Elastic Wavefield Decomposition at the Ocean Bottom
Including near-ocean bottom velocity estimation

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Abstract

Multi-component ocean-bottom seismic data offers the opportunity to have access to seismic P-waves and S-waves, (mode converted waves). The initial pre-processing step to retrieve reliable P- and S-wave sections is elastic decomposition of the multi-component measurements. This wavefield decomposition at the ocean bottom requires properties of the ocean floor in order to get reliable one-way wavefields. We consider the situation that via the JMI process, (Joint Migration Inversion), an estimation for the upgoing P- and S-waves at a certain depth level has been obtained. We develop an inversion scheme that estimates the P- and S-wave velocities in the remaining part of the subsurface, between the depth level below the water bottom and the bottom itself. In addition, we apply a composition of the resulting up-going wavefields at the bottom into the measured quantities by the 4-C receivers. Thus, a combined tomography and decomposition problem is solved. In this way, the unknown properties of the ocean bottom as well as the velocities in the near bottom layer are estimated based on comparing the predicted data with the true measurements. In the thesis, results of this methodology are shown under the assumption of a horizontally stratified medium. The results indicate that it is possible to develop an inversion scheme that estimates the near-bottom velocity model in combination with the optimum composition operators.
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August 13, 2014
# Table of Contents

Abstract v  
Acknowledgements vii  

1 Introduction 1  
1-1 Decomposition 1  
1-2 Parameters estimation 2  
1-3 Notations and transformations 3  

2 Acoustic and Elastic Wavefield Decomposition 5  
2-1 Acoustic case 5  
2-1-1 Two-way wave equation 5  
2-1-2 One-way wavefields 6  
2-2 Elastic case 8  
2-2-1 Two-way wave equation 8  
2-2-2 One-way wavefields 9  
2-3 Wavefield decomposition at the ocean bottom 12  
2-3-1 Acoustic decomposition 13  
2-3-2 Elastic decomposition 13  
2-4 Synthetic data 14  
2-4-1 Wavefield decomposition 14  
2-5 Decomposition with erroneous velocities 19  

3 Near-bottom velocity estimation 23  
3-1 Wavefield tomography 23  
3-1-1 Velocity perturbation theory 23  
3-1-2 Tomographic Inversion 25  

August 13, 2014
List of Figures

2-1 Logs used for creating synthetic data. ........................................................ 15
2-2 Synthetic ocean bottom data. ................................................................. 16
2-3 Decomposition of the total wavefield just above the ocean bottom to (a) up-going acoustic pressure and (b) down-going acoustic pressure. ................................................................. 17
2-4 Decomposition levels, just above the ocean bottom \((z_1 - \epsilon)\) and just below the ocean bottom \((z_1 + \epsilon)\). ................................................................. 17
2-5 Decomposition of the total wavefield just below the ocean bottom to (a) down-going P-waves, (b) up-going P-waves, (c) down-going S-waves and (d) up-going S-waves. ................................................................. 18
2-6 Elastic decomposition with 100 m/s error in velocities to (a) down-going P-waves, (b) up-going P-waves, (c) down-going S-waves and (d) up-going S-waves. ................................................................. 20
2-7 Elastic decomposition with (a-b) 200 m/s and (c-d) 400 m/s error in the velocities. ................................................................. 21
3-1 Illustration of the depth levels to update. ........................................... 26
3-2 Magnified view of the up-going measurements (blue), predicted (red) and residual (black), before applying wavefield tomography. Note that the up-going P- and S-wave residuals are computed using only one trace \((p = 0.2622 \times 10^{-3} \text{ s/m})\), for the sake of comparison. ................................................................. 29
3-3 The measurements (blue), predicted up-going wavefields (red) and residual (black), after applying wavefield tomography. Note how well the predicted up-going wavefields match the measured up-going wavefields. The trace we used to show the residual corresponds to \((p = 0.2622 \times 10^{-3} \text{ s/m})\). ................................................................. 30
3-4 The true (blue), estimated (red) and initial (green) velocity models for (a) P-wave and (b) S-wave. Note that these are also magnified velocity models to show the update levels. ................................................................. 31
3-5 Plot of \(J_p\) (blue) and \(J_s\) (red), versus the number of iterations. ................................................................. 31
3-6 The total measurements (blue), predicted data (red) and residual (black), after applying composition. Note the good match as well as the consistency of the events in the measured and predicted data. The trace chosen to compute the residual corresponds to \((p = 0.266 \times 10^{-3} \text{ s/m})\). ................................................................. 34
4-1 Up-going measurements (blue), predicted wavefields (red) and residual (black). (a) and (b) correspond to data before applying wavefield tomography, while (c) and (d) correspond to data after applying wavefield tomography. The difference might not be noticeable as it is in Figure 3-2 and Figure 3-3. 

4-2 True (blue), estimated (red) and initial (green) velocity profiles.
List of Tables
Chapter 1

Introduction

1-1 Decomposition

In marine seismic data acquisition, there are two types of medium to consider. The first medium is the ocean which only supports compressional waves (P)-waves. The second one is a solid medium (the Earth’s subsurface), in which longitudinal (P)-waves and shear (S)-waves can propagate. Air guns towed behind vessels are the common sources used to generate P-waves. The energy sent by the sources propagates in the water into the subsurface and gets transmitted and reflected. Wave conversion from P to S-waves or vise versa may occur at any point along the propagation path in the Earth’s subsurface.

Seismic receivers can be chosen to be streamers towed behind the vessels at the ocean surface. Hence, the receivers only record the P-waves. In shallow water areas, vessels with streamers cannot access. Another choice is to place the receivers at the ocean bottom, via ocean bottom cables (OBC) or ocean bottom nodes (OBN). Commonly multi-component receivers are used: 3 component-geophones and a hydrophone. This choice gives the opportunity to record P- waves and S-waves which are generated via wave conversion. Due to low S-wave velocities, S-wave data carry high resolution information. Moreover, the receivers can be left at the ocean bottom for a long period allowing for accurate time lapse measurements. Therefore, reservoirs can be well monitored with high repeatability, and cost reduction of receivers deployment. A good example is the monitoring of the Valhall Field [van Gestel et al., 2008]. Another advantage of ocean bottom technology over streamers is the possibility of making various acquisition designs with different source and receiver geometries.

S-wave seismic acquisition, processing and interpretation is getting more popular with the demand of finding unconventional reservoirs. They offer more attributes such as shear-wave interval attenuation for reservoir characterization [Shekar and Tsvankin, 2011] and shear-wave AVO which allowed detection of fractures in the Vacuum Field [DeVault et al., 2002]. Together, PP and PS data can enhance the interpretation and lead to direct access to elastic rock properties [Garotta et al., 2002]. Sometimes the S-wave seismic section has an

August 13, 2014
advantage over the P-wave seismic section. A well known example is the presence of a gas cloud in the subsurfaces which attenuates the P-waves while S-waves propagate through without attenuation. [Cafarelli et al., 2000] and [Mancini et al., 2005] showed field data examples where mode converted waves imaged the target below the gas clouds, while P-waves failed to do so.

Since it is not possible yet to record pure P- and S-waves, these two types of waves get recorded by the 4-C receivers. In order to have pure P- and S-wave records, it is required to perform proper elastic wavefield decomposition. The reliability of any further processing of the seismic data depends on whether the decomposition results are reliable or not. The terms elastic or acoustic wavefield decomposition are used to refer to the medium in which the decomposition is performed and the outcome of the decomposition process. While acoustic decomposition provides up- and down-going acoustic pressure, elastic decomposition delivers up- and down-going S-waves.

There exist different methods for decomposition. Data can be decomposed based on polarity, as P- and S-waves propagate with different polarities. Another decomposition method and the most known in the industry is PZ summation, where the scaled vertical geophone and hydrophone are summed together to get the decomposed up- and down-going P-waves. This method is relatively cheap, but it is only valid for close to normal angle of incidence. Wave equation based decomposition, both acoustic and elastic, is a more appropriate way of honoring the physics of wave propagation. The theory of this method has been explained by different authors including [Ursin, 1983] and [Wapenaar and Berkhout, 1989]. It is commonly derived in the wavenumber/rayparameter-frequency domains. This is the decomposition approach that we describe and apply in this thesis.

1-2 Parameters estimation

Acoustic wavefield decomposition only requires the ocean properties. On the other hand, elastic wavefield decomposition requires the properties of the first layer beneath the receivers. Therefore, it is important to know the properties of the near-ocean bottom. One of the current practice methods for estimating the near-bottom parameters is via refraction tomography surveys, which are based on first breaks picking. For large data sets, this method is time consuming. [Schalkwijk, 2001] and [Schalkwijk et al., 2003] suggested an adaptive wavefield decomposition scheme, with which the near-ocean bottom parameters can be estimated, and applied it to field data as well. This approach allows good decomposition results. However, it is based on different criteria that requires direct arrivals picking and events identification.

Another near-ocean bottom problem is the complex behavior of the S-wave velocity. This leads to distortion of the seismic image and requires redatuming the seismic data to a certain depth level below the weathering layer. This involves near-ocean bottom velocity estimation as well. It is achieved in practice very often by model building of the near-bottom with the analysis of refraction surveys.
In this thesis, we apply a waveform tomography inversion method to retrieve the near-ocean bottom properties. The waveform tomography can be regarded a subset of the Joint Migration Inversion method established by [Berkhout, 2012] and performed by [Staal and Verschuur, 2013] and [Staal and Verschuur, 2014] on acoustic data. The method is data-driven, capable of utilizing the total wavefield including all multiples and requires no user interaction. In this thesis we will consider the situation that via the JMI process (or another process), an estimation for the upgoing P-waves and S-waves at a certain depth level has been obtained. We develop an inversion scheme that estimates the velocities in the remaining part of the subsurface, between the depth level below the water bottom and the bottom itself, and in addition apply a composition of the resulting upgoing wavefields at the bottom into the measured quantities by the 4-C receivers. Thus, a combined tomography and decomposition problem is solved. In this way, the unknown properties at the bottom as well as the velocities in the near-bottom layer will be estimated based on comparing the predicted with the true measurements. This leads to having a detailed ocean floor model that provides exact decomposition results. Moreover, statics correction becomes possible to achieve.

Note that in practice, the JMI method cannot provide the perfect upgoing P-waves and S-waves if the near-bottom model is not yet available to large accuracy. However, this means that the JMI method including the described near-bottom estimation need to be run iteratively until all modelling results are in accordance to the measurements. But, for the sake of this thesis we assume perfect knowledge of the upgoing wavefields at the certain depth level below the water bottom.

**1-3 Notations and transformations**

We list in this section the transforms, conventions, symbols and definitions that are used in this thesis.

We define the forward and inverse temporal Fourier transforms as

\[
U(\omega) = \int_{-\infty}^{+\infty} e^{-j\omega t} u(t) \, dt \\
u(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{j\omega t} U(\omega) \, d\omega,
\]

respectively, where \(\omega\) denotes the angular frequency. For a real function \(u(t)\), the inverse Fourier transform become

\[
u(t) = \frac{1}{\pi} \text{Re} \int_{0}^{+\infty} e^{j\omega t} U(\omega) \, d\omega.
\]

We define the horizontal 1D forward and inverse spatial Fourier transforms as

\[
\tilde{U}(k_x, \omega) = \int_{-\infty}^{+\infty} e^{jk_x x} U(x, \omega) \, dx \\
U(x, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-jk_x x} \tilde{U}(k_x, \omega) \, dk_x,
\]
respectively, where \( k_x \) denotes the horizontal wavenumber.

Based on the definitions of the temporal and spatial Fourier transforms, differentiation with respect to time in the time domain is equivalent to multiplication with \((j\omega)\) in the frequency domain, \((\partial_t \rightarrow j\omega)\). Also, Differentiation with respect to the 1D space-frequency domain is equivalent to multiplication with \((-jk_x)\) in the wavenumber-frequency domain, \((\partial_x \rightarrow -jk_x)\).

We define the horizontal 2D forward and inverse Fourier transforms as

\[
\tilde{U}(k_x, k_y, \omega) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{j(k_x x + k_y y)} U(x, y, \omega) \, dx \, dy
\]  
(1-3a)

\[
U(x, y, \omega) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-j(k_x x + k_y y)} \tilde{U}(k_x, k_y, \omega) \, dk_x \, dk_y,
\]  
(1-3b)

respectively. Similarly, \((\partial_l \rightarrow -jk_l)\), where \((l = x, y)\).

We define the temporal forward and inverse Radon transform, respectively as

\[
\hat{u}(p, \tau) = \int_{-\infty}^{+\infty} u(x, \tau + px) \, dx
\]  
(1-4a)

\[
u(x, t) = \frac{1}{2\pi} \partial_t \int_{-\infty}^{+\infty} H[\hat{u}(p, t - px)] \, dp,
\]  
(1-4b)

where \( H \) denotes the Hilbert transform, \((p = \frac{k_x}{\omega})\) is the horizontal slownes, and \((\tau = t - px)\) is the time intercept.

The 1D Radon transform can also be defined in the frequency domain:

\[
\tilde{U}(p, \tau) = \int_{-\infty}^{+\infty} e^{j\omega px} U(x, \omega) \, dx
\]  
(1-5a)

\[
U(x, \omega) = \frac{\omega}{2\pi} \int_{-\infty}^{+\infty} e^{-j\omega px} \tilde{U}(\omega p, \omega) \, dp.
\]  
(1-5b)

The definition of the Kronecker delta is given by

\[
\delta_{ij} = \begin{cases} 
1, & \text{for } i = j, \\
0, & \text{for } i \neq j.
\end{cases}
\]  
(1-6)

They symbol (*) indicates complex conjugate.
In this chapter, we review the theory of acoustic and elastic wavefield decomposition following the work of [Wapenaar and Berkhout, 1989] and [Schalkwijk, 2001]. First, we explain acoustic wavefield decomposition in the wavenumber-frequency domain. Taking similar approach to the acoustic decomposition, we then explain the theory of elastic wavefield decomposition in an isotropic medium. Finally, we apply the theory to multi-component ocean-bottom synthetic data to get the up- and down-going acoustic pressure from the acoustic decomposition. While the output of the elastic decomposition is the up- and down-going P- and S-waves.

2-1 Acoustic case

To reach to an expression for the decomposed one-way wavefields, we need to establish a relationship between the recorded two-way wavefields and the desired one-way wavefields. We consider a horizontally layered medium where medium parameters vary with respect to depth only. The compressibility $\kappa = \kappa(z)$, and the density $\rho = \rho(z)$.

2-1-1 Two-way wave equation

The starting point is deriving the two-way wave equation from the linearized equation of motion:

$$\partial_k p + \rho \partial_t v_k = f_k$$  \hspace{2cm} (2-1)

and the linearized stress-strain relationship:

$$\partial_k v_k + \kappa \partial_t p = q.$$  \hspace{2cm} (2-2)

For a source-free situation, we write equations (2-1) and (2-2) in the space-frequency domain.
Acoustic and Elastic Wavefield Decomposition

\[
\nabla P(x, y, z, \omega) + j\omega \rho(z) V(x, y, z, \omega) = 0 \tag{2-3}
\]

\[
\nabla V(x, y, z, \omega) + j\omega \kappa(z) P(x, y, z, \omega) = 0, \tag{2-4}
\]

and in the wavenumber-frequency domain

\[
\begin{bmatrix}
-j k_x \tilde{P}(k_x, k_y, z, \omega) \\
-j k_y \tilde{P}(k_x, k_y, z, \omega) \\
\partial_z \tilde{P}(k_x, k_y, z, \omega)
\end{bmatrix} = -j \omega \rho(z) \begin{bmatrix}
\tilde{V}_x(k_x, k_y, z, \omega) \\
\tilde{V}_y(k_x, k_y, z, \omega) \\
\tilde{V}_z(k_x, k_y, z, \omega)
\end{bmatrix}, \tag{2-5}
\]

\[-j k_x \tilde{V}_x(k_x, k_y, z, \omega) - j k_y \tilde{V}_y(k_x, k_y, z, \omega) + \partial_z \tilde{V}_z(k_x, k_y, z, \omega) = -j \omega \kappa(z) \tilde{P}(k_x, k_y, z, \omega). \tag{2-6}\]

To reduce the notations, we omit \(k_x, k_y\) and \(\omega\) from the notations. Taking into account the following relationships and considering only homogenous propagating waves,

\[
k^2_z(z) = \frac{\omega^2}{c(z)^2} - k^2_x - k^2_y, \quad \text{for} \quad k^2_x + k^2_y \leq k^2_z(z) \tag{2-7a}
\]

\[
c(z)^2 = \frac{1}{\kappa(z) \rho(z)}. \tag{2-7b}
\]

We can reach to a first-order two-way wave equation by eliminating \(\tilde{V}_x(z)\) and \(\tilde{V}_y(z)\)

\[
\partial_z \begin{bmatrix}
\tilde{P}(z) \\
\tilde{V}_z(z)
\end{bmatrix} = \begin{bmatrix}
0 & -j \omega \rho(z) \\
k^2_z(z) & 0
\end{bmatrix} \begin{bmatrix}
\tilde{P}(z) \\
\tilde{V}_z(z)
\end{bmatrix}. \tag{2-8}
\]

In compact notations, we can written it as

\[
\partial_z \tilde{Q}(z) = \tilde{A}(z) \tilde{Q}(z). \tag{2-9}
\]

where \(\tilde{Q}\) contains the vertical component of the velocity and the acoustic pressure, which are continuous in a horizontally layered medium. Therefore, \(\tilde{Q}\) is continuous. The vertical derivative represents variations of the wavefields in the vertical direction.

### 2-1-2 One-way wavefields

To go from the two-way wavefields representation to one-way wavefields, we perform eigenvalue decomposition

\[
\tilde{A}(z) = \tilde{L}(z) \tilde{A}(z) \tilde{L}^{-1}(z), \tag{2-10}
\]

where \(\tilde{L}\) contains the vertical component of the velocity and the acoustic pressure, which are continuous in a horizontally layered medium.
where \( \tilde{L}(z) \) and \( \tilde{\Lambda}(z) \) contain the eigenvectors and the eigenvalues, respectively, which we find by solving the characteristic equation

\[
\det[\tilde{A} - \lambda I] = 0 \quad (2-11a)
\]

\[
\tilde{\Lambda}(z) = \begin{pmatrix} -jk_z(z) & 0 \\ 0 & jk_z(z) \end{pmatrix} \quad (2-11b)
\]

\[
\tilde{L}(z) = \begin{pmatrix} k_z(z) & -k_z(z) \\ \omega \rho(z) & \omega \rho(z) \end{pmatrix} \quad (2-11c)
\]

\[
\tilde{L}^{-1}(z) = \begin{pmatrix} 1/2 & \frac{\omega \rho(z)}{2k_z(z)} \\ 1/2 & \frac{-\omega \rho(z)}{2k_z(z)} \end{pmatrix}. \quad (2-11d)
\]

Substituting equation (2-10) in the two-way wave equation results in

\[
\partial_z \tilde{Q}(z) = \tilde{L}(z) \tilde{\Lambda}(z) \tilde{L}^{-1}(z) \tilde{Q}(z). \quad (2-12)
\]

Next, we define a vector \( \tilde{D}(z) \) that contains the one-way up- and down-going wavefields such that

\[
\tilde{Q}(z) = \tilde{L}(z) \tilde{D}(z) \quad (2-13a)
\]

\[
\tilde{D}(z) = \tilde{L}^{-1}(z) \tilde{Q}(z) \quad (2-13b)
\]

and substitute it in equation (2-12) to get

\[
\partial_z [\tilde{L}(z) \tilde{D}(z)] = \tilde{L}(z) \tilde{\Lambda}(z) \tilde{D}(z) \quad (2-14a)
\]

\[
\tilde{D}(z) \partial_z \tilde{L}(z) + \tilde{L}(z) \partial_z \tilde{D}(z) = \tilde{L}(z) \tilde{\Lambda}(z) \tilde{D}(z) \quad (2-14b)
\]

\[
\tilde{L}^{-1}(z) \tilde{D}(z) \partial_z \tilde{L}(z) + \partial_z \tilde{D}(z) = \tilde{\Lambda}(z) \tilde{D}(z) \quad (2-14c)
\]

which we write as

\[
\partial_z \tilde{D}(z) = \tilde{B}(z) \tilde{D}(z), \quad (2-15)
\]

with

\[
\tilde{B}(z) = \tilde{\Lambda}(z) - \tilde{L}^{-1}(z) \partial_z \tilde{L}(z). \quad (2-16)
\]

For a homogenous medium, \( \left( \partial_z \tilde{L}(z) = 0 \right) \), which reduces equation (2-16) to

\[
\partial_z \tilde{D}(z) = \tilde{\Lambda}(z) \tilde{D}(z). \quad (2-17)
\]

In this case, the up- and down-going wavefields are decoupled. By defining \( \tilde{D}(z) \) to represent the one-way acoustic pressure, equation (2-17) becomes

\[
\partial_z \begin{pmatrix} \tilde{P}^+(z) \\ \tilde{P}^-(z) \end{pmatrix} = \begin{pmatrix} -jk_z(z) & 0 \\ 0 & jk_z(z) \end{pmatrix} \begin{pmatrix} \tilde{P}^+(z) \\ \tilde{P}^-(z) \end{pmatrix}. \quad (2-18)
\]
2-2 Elastic case

We follow similar approach to the acoustic case to describe the theory of elastic wavefield decomposition. We also consider elastic horizontally layered medium where medium parameters vary with respect to depth only, the density \( \rho = \rho(z) \) and the stiffness tensor \( C_{ijkl} = C_{ijkl}(z) \).

2-2-1 Two-way wave equation

We derive the elastic two-way wave equation from the linearized equation of motion

\[
\rho \partial_t v_i - \partial_j \tau_{ij} = f_i
\]

and the linearized stress-strain relationship:

\[
\partial_t \tau_{ij} - C_{ijkl} \partial_l v_k = -\partial_t \sigma_{ij}.
\]

The stiffness tensor given by [Auld, 1973] for an isotropic medium reads

\[
C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}).
\]

Substituting this equation in the stress-strain equation results in

\[
\partial_t \tau_{ij} - [\lambda \delta_{ij} \partial_k v_k + \mu (\partial_j v_i + \partial_i v_j)] = -\partial_t \sigma_{ij}.
\]

In the wavenumber-frequency domain and for a source-free situation, equation (2-19) becomes

\[
j \omega \rho(z) \begin{pmatrix}
\tilde{V}_x(z) \\
\tilde{V}_y(z) \\
\tilde{V}_z(z)
\end{pmatrix} = \begin{pmatrix}
- j k_x \tilde{T}_{xx}(z) - j k_y \tilde{T}_{xy}(z) & \partial_z \tilde{T}_{xz}(z) \\
- j k_x \tilde{T}_{yx}(z) - j k_y \tilde{T}_{yy}(z) & \partial_z \tilde{T}_{yz}(z) \\
- j k_x \tilde{T}_{zx}(z) - j k_y \tilde{T}_{zy}(z) & \partial_z \tilde{T}_{zz}(z)
\end{pmatrix}
\]

and the components of equation (2-22) become

\[
\begin{align*}
 j \omega \tilde{T}_{xx} &= - [\lambda + 2\mu] j k_x \tilde{V}_x - \lambda j k_y \tilde{V}_y + \lambda \partial_z \tilde{V}_z \\
 j \omega \tilde{T}_{yy} &= - \lambda j k_x \tilde{V}_x - [\lambda + 2\mu] j k_y \tilde{V}_y + \lambda \partial_z \tilde{V}_z \\
 j \omega \tilde{T}_{zz} &= - \lambda j k_x \tilde{V}_x - \lambda j k_y \tilde{V}_y + [\lambda + 2\mu] \partial_z \tilde{V}_z \\
 j \omega \tilde{T}_{xy} &= j \omega \tilde{T}_{yx} = \mu [\partial_z \tilde{V}_x - j k_x \tilde{V}_z] \\
 j \omega \tilde{T}_{yz} &= j \omega \tilde{T}_{zy} = \mu [\partial_z \tilde{V}_y - j k_y \tilde{V}_z] \\
 j \omega \tilde{T}_{zx} &= j \omega \tilde{T}_{xz} = \mu [- j k_y \tilde{V}_x - j k_x \tilde{V}_y]
\end{align*}
\]

Note that the z-dependency is still there, but we dropped it to reduce the notations. We eliminate \( \tilde{T}_x \) and \( \tilde{T}_y \) and obtain expressions for \( \partial_z \tilde{T}_z \) and \( \partial_z \tilde{V} \) to reach to a first order two-way wave equation

\[
\partial_z \tilde{Q}_p(z) = \tilde{A}_p(z) \tilde{Q}_p(z).
\]
where

\[ \tilde{Q}^p(z) = \begin{pmatrix} \tilde{V}_x(z) \\ \tilde{V}_y(z) \\ -\tilde{T}_{zz}(z) \\ -\tilde{T}_{xz}(z) \\ -\tilde{T}_{yz}(z) \\ \tilde{V}_z(z) \end{pmatrix} \]  

(2-26a)

\[ \tilde{A}^p(z) = \begin{pmatrix} 0 & \tilde{A}_{12}(z) \\ \tilde{A}_{21}(z) & 0 \end{pmatrix} \]  

(2-26b)

\[ \tilde{A}_{12} = \begin{pmatrix} -j\frac{\omega}{\mu} & 0 & jk_x \\ 0 & -j\frac{\omega}{\mu} & jk_y \\ jk_x & jk_y & -j\omega \rho \end{pmatrix} \]  

(2-26c)

\[ \tilde{A}_{12} = \begin{pmatrix} -j\omega \rho - \frac{1}{j\omega} [\alpha_1 k_x^2 + \mu k_y^2] & -\frac{1}{j\omega} [\alpha_2 k_x k_y] & \frac{\lambda}{\lambda + 2\mu} jk_x \\ -\frac{1}{j\omega} [\alpha_2 k_x k_y] & -j\omega \rho - \frac{1}{j\omega} [\mu k_x^2 + \alpha_1 k_y^2] & \frac{\lambda}{\lambda + 2\mu} jk_y \\ \frac{\lambda}{\lambda + 2\mu} jk_x & \frac{\lambda}{\lambda + 2\mu} jk_y & -j\omega \rho \end{pmatrix} \]  

(2-26d)

\[ \alpha_1 = 4\mu \left( \frac{\lambda + \mu}{\lambda + 2\mu} \right) \]  

(2-26e)

\[ \alpha_2 = \mu \left( \frac{3\lambda + 2\mu}{\lambda + 2\mu} \right) \]  

(2-26f)

Note the arrangement of \( \tilde{Q}^p \) and \( \tilde{A}^p \) such that it is possible to apply eigenvalue decomposition similar to the acoustic case. The original form of \( \tilde{Q} \) is

\[ \tilde{Q}(z) = \begin{pmatrix} -\tilde{T}_x(z) \\ \tilde{V}(z) \end{pmatrix} \]  

(2-27)

2-2-2 One-way wavefields

To have a representation of the one-way wavefields, we apply eigenvalue decomposition

\[ \tilde{A}^p(z) = \tilde{L}^p(z)\tilde{\Lambda}(z)\tilde{L}^p(z)^{-1} \]  

(2-28)

Solving the characteristic equation

\[ det[\tilde{A}^p - \lambda I] = 0 \]  

(2-29)

gives the following eigenvalues

\[ \tilde{\Lambda}(z) = \begin{pmatrix} -jk_{z,p}(z) & 0 & 0 & 0 & 0 & 0 \\ 0 & -jk_{z,s}(z) & 0 & 0 & 0 & 0 \\ 0 & 0 & -jk_{z,s}(z) & 0 & 0 & 0 \\ 0 & 0 & 0 & jk_{z,p}(z) & 0 & 0 \\ 0 & 0 & 0 & 0 & jk_{z,s}(z) & 0 \\ 0 & 0 & 0 & 0 & 0 & jk_{z,s}(z) \end{pmatrix} \]  

(2-30a)
The parameters (Lamé parameters with respect to velocity in an isotropic medium is which implies
\[ φ \text{ and } ψ \text{ represent the P- and S-wave potentials, } \varphi \text{ and } ψ, \text{ respectively.} \]

\[ \text{and the following eigenvectors} \]

\[ \mathbf{L}^p(z) = \begin{pmatrix} \tilde{L}^p_1(z) & \tilde{L}^p_2(z) & -\tilde{L}^p_2(z) \end{pmatrix}, \quad (2-31a) \]

\[ \text{where} \]

\[ \tilde{L}^p_1(z) = \begin{bmatrix} \ell_{1p} & \eta_1 + \eta_2 & \eta_1' + \eta_2' \\ k_y & k_x & -2k_x \mu \\
\frac{\mu}{k_x} [k_z^2 - 2k_x^2 - 2k_y^2] \ell_{1p} & \frac{\omega}{\eta_2} & \frac{\omega}{\eta_2'} \end{bmatrix}, \quad (2-31b) \]

and

\[ \tilde{L}^p_2(z) = \begin{bmatrix} \ell_{2p} & \xi_1 + \xi_2 & \xi_1' + \xi_2' \\ \frac{k_y}{k_x} & \xi_1 & -2k_x \xi_1' \\
\frac{\omega}{k_x} & \xi_2 & \frac{\omega}{\xi_2'} \quad (2-31c) \]

The parameters \((\ell_{1p}, \eta_1, \eta_2, \eta_1', \eta_1')\) and \((\ell_{2p}, \xi_1, \xi_2, \xi_1', \xi_1')\) are normalization parameters which vary depending on the desired one-way wavefields choice. We chose the desired wavefields to represent the P- and S-wave potentials, \(\varphi\) and \(ψ\), respectively. The definition for the so called Lamé parameters with respect to velocity in an isotropic medium is

\[ \partial_t \vec{v} = \frac{-1}{ρ} [\nabla \varphi + \nabla \times \vec{ψ}], \quad (2-32) \]

which implies

\[ -\rho \partial_t \vec{v}_p = \nabla \varphi \quad (2-33a) \]

\[ -\rho \partial_t \vec{v}_s = \nabla \vec{ψ}, \quad (2-33b) \]

where

\[ \vec{ψ} = (ψ_x, ψ_y, ψ_z)^T. \quad (2-33c) \]

Note that \(ψ_x\), \(ψ_y\) and \(ψ_z\) denote the S-wave potentials polarized in the \(y - z\), \(x - z\) and \(x - y\) planes, respectively. The eigenvectors for the desired one-way wavefields become

\[ \tilde{L}^p_1(z) = \frac{\mu}{\omega^2 ρ} \begin{bmatrix} \omega k_x' & -\omega k_x k_y & -\omega (k_z^2 - k_x^2) \\ \mu & \omega k_y & \omega (k_z^2 - k_y^2) \\ k_z^2 - 2k_x^2 - 2k_y^2 & -2k_y k_z & 2k_x k_z \end{bmatrix}, \quad (2-34a) \]
and

$$\tilde{L}_2^p(z) = \frac{\mu}{\omega^2 \rho} \begin{bmatrix} 2k_z k_{z,p} & -2k_z k_y & -(k_s^2 - 2k_y^2) \\ 2k_z k_{z,p} & k_z^2 - 2k_y^2 & 2k_z k_y \\ \frac{\omega_k z_p}{\mu} & \frac{\omega_k y}{\mu} & \frac{\omega_z}{\mu} \end{bmatrix}.$$ \hspace{1cm} (2-34b)

Analogous to the acoustic case, define a vector $\tilde{D}^p$ that contains a permutation of the one-way up- and down-going wavefields such that

$$\tilde{Q}^p(z) = \tilde{L}^p(z) \tilde{D}^p(z) \hspace{1cm} (2-35)$$

$$\begin{pmatrix} \tilde{V}_x(z) \\ \tilde{V}_y(z) \\ -T_{zz}(z) \\ -T_{xz}(z) \\ -T_{yz}(z) \\ \tilde{V}_z(z) \end{pmatrix} = \begin{pmatrix} \tilde{L}_1^p(z) & \tilde{L}_1^p(z) & \tilde{L}_1^p(z) & \tilde{L}_2^p(z) & \tilde{L}_2^p(z) & \tilde{L}_2^p(z) \end{pmatrix} \begin{pmatrix} \tilde{\Phi}^+(z) \\ \tilde{\Psi}^+(z) \\ \tilde{\Psi}^+(z) \\ \tilde{\Phi}^-(z) \\ -\tilde{\Psi}^+(z) \\ -\tilde{\Psi}^+(z) \end{pmatrix}. \hspace{1cm} (2-36)$$

From the arrangement of equation (2-36), it is possible to write $\tilde{L}^p$ in the following notation

$$\tilde{L}^p(z) = \begin{pmatrix} \tilde{L}_1^{+p}(z) & \tilde{L}_1^{-p}(z) \\ \tilde{L}_2^{+p}(z) & \tilde{L}_2^{-p}(z) \end{pmatrix}, \hspace{1cm} (2-37)$$

such that $[\tilde{L}_1^{+p} = \tilde{L}_1^p]$ and $[\tilde{L}_2^{-p} = \tilde{L}_2^p(-k_z, s)]$ form the down-going wavefield composition sub matrices and $[\tilde{L}_1^{-p} = \tilde{L}_1^p]$ and $[\tilde{L}_2^{+p} = \tilde{L}_2^p(-k_z, p)]$ form the up-going wavefield composition sub matrices. The vertical wavenumber changes to negative for the up-going wavefield composition sub matrices to match the sign convention, the positive vertical wavenumber corresponds to the down-going wavefield and the negative vertical wavenumber corresponds to the up-going wavefield.

We return back to use the original form of $\tilde{Q}$ and arrange the matrices accordingly

$$\tilde{Q}(z) = \begin{pmatrix} -\tilde{T}_z(z) \\ \tilde{V}(z) \end{pmatrix} = \begin{pmatrix} \tilde{L}_1^{+}(z) \\ \tilde{L}_1^{-}(z) \end{pmatrix} \tilde{D}^+(z) + \begin{pmatrix} \tilde{L}_1^{+}(z) \\ \tilde{L}_1^{-}(z) \end{pmatrix} \tilde{D}^-(z) = \begin{pmatrix} \tilde{\Phi}^+(z) \\ \tilde{\Psi}^+(z) \\ \tilde{\Psi}^+(z) \\ \tilde{\Phi}^-(z) \\ -\tilde{\Psi}^+(z) \\ -\tilde{\Psi}^+(z) \end{pmatrix} = \tilde{L}(z) \tilde{D}(z), \hspace{1cm} (2-38)$$

where

$$\tilde{L}_1^{+}(z) = \frac{\mu}{\omega^2 \rho} \begin{bmatrix} \pm 2k_z k_{z,p} & -2k_z k_y & -(k_s^2 - 2k_y^2) \\ \pm 2k_y k_{z,p} & k_s^2 - 2k_y^2 & 2k_z k_y \\ k_s^2 - 2k_x^2 - 2k_y^2 & \mp 2k_y k_{z,s} & \pm 2k_x k_{z,s} \end{bmatrix}. \hspace{1cm} (2-39a)$$
and

\[
\tilde{L}^\pm_2(z) = \frac{1}{\omega} \begin{bmatrix}
k_x & \pm k_x k_y & \pm \frac{k_y^2 - k_x^2}{k_z s} \\
-\frac{k_y^2 - k_x^2}{k_z s} & k_y & \pm \frac{k_x k_y}{k_z} \\
\pm k_{z,p} & 0 & -k_y
\end{bmatrix}.
\]

(2-39b)

The decomposition equation reads

\[
\tilde{D}(z) = \tilde{L}^{-1}(z)\tilde{Q}(z) = \tilde{N}(z)\tilde{Q}(z),
\]

(2-40)

where

\[
\tilde{N}^\pm_1(z) = \frac{1}{2} \begin{bmatrix}
\pm \frac{k_x}{k_{z,p}} & \pm \frac{k_y}{k_{z,p}} & 1 \\
0 & 1 & \pm \frac{k_y}{k_{z,s}} \\
-1 & 0 & \pm \frac{k_x}{k_{z,s}}
\end{bmatrix}
\]

(2-41a)

and

\[
\tilde{N}^\pm_2(z) = \frac{\mu}{2\omega} \begin{bmatrix}
2k_x & 2k_y & \pm \frac{k_y^2 - 2k_x^2 - 2k_y^2}{k_{z,p}} \\
\pm \frac{k_y^2 - 2k_x^2}{k_z s} & \pm \frac{k_x k_y}{k_z s} & 2k_x \\
\pm \frac{k_x^2 - k_y^2}{k_z s} & \pm \frac{k_y k_z}{k_z s} & -2k_y
\end{bmatrix}.
\]

(2-41b)

The derivation details of the composition/decomposition operators can also be found in [Kennett, 1983]. Following the same steps as in the acoustic case, we reach to the one-way wave equation

\[
\partial_z \tilde{D}(z) = \tilde{B}(z)\tilde{D}(z),
\]

(2-42)

with

\[
\tilde{B}(z) = \tilde{A}(z) - \tilde{N}(z) \partial_z \tilde{L}(z).
\]

(2-43)

For a homogenous medium \( \left( \partial_z \tilde{L}(z) = 0 \right) \) and

\[
\partial_z \tilde{D}(z) = \tilde{A}(z)\tilde{D}(z).
\]

(2-44)

In this case, the up- and down-going wavefields are again decoupled.

### 2-3 Wavefield decomposition at the ocean bottom

At the ocean bottom, \( z = z_1 \) the following boundary conditions apply:

1. \(-\tau_{zz}(z_1) = p(z_1)\);
2. \(\tau_{xz}(z_1) = \tau_{yz}(z_1) = 0\).
2-3 Wavefield decomposition at the ocean bottom

2-3-1 Acoustic decomposition

Equation (2-13) defines the relationship between the two-way wavefields in $\tilde{Q}(z)$ and the one-way wavefields in $\tilde{D}(z)$. They are related by the composition matrix $\tilde{L}(z)$, which composes the total wavefields from the one-way wavefields.

$$
\begin{pmatrix}
\tilde{P}^+(z_1) \\
\tilde{V}^+(z_1)
\end{pmatrix} =
\begin{pmatrix}
1 & 1 \\
\omega\rho_0(z_1) & \omega\rho_0(z_1)
\end{pmatrix}
\begin{pmatrix}
\tilde{P}^+(z_1) \\
\tilde{P}^-(z_1)
\end{pmatrix}.
$$

(2-45)

Using the decomposition matrix $\tilde{N}(z)$, we can decompose the total wavefield to the one-way wavefields

$$
\begin{pmatrix}
\tilde{P}^+(z_1) \\
\tilde{P}^-(z_1)
\end{pmatrix} =
\begin{pmatrix}
1/2 & \omega\rho_0(z_1) \\
1/2 & 2k_{z,0}(z_1)
\end{pmatrix}
\begin{pmatrix}
\tilde{P}^+(z_1) \\
\tilde{V}^+(z_1)
\end{pmatrix},
$$

(2-46)

and in compact notation this reads

$$
\tilde{P}^\pm(z_1) = \frac{1}{2} \tilde{P}(z_1) \pm \frac{\omega\rho_0(z_1)}{2k_{z,0}(z_1)} \tilde{V}^+(z_1).
$$

(2-47)

This equation shows that by scaling the frequency-wavenumber transformed acoustic pressure and vertical velocity component measurements, it is possible to decompose the total wavefield to up- and down-going wavefields just above the ocean bottom ($z_1 = 0$, $\epsilon \to 0$). Since decomposition is done just above the ocean bottom, it only requires the properties of the ocean ($\rho_0(z_1), k_{z,0}(z_1)$).

2-3-2 Elastic decomposition

For the elastic decomposition, we consider waves that are polarized in the $(x-z)$ plane where $k_y = 0$. We also neglect SH waves and take into account only P-SV waves. Hence, the composition and decomposition equations become

$$
\begin{pmatrix}
-\tilde{T}_{xz}(z_1) \\
-\tilde{T}_{zz}(z_1) \\
\tilde{V}_x(z_1) \\
\tilde{V}_z(z_1)
\end{pmatrix} =
\begin{pmatrix}
\tilde{L}_{1,p-st}^+(z_1) & \tilde{L}_{1,p-st}^-(z_1) \\
\tilde{L}_{2,p-st}^+(z_1) & \tilde{L}_{2,p-st}^-(z_1)
\end{pmatrix}
\begin{pmatrix}
\tilde{\Phi}^+(z_1) \\
\tilde{\Phi}^-(z_1)
\end{pmatrix},
$$

(2-48)

and

$$
\begin{pmatrix}
\tilde{\Phi}^+(z_1) \\
\tilde{\Phi}^-(z_1) \\
\tilde{\Psi}^+(z_1) \\
\tilde{\Psi}^-(z_1)
\end{pmatrix} =
\begin{pmatrix}
\tilde{N}_{1,p-st}^+(z_1) & \tilde{N}_{1,p-st}^-(z_1) \\
\tilde{N}_{2,p-st}^+(z_1) & \tilde{N}_{2,p-st}^-(z_1)
\end{pmatrix}
\begin{pmatrix}
-\tilde{T}_{xz}(z_1) \\
-\tilde{T}_{zz}(z_1) \\
\tilde{V}_x(z_1) \\
\tilde{V}_z(z_1)
\end{pmatrix},
$$

(2-49)

where

$$
\tilde{L}_{1,p-st}^\pm(z_1) = \frac{\mu}{\omega^2 \rho}
\begin{pmatrix}
\pm 2k_x k_{x,0} & -(k_x^2 - 2k_z^2) \\
(k_x^2 - 2k_z^2) & \pm 2k_x k_{z,0}
\end{pmatrix}.
$$

(2-50a)
\[ \tilde{L}_{x,p-sv}(z_1) = \frac{1}{\omega \rho} \begin{bmatrix} k_x & \pm k_{z,s} \\ \pm k_{z,p} & k_x \end{bmatrix} \]  

and

\[ \tilde{N}_{1, p-sv}(z_1) = \frac{1}{2} \begin{bmatrix} \pm \frac{k_x}{k_{z,p}} & 1 \\ -1 & \pm \frac{k_x}{k_{z,s}} \end{bmatrix} \]  

\[ \tilde{N}_{2, p-sv}(z_1) = \frac{\mu}{2\omega} \begin{bmatrix} 2k_x & \pm \frac{k_s^2 - 2k_x^2}{k_{z,p}} \\ \pm \frac{k_s^2 - k_x^2}{k_{z,s}} & 2k_x \end{bmatrix} \]  

Using the OBC measured data \((p(z_1), v_x(z_1), v_z(z_1))\), and taking into account the boundary conditions, the total wavefield can be decomposed into up- and down-going P- and S-waves, just below the ocean bottom. Hence, the decomposition and composition matrices depend on the properties of the layer just below the receivers.

### 2-4 Synthetic data

We show synthetic data to demonstrate the wavefield decomposition theory. The receivers measure the acoustic pressure \((p)\), vertical particle velocity \((v_z)\) and the horizontal particle velocity \((v_x)\). They are placed at the ocean bottom \((z_1 = 500 \text{ m})\). The source is a dipole P-wave zero-phase wavelet positioned at the water surface \((z = 0)\). The medium below the ocean bottom consists of a gradient mimicked by multi thin layers to resemble the earth’s subsurface where velocities and density often increase gradually close the sea bottom. Below that, there is a horizontally layered medium with sharp boundaries. We performed 1.5 D elastic modeling on the provided density, P and S velocity logs, Figure 2-1. We used the reflectivity method [Müller, 1985] to compute the synthetic data in the wavenumber-frequency domain. The synthetic data includes the primaries as well as the surface and internal multiples, Figure 2-2.

#### 2-4-1 Wavefield decomposition

Applying equation (2-47) to the acoustic pressure and the vertical component of the particle velocity gives the up- and down-going acoustic pressure just above the ocean bottom, Figure 2-3. \((p^+)\) does not contain the up-going primary reflections or any source-side multiples. On the other hand, \((p^-)\) contains the up-going wavefield, the direct arrival and any receiver-side multiple. This is because decomposition is performed just above the ocean bottom. Any down-going energy reflected from the ocean bottom is mapped into the up-going wavefield component, Figure 2-4.

Using the three components of the synthetic data, we apply equation (2-49) to decompose the data into up- and down-going P- and S-waves, Figure 2-5. Since the decomposition level is just below the ocean bottom, the up-going wavefields are free from the direct arrival and any receiver-side multiples.
2-4 Synthetic data

Figure 2-1: Logs used for creating synthetic data.
Figure 2-2: Synthetic ocean bottom data.
Figure 2-3: Decomposition of the total wavefield just above the ocean bottom to (a) up-going acoustic pressure and (b) down-going acoustic pressure.

Figure 2-4: Decomposition levels, just above the ocean bottom ($z_1 - \epsilon$) and just below the ocean bottom ($z_1 + \epsilon$).
Figure 2-5: Decomposition of the total wavefield just below the ocean bottom to (a) down-going P-waves, (b) up-going P-waves, (c) down-going S-waves and (d) up-going S-waves.
2-5 Decomposition with erroneous velocities

In order to get the true one-way wavefields based on the theory of elastic wavefield decomposition, the ocean floor parameters need to be exact. To test the sensitivity of decomposition to the ocean floor parameters, we apply decomposition to the synthetic data with erroneous velocities. Instead of using $c_p(1) = 1550\, \text{m/s}$ and $c_s(1) = 100\, \text{m/s}$, we first test decomposition with velocities 100 m/s higher than the true ones, see Figure 2-6. In this case, the up-going P- and S-waves start to contain some of the down-going energy of the direct arrival. This is reasonable since the amplitude of the direct arrival is relatively low at the apex compared with the other events. Increasing the error to 200 m/s leads to boosting the direct down-going arrival in the up-going sections, see Figure 2-7a and Figure 2-7b. Further increasing the velocity error to 400 m/s, see Figure 2-7c and Figure 3-6d, leads to interchanging of the up- and down-going events and almost no difference between the up-going wavefields and the total measurements. From here, we conclude the importance of a reasonable estimate of the near-bottom velocities, such that reliable one-way wavefields are obtained.
Figure 2-6: Elastic decomposition with 100 m/s error in velocities to (a) down-going P-waves, (b) up-going P-waves, (c) down-going S-waves and (d) up-going S-waves.
Figure 2-7: Elastic decomposition with (a-b) 200 m/s and (c-d) 400 m/s error in the velocities.
Since elastic wavefield decomposition at the ocean bottom requires properties of the ocean floor, an accurate near-ocean bottom model is essential. In this chapter, we perform wavefield tomography to build P- and S-wave velocity models of the near-ocean bottom layers. [Berkhout, 2012] proposed the Joint Migration Inversion scheme, which aims at estimating the velocity and reflectivity. Total wavefield tomography is the part of JMI that estimates the velocities. This approach does not require first arrivals picking as in travel time tomography, but rather finds a distribution of velocities and reflections such that the two-way modeled wavefield matches the observed data. The methodology was implemented by [Staal and Verschuur, 2013] and [Staal and Verschuur, 2014] on acoustic data considering the total wavefield. Like stated in the introduction, we assume that via the JMI method (or another approach), we have obtained the upgoing P- and S-waves at a known depth in the vicinity of the water bottom (e.g. few 100 meters below the bottom) and our task is to find the velocities of the layers between this depth and the bottom. With those estimated velocities, we compose the up-going P- and S-waves to form the modeled vertical and horizontal velocity components which can then be matched with the measurements. Since composition requires the down-going P- and S-waves as well, we express them in terms of the acoustic pressure measurement. To retrieve the velocities of the first layer below the ocean bottom, we apply another inversion loop that minimizes the mismatch between the measured data and the modeled data that we compose with the retrieved velocities from the tomography part. For both inversion loops, we apply the gradient descent method. All the derivations are in the rayparameter - frequency domain, and are based on a 1.5D model.

3-1 Wavefield tomography

3-1-1 Velocity perturbation theory

If the wavefield propagates between two levels, \( z_n \) and \( z_m \), in two different velocity models, the wavefields experience two different extrapolations. Assume that the velocity models form
the true and background velocity models with their corresponding extrapolation operators $\hat{W}^-$ and $\hat{W}_0^-$, respectively. The expression of the true extrapolation operator in terms of the background extrapolation operator reads

$$\hat{W}^- (p, z_m, z_n, \omega) = \hat{W}_0^- (p, z_m, z_n, \omega) + \Delta \hat{W}^- (p, z_m, z_n, \omega). \quad (3-1)$$

These extrapolation operators are phase shift operators expressed as

$$\hat{W}^- (p, z_m, z_n, \omega) = e^{-j\omega q \Delta z}, \quad (3-2)$$

where $q$ is dependent on considering P- or S-wave propagation:

$$q_p = \sqrt{\frac{1}{c_p^2} - p^2}, \quad q_s = \sqrt{\frac{1}{c_s^2} - p^2}. \quad (3-3)$$

For the rest of the derivations, we drop the rayparameter $p$ and $\omega$ from the equations for notations reduction. The subscript $p$ is not to be confused with the rayparameter $p$, the earlier denotes P-wave.

Define $\beta$ to be the P- and S-wave velocity contrasts at each depth level

$$\beta_p(z) = 1 - \frac{c_0(z)^2}{c_p(z)^2}, \quad \beta_s(z) = 1 - \frac{c_0(z)^2}{c_s(z)^2}. \quad (3-4)$$

Perform Taylor series expansion for $\hat{W}$ at $\beta = 0$ and ignore higher order terms:

$$\hat{W}^- (z_m, z_n) \approx \hat{W}_0^- (z_m, z_n) + \frac{\partial \hat{W}_0^- (z_m, z_n)}{\partial \beta} \beta(z) + ... \quad (3-5a)$$

$$\Delta \hat{W}^- (z_m, z_n) \approx \frac{\partial \hat{W}_0^- (z_m, z_n)}{\partial \beta} \beta(z) \quad (3-5b)$$

$$\Delta \hat{W}^- (z_m, z_n) \approx \hat{G}_0^- (z_m, z_n) \beta(z) \quad (3-5c)$$

with

$$\hat{G}_0^- (z_m, z_n) = j\omega \Delta z \frac{q_0^2}{2q} e^{-j\omega q \Delta z} \quad (3-6)$$

$$q = \sqrt{q_0^2(1 - \beta(z)) - p^2} \quad (3-7)$$

A stabilization parameter $\epsilon$ is necessary for $90^\circ$ angle of propagation, where $\hat{G}_0$ becomes undefined, ($q = 0$).

$$\hat{G}_0^- (z_m, z_n) \approx \frac{q^2}{2q} e^{-j\omega q \Delta z}. \quad (3-8)$$

From expressions (3-1) and (3-5c), it can be seen that it is possible to retrieve the true extrapolation operator from the known background extrapolation operator and the linearized difference extrapolation operator. Note that $\beta = 0$ and $\hat{W} = \hat{W}_0$ is only valid when the background model matches the true model, $c_0 = c$. This implies that the linearization of the difference extrapolation operator requires a good initial model that is close enough to the true model.

August 13, 2014
Define $\tilde{\Phi}_{p}^{-}(z_{n})$ and $\tilde{\Phi}_{s}^{-}(z_{n})$ to be the upgoing wavefields at a certain depth level. These upgoing wavefields are the assumed-known $\tilde{\Phi}_{p}^{-}(z_{N+1})$ and $\tilde{\Phi}_{s}^{-}(z_{N+1})$, forward extrapolated with $W_{0}^{-}(z_{n}, z_{N+1})$. This is just an assumption to reduce the wavefield tomography problem to a subset of a larger scheme, in which the up-going wavefields are actually estimated. The cumulative effect of the velocity perturbations from all the depth levels on the upgoing wavefields recorded at the ocean bottom ($z_{1}$) are given by

$$\Delta \tilde{\Phi}(z_{1}) = \sum_{n=2}^{N} \tilde{G}_{p0}^{-}(z_{1}, z_{n}) \beta_{p}(z_{n}) \tilde{\Phi}_{p}^{-}(z_{n}), \quad (3-9a)$$

$$\Delta \tilde{\Psi}(z_{1}) = \sum_{n=2}^{N} \tilde{G}_{s0}^{-}(z_{1}, z_{n}) \beta_{s}(z_{n}) \tilde{\Phi}_{s}^{-}(z_{n}), \quad (3-9b)$$

where

$$\tilde{G}_{p0}^{-}(z_{1}, z_{n}) = W_{p0}^{-}(z_{1}, z_{n-1}) \tilde{\Phi}_{p0}^{-}(z_{n-1}, z_{n}), \quad (3-9c)$$

$$\tilde{G}_{s0}^{-}(z_{1}, z_{n}) = W_{s0}^{-}(z_{1}, z_{n-1}) \tilde{\Phi}_{s0}^{-}(z_{n-1}, z_{n}), \quad (3-9d)$$

and

$$\tilde{\Phi}_{p}^{-}(z_{n}) = W_{p0}^{-}(z_{n}, z_{N+1}) \tilde{\Phi}_{p}^{-}(z_{N+1}), \quad (3-9e)$$

$$\tilde{\Phi}_{s}^{-}(z_{n}) = W_{s0}^{-}(z_{n}, z_{N+1}) \tilde{\Phi}_{s}^{-}(z_{N+1}). \quad (3-9f)$$

Note that $\tilde{\Psi} = \tilde{\Psi}_{y}$ is the S-wave potential polarized in the ($x-z$) plane. An illustration of the depth levels to update is shown in Figure 3-1. Note that we assumed correct velocity between depth level $z_{N} + 1$ and $z_{N}$. The tomography takes place from depth levels $z_{N}$ to $z_{1}$.

Assume that there is a velocity contrast $\beta$ between the background and the true velocity models, then equation (3-9) can predict the difference between the two wavefields propagating in each model. This estimated residual might be different from the actual residual in the case that the true wavefield experience multiple reflections that are not experienced by the estimated wavefield. This is because the estimated wavefield takes into account only the transmission effects.

### 3-1-2 Tomographic Inversion

In order to find an estimated velocity model of the near ocean bottom, we implement the gradient descent method. The method minimizes the difference between the measured wavefield and the modeled wavefield in the estimated model in a least-squares sense. The wavefields represent the decomposed P- and S-wave potentials which we use to retrieve the P- and S-wave velocity profiles, independently.

The two objective functions are

$$J_{p} = \sum_{p1}^{p2} \sum_{\omega_{1}}^{\omega_{2}} ||\tilde{\Phi}_{obs}^{-}(z_{1}) - \tilde{\Phi}_{mod}^{-}(z_{1})||^{2} = \sum_{p1}^{p2} \sum_{\omega_{1}}^{\omega_{2}} ||\Delta \tilde{\Phi}^{-}(z_{1})||^{2} \quad (3-10a)$$

$$J_{s} = \sum_{p1}^{p2} \sum_{\omega_{1}}^{\omega_{2}} ||\tilde{\Psi}_{obs}^{-}(z_{1}) - \tilde{\Psi}_{mod}^{-}(z_{1})||^{2} = \sum_{p1}^{p2} \sum_{\omega_{1}}^{\omega_{2}} ||\Delta \tilde{\Psi}^{-}(z_{1})||^{2}. \quad (3-10b)$$
Substitute equations (3-9a) and (3-9b) in the objective functions and take the derivative with respect to $\beta$ to get the following gradients

$$
\nabla \beta_p(z_n) = \sum_{p_1}^{p_2} 2[\tilde{G}_{p0}(z_1, z_n)]^* \Delta \tilde{\Phi}^-(z_1)[\tilde{Q}_p^-(z_n)]^* 
$$

(3-11a)

$$
\nabla \beta_s(z_n) = \sum_{p_1}^{p_2} 2[\tilde{G}_{s0}(z_1, z_n)]^* \Delta \tilde{\Psi}^-(z_1) [\tilde{Q}_s^-(z_n)]^*. 
$$

(3-11b)

Note that $\nabla \beta(z_n)$ contains the velocity contrasts based on wavefields that propagate with a range of angles, $p$ values. The angle-dependent contributions at a certain depth level are combined to result in one $\nabla \beta(z_n)$ describing the velocity variations as a function of depth.

Using the computed gradients, we predict the wavefield perturbations for each $p$ value

$$
\Delta \tilde{\Phi}_{\nabla \beta}(z_1) = \sum_{n=2}^{N} \tilde{G}_{p0}(z_1, z_n) \nabla \beta_p(z_n) \tilde{Q}_p^-(z_n) 
$$

(3-12a)

$$
\Delta \tilde{\Psi}_{\nabla \beta}(z_1) = \sum_{n=2}^{N} \tilde{G}_{s0}(z_1, z_n) \nabla \beta_s(z_n) \tilde{Q}_s^-(z_n). 
$$

(3-12b)

Now, we can update the velocity contrasts $\beta$

$$
\beta_p(z)_{\text{new}} = \beta_p(z)_{\text{old}} + \alpha_p \nabla \beta_p(z) 
$$

(3-13a)

$$
\beta_s(z)_{\text{new}} = \beta_s(z)_{\text{old}} + \alpha_s \nabla \beta_s(z), 
$$

(3-13b)

by matching the predicted wavefield updates, $\Delta \tilde{\Phi}_{\nabla \beta}$ and $\Delta \tilde{\Psi}_{\nabla \beta}$ with the observed data mismatches, $\Delta \tilde{\Phi}^-$ and $\Delta \tilde{\Psi}^-$ and by finding the corresponding scale factors $\alpha_p$ and $\alpha_s$. In fact, $\alpha_p$ and $\alpha_s$ are calculated by solving
\[ \min (||\Delta \tilde{\Phi} - \alpha p \Delta \tilde{\Phi} \nabla \beta p||^2) \] 

(3-14)

\[ \min (||\Delta \tilde{\Psi} - \alpha s \Delta \tilde{\Psi} \nabla \beta s||^2) \] 

(3-15)

after which with the new \( \beta \)'s, the update of the velocities can be represented as:

\[ c_p(z)^{\text{new}} = \frac{c_p(z)^{\text{old}}}{\sqrt{1 - \alpha p \nabla \beta_p(z)}} \] 

(3-16a)

\[ c_s(z)^{\text{new}} = \frac{c_s(z)^{\text{old}}}{\sqrt{1 - \alpha s \nabla \beta_s(z)}} \] 

(3-16b)

In summary, the inversion loop that minimizes the difference between the true and the predicted decomposed data by estimating the P and S velocities is as follows:

1. Forward model \( \tilde{\Phi}^{\text{mod}}(z_1) \) and \( \tilde{\Psi}^{\text{mod}}(z_1) \) in the initial/current velocity models.
2. Compute the gradients.
3. Compute the effect of the gradients (prediction of the residual).
4. Compute the step lengths \( \alpha \).
5. Update the velocities.
6. Repeat steps 1 to 5 until the error reaches to an absolute minimum or the maximum number of iterations is reached.

3-1-3 Examples

To test the wavefield tomography method, we apply it to multi-component ocean bottom data. The modeled data consists of the up-going P- and S-waves at depth level \( z = 300 \) m. They are modeled in the tau-p domain considering a 1.5D elastic model. In this domain, each slowness value (p) corresponds to a certain angle of propagation of a plane wave. The synthetic data was computed based on a horizontally layered Earth, where the medium parameter vary with respect to depth only. We create a velocity gradient model which resembles a true subsurface. The measurements, the modeled data and the residual before applying wavefield tomography are shown in Figure 3-2. The mismatch between the measurements and the modeled data with the initial velocity models is clear. At this stage, the residual is in the same order of the measurements. Note that we only show a portion of the data for illustration purposes. The complete data set is shown in the appendix.

Applying wavefield tomography following equations (3-10) to (3-16) gives the results shown in Figure 3-3. Note the great improvement of the predicted data, to the point that the residual is almost negligible compared with the measurements’ amplitudes. The retrieved velocity models, that minimized the objective functions \( J_p \) and \( J_s \), are smooth.
representations of the true velocities, see Figure 3-4.

In Figure 3-5, the behaviors of the objective functions are shown. Conversion of the objective functions to absolute minima takes place at the 8th iteration for P-waves and at the 5th iteration for S-waves. This is explained by looking at the initial error, which is higher for the P-waves due to the larger velocity difference between the true and initial P-wave velocity models. The initial S-wave velocity model was chosen to be close to the true velocity model since S-wave velocities are usually very small in the near-bottom and their corresponding two-way travel times are relatively large. If the initial model of the S-wave velocity is not close enough to the true model, cycle skipping may occur and wavefield tomography will not be able to retrieve a reliable velocity model. Hence, careful design of the initial S-wave velocity model should be considered. Another option to reduce the risk of getting trapped in a local minimum is to start the tomography process with low frequencies first, so that cycle skipping is avoided.

The argument that the estimated velocity model is not exactly the same as the true velocity model is valid. This is because the utilized data in the tomography process does not contain any reflection information from the near-bottom layers. Better velocity estimations can be found if the primary reflections from within the near-bottom layer were incorporated. This is justified by the higher information content of the reflected waves which propagate more in the subsurface with different angles. In addition, adding multiples should provide more detailed velocities as shown by [Staal and Verschuur, 2013] for the acoustic case.

### 3-2 Wavefield composition

Since the retrieved velocities of the ocean floor did not exactly match the actual velocities, we propose in this section another inversion loop that results in a better estimate of the seafloor velocities.

The elastic composition equation, (2-48), reads in the rayparameter-frequency domain

\[
\begin{pmatrix}
-\tilde{T}_{xz}^2(z_1) \\
-\tilde{T}_{zz}^2(z_1)
\end{pmatrix} = c_s^2(z_1) \begin{pmatrix}
2 pq p(z_1) & -(c_s^{-2}(z_1) - 2p^2) \\
-c_s^{-2}(z_1) - 2p^2 & 2 pq_s(z_1)
\end{pmatrix} \begin{pmatrix}
\tilde{\Phi}^+(z_1) \\
\tilde{\Psi}^y(z_1)
\end{pmatrix} + c_s^2(z_1) \begin{pmatrix}
-2 pq p(z_1) & -(c_s^{-2}(z_1) - 2p^2) \\
c_s^{-2}(z_1) - 2p^2 & -2 pq_s(z_1)
\end{pmatrix} \begin{pmatrix}
\tilde{\Phi}^-(z_1) \\
\tilde{\Psi}^{-y}(z_1)
\end{pmatrix},
\]

\[\text{(3-17a)}\]

\[\begin{pmatrix}
\tilde{V}_x(z_1) \\
\tilde{V}_z(z_1)
\end{pmatrix} = \frac{1}{\rho(z_1)} \begin{pmatrix}
p & -q_s(z_1) \\
q_p(z_1) & p
\end{pmatrix} \begin{pmatrix}
\tilde{\Phi}^+(z_1) \\
\tilde{\Psi}^y(z_1)
\end{pmatrix} \]

\[+ \frac{1}{\rho(z_1)} \begin{pmatrix}
p & +q_s(z_1) \\
-q_p(z_1) & p
\end{pmatrix} \begin{pmatrix}
\tilde{\Phi}^-(z_1) \\
\tilde{\Psi}^{-y}(z_1)
\end{pmatrix}.
\]

\[\text{(3-17b)}\]

In order to compose \(\tilde{V}_x\) and \(\tilde{V}_z\) from their decomposed components using equation (3-17b), it is required to have the 4 potentials as well as the ocean floor parameters. In the tomography process, we only utilized the up-going P- and S-waves to estimate their velocity profiles. For
(a) $\varphi_{\text{true}}^{-}$ and $\varphi_{\text{mod}}^{-}$

(b) $\psi_{\text{true}}^{-}$ and $\psi_{\text{mod}}^{-}$

(c) $\Delta \varphi_{\text{true}}^{-}$

(d) $\Delta \psi_{\text{true}}^{-}$

**Figure 3-2:** Magnified view of the up-going measurements (blue), predicted (red) and residual (black), before applying wavefield tomography. Note that the up-going P- and S-wave residuals are computed using only one trace ($p = 0.2622 \times 10^{-3}$ s/m), for the sake of comparison.
Figure 3-3: The measurements (blue), predicted up-going wavefields (red) and residual (black), after applying wavefield tomography. Note how well the predicted up-going wavefields match the measured up-going wavefields. The trace we used to show the residual corresponds to ($p = 0.2622 \times 10^{-3}$ s/m).
Figure 3-4: The true (blue), estimated (red) and initial (green) velocity models for (a) P-wave and (b) S-wave. Note that these are also magnified velocity models to show the update levels.

Figure 3-5: Plot of $J_p$ (blue) and $J_s$ (red), versus the number of iterations.
Near-bottom velocity estimation

In the sake of simplicity, we assume a known density profile. Using equation (3-17a) with the assumption that the acoustic pressure component \(-\tilde{T}_{zz}(z_1)\) is known, we compute the down-going P- and S-waves in terms of the up-going P- and S-waves and the acoustic pressure, to get

\[
\tilde{\Psi}^+ = \left[ \frac{-\tilde{T}_{zz}}{Ac_s^2} - 2\tilde{\Phi}^- - \frac{A}{2pq_p} \tilde{\Psi}^- + \frac{2pq_s}{A} \tilde{\Psi}^- \right] \left[ \frac{A^2 + 4p^2q_sq_p}{2Apq_p} \right]^{-1}
\]

(3-18a)

\[
\tilde{\Phi}^+ = \left[ A\tilde{\Psi}^+ + 2pq_p \tilde{\Phi}^- + A\tilde{\Psi}^- \right] [2pq_p]^{-1},
\]

(3-18b)

where

\[
A = c_s^{-2} - 2p^2.
\]

(3-18c)

For waves propagating at normal incidence and at 90°, where \(p = 0\) and \(q = 0\), the expressions become unstable. Hence, we use a stabilization factor \(\epsilon\) to get

\[
\tilde{\Psi}^+ = \left[ \frac{-\tilde{T}_{zz}}{Ac_s^2} - 2\tilde{\Phi}^- - \left( \frac{A}{2p^2 + \epsilon^2 q_p^2 + \epsilon^2} \right) \frac{q_p}{Apq_p} \right] \tilde{\Psi}^- \left[ \frac{A^2 + 4p^2q_sq_p}{2Apq_p} \right]^{-1}
\]

(3-19a)

\[
\tilde{\Phi}^+ = \left[ A\tilde{\Psi}^+ + 2pq_p \tilde{\Phi}^- + A\tilde{\Psi}^- \right] \left[ \frac{1}{2p^2 + \epsilon^2 q_p^2 + \epsilon^2} \right].
\]

(3-19b)

This means that we can now relate the calculated up-going wavefields \(\tilde{\Phi}^-(z_1)\) and \(\tilde{\Psi}^-(z_1)\) to the measured particle velocities, \(\tilde{V}_x(z_1)\) and \(\tilde{V}_z(z_1)\).

### 3-2-1 Composition/decomposition Inversion

In this inversion loop, we minimize the difference between the observed and modeled vertical and horizontal velocity components in a least-squares sense, using the gradient descent method. The true data are obtained from the observations in the true model, while the modeled data are composed with tomographic estimated velocities of the ocean floor. Hence, the error is converted to an update to the ocean floor velocities. The objective function reads

\[
J = ||\tilde{V}_{xobs}(z_1) - \tilde{V}_{xmod}(z_1)||^2 + ||\tilde{V}_{zobs}(z_1) - \tilde{V}_{zmod}(z_1)||^2.
\]

(3-20)

The analytical gradients of \(J\) with respect to \(c_p\) and \(c_s\) are cumbersome to compute and require linearization. Therefore, we estimate the derivatives numerically using the central finite difference approximation,

\[
\nabla J_p = \frac{J(c_p + \Delta c_p) - J(c_p - \Delta c_p)}{2\Delta c_p}
\]

(3-21a)

\[
\nabla J_s = \frac{J(c_s + \Delta c_s) - J(c_s - \Delta c_s)}{2\Delta c_s}.
\]

(3-21b)
We update the ocean floor velocities with the following equations

\[
\begin{align*}
    c_p(z_1)^{new} &= c_p(z_1)^{old} + \gamma_p \Delta c_p \\
    c_s(z_1)^{new} &= c_s(z_1)^{old} + \gamma_s \Delta c_s.
\end{align*}
\]  

(3-22a) (3-22b)

Again, scale factors \(\gamma_p\) and \(\gamma_s\) are obtained by minimizing \(J_p\) and \(J_s\).

The resulting composition/decomposition inversion loop, which minimizes the difference between the true and the predicted composed data by estimating \(c_p(z_1)\) and \(c_s(z_1)\) is as follows:

1. Composition of \(\tilde{V}_z(z_1)\) and \(\tilde{V}_x(z_1)\) using the current estimated velocities from the wavefield tomography.
2. Compute the gradients.
3. Compute the step lengths \(\gamma\).
4. Update the velocities.
5. Repeat steps 1 to 4 until the error reaches to an absolute minimum or the maximum number of iterations is reached.

### 3-2-2 Examples

We use the same example we have used for wavefield tomography to implement the composition/decomposition inversion and obtain better estimates of the ocean floor velocities. We compose the total wavefields \(v_x\) and \(v_z\) with the current estimates from wavefield tomography, where \(c_{est}(1) = 200\) m/s and \(c_{est}(1) = 1750\) m/s. The true ocean floor P- and S- wave velocities are 1700 m/s and 200 m/s, respectively. Following the composition/decomposition inversion scheme, the objective functions reached to an absolute minima after the first iteration, which is expected since the S-wave velocity of the ocean floor is the same as the estimated one. On the other hand, the P-wave velocity is approximately 50 m/s different from the actual ocean floor velocity. Recall from section 2-5, where we tested elastic decomposition with erroneous velocities, that \(\pm 100\) m/s error in the decomposition velocity still provides reasonable decomposition results. Similarly, composition with a 50 m/s error should provide good composition results. Figure 3-6 shows the results of composition with the current estimated velocities, the total measurements, and the residual. It is proven that composition with the current velocity estimates provided similar results to the total measurements to high extent. The discrepancy is due to events that do not originally exist in the modeled up-going wavefields, but are present in the measurements. This is because of capturing the wavefields at a deeper depth level below the bottom. An example is the small reflections between 1.9 and 2.9 seconds in the measured \(v_x\) due to the small layers in the near bottom.
Near-bottom velocity estimation

Figure 3-6: The total measurements (blue), predicted data (red) and residual (black), after applying composition. Note the good match as well as the consistency of the events in the measured and predicted data. The trace chosen to compute the residual corresponds to \( p = 0.266 \times 10^{-3} \text{ s/m} \).
In this thesis, we have reviewed the acoustic and elastic wavefield decomposition theory following [Wapenaar and Berkhout, 1989] and [Schalkwijk, 2001]. By scaling the two-way wavefields, we reached to expressions for the one-way wavefields. Applying the decomposition theory to synthetic multi-component ocean bottom data provided up- and down-going acoustic pressure for the acoustic case, and up- and down-going P- and S-waves for the elastic case. The sensitivity of the elastic decomposition theory to the ocean floor velocities can be neglected for a range of ±100 m/s velocity errors. Larger velocity errors resulted in interchanging of the up- and down-going events to the point that the one-way wavefields are not reliable anymore. We conclude the importance of valid ocean floor velocity models such that the decomposed wavefields resemble the actual up- and down-going P- and S-waves.

In the third chapter, we have implemented the wavefield tomography, proposed by [Berkhout, 2012] and [Staal and Verschuur, 2013], in the rayparameter-frequency domain based on a 1.5D model. The methodology showed its ability to retrieve a smooth version of the near-bottom velocities, which minimized the discrepancy between the measured and the predicted data. Using the best velocity estimates from wavefield tomography, we applied composition to form the predicted multi-component ocean bottom data, which we compare with the measurements to find an optimum velocity estimate of the ocean bottom. From the examples we have shown, the best seafloor estimates are already provided by the wavefield tomography. This is expected since these estimates are around (±50 m/s) different from the actual seafloor velocities. Based on the tests of elastic decomposition with erroneous velocities, 100 m/s velocity error is acceptable. This is also proven by having the true composition results, when composing the one-way wavefields with slightly erroneous velocities. However, if there is a large difference between the estimated and the true seafloor velocities, the composition/decomposition inversion loop will be able to correct for that. Then the wavefield tomography and the composition/decomposition inversions can be run in a flip-flop mode, where the two loops contribute to each others leading to the best near-bottom and ocean floor velocity estimates. The results indicate that such combined inversion scheme will be feasible in practice.


Appendix

For completion, we include in this appendix the complete set of the measured and modeled up-going wavefields before and after applying wavefield tomography, since we only showed a magnified portion for illustration in section 3-1-3. We also show the complete velocity profiles as well.
Figure 4-1: Up-going measurements (blue), predicted wavefields (red) and residual (black). (a) and (b) correspond to data before applying wavefield tomography, while (c) and (d) correspond to data after applying wavefield tomography. The difference might not be noticeable as it is in Figure 3-2 and Figure 3-3.
Figure 4-2: True (blue), estimated (red) and initial (green) velocity profiles.