r_0 b_{n,s}^-) - \frac{1}{2} S_n(\omega) \left( b_{n,s}^+ \right)^2 - r_0 \left( b_{n,s}^- \right)^2 \right) \right]_{\omega=0}
\]

\[
W_3 = \left[ \sum_{n=1}^{N} \left( (1 + r_0) \frac{c^3 \partial^3 S_n(\omega)}{6 \partial \omega^3} + i c^2 \frac{\partial^2 S_n(\omega)}{\partial \omega^2} (b_{n,s}^+ + r_0 b_{n,s}^-) - \frac{c}{2} \frac{\partial S_n(\omega)}{\partial \omega} \left( b_{n,s}^+ \right)^2 - r_0 \left( b_{n,s}^- \right)^2 \right) + i \frac{c}{6} S_n(\omega) \left( b_{n,s}^+ \right)^3 - r_0 \left( b_{n,s}^- \right)^3 \right]_{\omega=0}
\]

Where:

\[
\begin{align*}
   b_{n,s}^+ - b_{n,s}^- &= 2 z_n \cos \theta^s \\
   (b_{n,s}^+)^2 - (b_{n,s}^-)^2 &= 4 z_n \cos \theta^s (b_{n,s}^+ - z_n \cos \theta^s) \\
   (b_{n,s}^+)^3 - (b_{n,s}^-)^3 &= 6 z_n \cos \theta^s (b_{n,s}^+)^2 + 12 z_n^2 (\cos \theta^s)^2 b_{n,s}^- + 8 z_n^3 \cos \theta^3) 
\end{align*}
\]

Thus, at low frequencies \( W \approx W_1 k + W_2 k^2 + W_3 k^3 \) and one can write:
The latter is again approximated at low frequencies as $W_1$, $W_2$ and $W_3$ are frequency independent:

$$F \approx \frac{1}{W_1k + W_2k^2 + W_3k^3} \quad (B-13)$$

Thus:

$$1 \approx \left( \frac{A'_{-1}}{k} + A_0 + A'_1k \right) (W_1k + W_2k^2 + W_3k^3) \quad (B-15)$$

Equating coefficients for $1$, $k$ and $k^2$:

$$\begin{align*}
1 &= A'_{-1}W_1 \\
0 &= A_0W_1 + A'_{-1}W_2 \\
0 &= A_0W_2 + A'_1W_1 + A'_{-1}W_3
\end{align*} \quad (B-16)$$

Solving the previous linear system of equations, one obtains:

$$A'_{-1} = \frac{1}{W_1}; \quad A_0 = -\frac{W_2}{W_1}; \quad A'_1 = \frac{W_2^2}{W_1^2} - \frac{W_3}{W_1} \quad (B-17)$$

Where $W_1$, $W_2$ and $W_3$ are the coefficients in equations (B-9), (B-10) and (B-11), respectively. Alternatively, equation (B-14) can be written in the following fashion:

$$F \approx \frac{A'_{-1}}{i\omega} + A_0 + i\omega A_1 \quad (B-18)$$

Where the previous coefficients are related to those in equation (B-17) such that $A_{-1} = icA'_{-1}$ and $A_1 = A'_1/ic$ and, hence, are take-off angle dependent.

In the time domain and on a trace-by-trace basis, the first term of the low frequency approximation in equation (B-18) corresponds to an integral, the second one is a constant term and the third one is a time derivative.

Yet, this approach is not practical as angle information is not directly available.

Thus, the previous coefficients are particularised for $\theta^s = 0^\circ$, in which $b_{n,n}^\pm(\theta^s = 0^\circ) \equiv c_n^\pm = \pm z_n - z_{ref} - \tau_n c$ and:

$$\begin{align*}
A_{-1} &= \frac{ic}{W_1} = \frac{c}{2\left[ \sum_{n=1}^N S_n(\omega)z_n \right]_{\omega=0}} \\
A_0 &= -\frac{W_2}{W_1} = \frac{\left[ \sum_{n=1}^N \left( i\omega \partial S_n(\omega) / \partial \omega \right) z_n + S_n(\omega)z_n(\tau_n + \tau_{c}) \right]_{\omega=0}}{2\left[ \sum_{n=1}^N S_n(\omega)z_n \right]_{\omega=0}^2} \\
A_1 &= \frac{W_2^2}{icW_1} - \frac{W_1}{icW_1} = \frac{\left[ \sum_{n=1}^N \left( i\omega \partial S_n(\omega) / \partial \omega \right) z_n + S_n(\omega)z_n(\tau_n + \tau_{c}) \right]_{\omega=0}^2}{4ic\left[ \sum_{n=1}^N S_n(\omega)z_n \right]_{\omega=0}^2} + \frac{\left[ \sum_{n=1}^N \left( i\omega^2 \partial S_n(\omega) / \partial \omega^2 \right) z_n + 2i\omega \partial S_n(\omega) / \partial \omega \right) z_n + S_n(\omega)z_n(\tau_n + \tau_{c}) \right]_{\omega=0}^2}{4ic\left[ \sum_{n=1}^N S_n(\omega)z_n \right]_{\omega=0}^2} + \frac{\left[ \sum_{n=1}^N \left( i\omega^2 \partial S_n(\omega) / \partial \omega^2 \right) z_n + 2i\omega \partial S_n(\omega) / \partial \omega \right) z_n + S_n(\omega)z_n(\tau_n + \tau_{c}) \right]_{\omega=0}^2}{4ic\left[ \sum_{n=1}^N S_n(\omega)z_n \right]_{\omega=0}^2} \quad (B-19)
\end{align*}$$

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Furthermore, the latter considerably simplify when impulsive sources are considered:

\[
\begin{align*}
S_n(\omega) = 1 & \quad \theta_s = 0^\circ \\
A_{-1} &= \frac{c}{2\sum_{n=1}^{N} z_n} \\
A_0 &= \frac{\sum_{n=1}^{N} z_n(z_{ref} + \tau_n c)}{2(\sum_{n=1}^{N} z_n)^2} \\
A_1 &= \frac{(\sum_{n=1}^{N} z_n(z_{ref} + \tau_n c))^2}{2c(\sum_{n=1}^{N} z_n)^3} - \frac{\sum_{n=1}^{N} z_n(z_n^2 + 3z_{ref}^2 + 6z_{ref}\tau_n c + 3\tau_n^2 c^2)}{12c(\sum_{n=1}^{N} z_n)^2}
\end{align*}
\]

This is a generalisation of the low frequency filter presented in Amundsen and Zhou [2013, equation 10] to any marine source array with \(N\) impulsive sources (in fact, the coefficients in equation (B-17) consider any source signature and any take-off angle).

In order to assess the differences with the latter, the amplitude and phase plots of both filters are again compared with the spectra of \(F\) for impulsive sources, similarly as in figure 3-6. Impulsive sources of equal strength are considered and \(\theta^s = 0^\circ\), i.e. the coefficients in equation (B-20) are used. The result is shown in figure B-1.

The low frequency filter suggested here has slightly less error when compared to \(F\) (first row in figure 3-6) than the filter suggested in Amundsen and Zhou [2013, equation 10], but the differences are not dramatic (compare the second and the third plots of each row). This is because, although the current approach considers the different source depths and the synchronisation delay in a Delta 3 source configuration, it has been particularised for impulsive sources and \(\theta^s = 0^\circ\).

Thus, the only improvement possible is around \(\theta^s = 0^\circ\) due to the different source ghosts of the \(N\) sources. Amplitude is only slightly modified, as the plots in the second column are almost the same for both low frequency filters. However, there is an improvement when also the phase is considered, as it can be seen in the third column (the dark blue area is extended, meaning little differences with the exact). Thus, the current approach mainly improves the result due to phase correction after considering the depths of the different sources.

This area of improvement could be extended by including the notionals and considering more angles, but the latter is impractical. For that reason, the small take-off angle approach is preferred, which is more accurate for more take-off angles and for all frequencies.

Additionally, note that the current approach converges\(^1\) to Amundsen and Zhou [2013, equation 10] under the following conditions:

\[
\begin{align*}
z &= z_{ref} = z_{min} \\
N &= 1 \\
z_1 &= z
\end{align*}
\Rightarrow
\begin{align*}
A_{-1} &= \frac{c}{2z} \\
A_0 &= \frac{1}{2} \\
A_1 &= \frac{z}{6c}
\end{align*}
\Rightarrow
\begin{align*}
F &\approx \frac{1}{2} \left[ 1 + \frac{1}{ikz} + \frac{ikz}{3} \right]
\end{align*}
\]

\(^1\)The sign difference of the coefficients is due to the chosen sign convention for the Fourier transforms.
Figure B-1: Error plots for the exact inverse of the far field for impulsive sources (top row), the low-frequency filter as per [Amundsen and Zhou, 2013] (second row) and the low frequency filter derived here for $\theta_s = 0^\circ$ (third row) with 24 sources in a Delta 3 source configuration. Impulsive sources of equal strength are considered.
Appendix C

Real data: error plots and parameters

In this chapter, the acquisition parameters of interest from the real data studied in chapters 5 and 6 are first presented. Table C-1 summarises several of them.

<table>
<thead>
<tr>
<th>Acquisition parameter</th>
<th>Nomenclature</th>
<th>Survey and attributes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td></td>
<td>UKCS Bruce field</td>
</tr>
<tr>
<td>Client</td>
<td></td>
<td>WesternGeco Multiclient</td>
</tr>
<tr>
<td>Sail line name</td>
<td></td>
<td>1079</td>
</tr>
<tr>
<td>Source array type</td>
<td></td>
<td>Delta 3 source</td>
</tr>
<tr>
<td>Number of sources per array</td>
<td>N</td>
<td>18 (clusters counted as 1 source)</td>
</tr>
<tr>
<td>Source signatures</td>
<td></td>
<td>Obtained from NFH measurements</td>
</tr>
<tr>
<td>Shot spacing</td>
<td></td>
<td>25 m</td>
</tr>
<tr>
<td>Receiver spacing</td>
<td>( \Delta x )</td>
<td>3.125 m</td>
</tr>
<tr>
<td>Receiver depth</td>
<td></td>
<td>17 m</td>
</tr>
<tr>
<td>Number of receivers per line</td>
<td></td>
<td>995</td>
</tr>
<tr>
<td>Streamers considered</td>
<td></td>
<td>1, aligned with the boat</td>
</tr>
<tr>
<td>Source to 1st receiver distance</td>
<td></td>
<td>( \sim 63.1 \text{m} )</td>
</tr>
<tr>
<td>Time sampling</td>
<td>( \Delta t )</td>
<td>2 ms</td>
</tr>
<tr>
<td>Nyquist frequency</td>
<td>( f_N )</td>
<td>250 Hz</td>
</tr>
<tr>
<td>Swell height/direction</td>
<td></td>
<td>1.0/S</td>
</tr>
<tr>
<td>Water depth</td>
<td></td>
<td>115.6 m average across the line</td>
</tr>
</tbody>
</table>

Table C-1: Acquisition parameters of interest from the survey performed at the UKCS Bruce field by WesternGeco Multiclient.

Next, the notionals are stacked and the acoustic center of the array is found as in section 2-2-5. Thus, the phase difference between the far field at zero source take-off angle \( (\theta_s = 0^\circ) \) and the maximum source take-off angle \( (\theta_{max} = | \pm 20^\circ |) \) is computed for different reference depths, as shown in figure C-1.

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A reference depth of $z_{ref} = 6 \text{ m}$ is chosen as a compromise to minimise the phase difference between these angles.

For this reference depth, the amplitude and phase spectra of the exact far field as in equation (3-2) are computed for all take-off angles and for zero azimuth with the stacked notionals. As in section 3-2-3, the corresponding amplitude and phase spectra of the different small take-off angle terms, their amplitude difference with the exact filter and the amplitude of the difference with the exact are computed and shown in figure C-2 and figure C-3 for target wavelets equal to $T_w = 1$ and $T_w = T_{CMS}$, respectively. The coordinate system used is the one in figure 1-2.

Interestingly, the coefficients seem to amplify more the amplitudes on the boat side (negative take-off angles). These plots are used to decide the high-cut frequency for the $C$ coefficient, as this boosts too much signal and noise at high frequencies and large take-off angles. By inspection of the second and the third plots in the last row of figures C-2 and C-3, this frequency is set to 90 Hz, although other similar values are possible. Thus, only $A$ and $B$ coefficients have an impact within the whole bandwidth, whereas $C$ is limited to the first 90 Hz. The latter is the so-called hybrid filter.

It is worth mentioning that the third and fourth rows in figure C-3 show the improvement in terms of phase and amplitude when including the small take-off angle coefficients with respect to the CMS matching filter, which corresponds to the second row in this figure (as the stacked notionals have been used both to generate the exact filter in the first row and also the different coefficients, in which $A$ is the inverse of the far field signature at zero take-off angle as used in CMS).

Finally, the different parameters used to generate coefficients $A$, $B$ and $C$ as shown in figure 5-6 according to equation (3-6) are shown in table C-2.
Figure C-2: Error plots for the exact inverse of the far field, \( A \), \( A + B'\theta^s \) and \( A + B'\theta^s + C'/(\theta^s)^2 \) considering stacked notionals obtained from NFH measurements and \( T_w = 1 \).
Figure C-3: Error plots for the exact inverse of the far field, $A$, $A + B'\theta^*$ and $A + B'\theta^* + C'(\theta^*)^2$ considering stacked notionals obtained from NFH measurements and $T_w = T_{CMS}$. 
<table>
<thead>
<tr>
<th>Attribute (units)</th>
<th>Nomenclature</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water velocity (m/s)</td>
<td>c</td>
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<td>1500</td>
<td>1500</td>
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<tr>
<td>Sea surface reflection coefficient</td>
<td>$r_0$</td>
<td>$-1$</td>
<td>$-1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>Number of sources</td>
<td>$N$</td>
<td>18</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>Source signatures</td>
<td>$s_n(t)$</td>
<td>Stacked</td>
<td>Stacked</td>
<td>Stacked</td>
</tr>
<tr>
<td>Source positions</td>
<td>$x_n, y_n, z_n$</td>
<td>Nominal*</td>
<td>Nominal*</td>
<td>Nominal*</td>
</tr>
<tr>
<td>Time sampling (ms)</td>
<td>$\Delta t$</td>
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<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Reference depth (m)</td>
<td>$z_{\text{ref}}$</td>
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<td>6</td>
<td>6</td>
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<tr>
<td>Band-pass filter window type</td>
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<td>Cosine</td>
<td>Cosine</td>
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<td>Band-pass filter order</td>
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<td>4**</td>
<td>4**</td>
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<td>Low-cut frequency (Hz)</td>
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<td>Zero time sample</td>
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<td>501</td>
</tr>
</tbody>
</table>

Table C-2: Parameters used to generate coefficients $A$, $B$ and $C$ with the stacked notionals plotted in figure 5-6.

*See figure 1-2.

**Alternative Blackman, [De Levie, 2004].