Development of the STEALTH-code and Investigation of the Effects of Feedwater Sparger positioning on the Thermal-Hydraulic Stability of Natural Circulation Boiling Water Reactors

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"All science is either physics or stamp collecting"

- Ernest Rutherford
Summary

This report represents a MSc thesis, written as part of the graduation project concluding the Master Applied Physics at the Delft University of Technology. It deals with research on natural circulation reactor stability performed at the section Physics of Nuclear Reactors of the Radiation, Radionuclides & Reactors department. The project comprises of the development of the STEALTH-code\(^1\) for simulating coolant flow in natural circulation reactors; benchmarking the code; and using it for the investigation of the (de-)stabilizing effects of feedwater sparger positioning. STEALTH was written specifically to simulate the coolant flow in GENESIS; an experimental facility constituting a scaled down version of the ESBWR. The development of the code deals only with the thermal-hydraulics; void reactivity feedback was not accounted for. The development of STEALTH, including numerical issues; in general and regarding the GENESIS facility, are extensively treated in this report. The code was benchmarked using data from Marcel \cite{1} who was working with GENESIS a few years earlier, and with measurements performed on the revised facility\(^2\).

In comparing the power-to-flow map generated with STEALTH to measurements by Marcel, it was shown that STEALTH over-estimated the mass flow. This was contributed to an incorrect friction distribution in STEALTH and the simplicity of the applied two-phase flow model. For the stability analysis the system was destabilized by increasing the riser outlet friction and was perturbed using power peaks of 3 kW with a duration of 1 second. The impact of the temperature feedback from the feedwater inlet on the system resonance frequency was discussed. The decay ratios produced by auto-correlating the mass flow from STEALTH for different sparger positions was plotted against their distance from the core inlet and against their phase difference from a cross-correlation with the core inlet temperature. These plots showed a clear trend in which the decay ratio increased up to six-fold for phase differences of \((2k - 1)\pi\), compared to those for phase differences of \(2k\pi\). Even for the non-destabilized system the results were significant. The same trend was displayed by the experimental data from GENESIS for a destabilized reactor; for the non-destabilized system the results were unclear. The system’s transfer functions were treated shortly and the practical relevance of the results to industrial-type reactors was discussed.

It was concluded that the STEALTH-code is a valuable tool for a qualitative analysis of thermal-hydraulic effects in natural circulation reactors. For quantitative research more benchmarking would need to be done, but the evolution of STEALTH to a program capable of simulating coolant flow for both transients and steady-state solutions is promising and should be given serious consideration.

\(^1\)Simulation of Time-dependent Effects on and AnaLysis of Thermal-Hydraulic stability in natural circulation reactors
\(^2\)Ten more feedwater-inlets were included in the GENESIS facility to facilitate a more detailed analysis.
Samenvatting

Dit verslag is onderdeel van het eindproject van de studie Technische Natuurkunde en de Master Applied Physics van de Technische Universiteit Delft. Het behandelt onderzoek naar natuurlijke circulatie reactoren, uitgevoerd bij de sectie *Physics of Nuclear Reactors* van de vakgroep *Radiation, Radionuclides & Reactors*. Het project beslaat de totstandkoming van de STEALTH-code voor het simuleren van koelwaterstroming in natuurlijke circulatie reactoren; de validatie van de code; en haar gebruik voor onderzoek naar de (de)stabiliserende effecten van repositionering van de koelwaterinlaat. STEALTH is specifiek geschreven voor de simulatie van koelwaterstroming in GENESIS; een experimentele opstelling welke als schaalmodel van de ESBWR ontworpen is. Het ontwerp van de code focuseert op thermisch-hydraulische effecten; de invloed van dampfracties op de reactiviteit wordt niet in acht genomen. Het ontwikkelen van STEALTH, inclusief numeriele problemen; in het algemeen en aangaande GENESIS, worden uitvoerig behandeld in dit verslag. De validatie van de code is gedaan aan de hand van data van Marcel [1], die een paar jaar eerder aan GENESIS werkte, en meetdata van de huidige gereviseerde opstelling.

In de vergelijking van power-to-flow grafieken, van STEALTH en van data van Marcel, kwam naar voren dat de massa debieten verkregen met STEALTH een te hoge waarde hadden. Dit werd toegeschreven aan een incorrecte distributie van wrijving in STEALTH en aan de simpliciteit van het gebruikte model voor de bepaling van het stromingsregime. Het systeem werd gedestabiliseerd door de frictie aan het eind van de riser te verhogen en gepertubeerd door middel van pieken in het vermogen van 3 kW met een lengte van 1 seconde. De invloed van perturbaties in de koelwatertemperatuur op de resonantie frequentie is behandeld. De vervalconstanten, verkregen uit een auto-correlatie van de koelwaterstroming voor verschillende inlaten, werden uitgezet tegen hun afstand tot de reactorkern en tegen hun faseverschil, verkregen uit een kruis-correlatie met de kerninlaattemperatuur. Deze grafieken toonden een duidelijke trend waarin de vervalconstanten tot 6 keer zo hoog waren voor faseverschillen gelijk aan $(2k - 1)\pi$, in vergelijking met die voor faseverschillen gelijk aan $2k\pi$. Zelfs voor het ongedestabiliseerde geval was het effect significant. De experimentele data voor het gedestabiliseerde systeem vertoonde dezelfde trend; voor het ongedestabiliseerde systeem was de uitkomst niet duidelijk. Tot slot zijn de overdrachtsfuncties van het systeem genoemd en is de relevantie van de resultaten voor industriële reactoren behandeld.

Er is geconcludeerd dat de STEALTH-code zeer bruikbaar is voor een kwalitatieve analyse van thermisch-hydraulische effecten in natuurlijke circulatie reactoren. Voor kwantitatief onderzoek is de validatie van de code onvoldoende, maar de verdere ontwikkeling van STEALTH tot een programma dat in staat is koelwaterstroming te simuleren tijdens steady-state oplossingen alsook transienten is veelbelovend en dient serieus overwogen te worden.

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[3]De opstelling is recentelijk uitgebreid met tien extra koelwaterinlaten voor een meer gedetailleerde analyse.
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Chapter 1

Introduction

1.1 Previous work

In the late nineties Van Bragt [2] was working on analytical modeling of boiling water reactors and designing a computer model for stability calculations. As a consequence of cancelation of inlet mass flow and inlet temperature fluctuations, the model was (theoretically) unconditionally stable when the feedwater sparger coincided with the core inlet. In 2002 Zboray et al. [3] were working on improving the stability of natural circulation boiling water reactors by optimising the position of the feedwater sparger. The stability maps produced with Van Bragt’s reduced-order model showed increasing stability with decreasing sparger height, but they weren’t able to support their theory with calculations using the MONA-code¹ nor with measurements on the DESIRE facility². Due to the large cross-sectional area of the downcomer in DESIRE the coolant velocity was small. The wavelength of perturbations was therefore also small, which complicated the analysis.

In 2007 Marcel was also working on numerical stability of natural circulation boiling water reactors [1]. He developed a down-scaled version of the ESBWR called GENESIS³. The coolant velocity in the downcomer of the GENESIS facility is a lot higher than in DESIRE, so that the wavelength of perturbations in GENESIS is of the order of the entire length of the downcomer. Marcel performed a few measurements with this facility using three different sparger heights and showed clear influence on the decay ratio. He also showed that there was no correlation between temperature fluctuations in the core and in the feedwater; the correlation existed only with the actual inlet mass flow. Having only three different sparger heights available, he was unable to give solid conclusions about the apparent influence of positioning the feedwater sparger. More specifically, he was unable to investigate cases in which the feedwater sparger would be thus positioned that the temperature and mass flow fluctuations would have a phase difference of $2\pi$.

1.2 Current project

The incentive to do this research was not only to further investigate the effect of feedwater sparger positioning on the stability of natural circulation reactors experimentally, but also to

¹A commercial thermal-hydraulic code developed by Scandpower [4].
²DESIRE is an older experimental facility at the department of Radiation, Radionuclides and Reactors of the Delft University of Technology.
³For a description of the GENESIS facility the reader is referred to Appendix C. Further details about the facility can be found in [1].
create a code that was capable of simulating the coolant flow in GENESIS, in order to do a numerical analysis. This code was to be based on a grid, with a large number of nodes, rather than a division of the system into a few variable-sized nodes; like the model used by Van Bragt [2]. It was also the aim to depend only on the more general (thermal-hydraulic) laws of physics; eliminating the explicit determination of the boiling boundary, and ensuring a more natural dependence on local properties of the physics involved. This led to the development of the STEALTH-code. STEALTH was written specifically for the simulation of the coolant flow in GENESIS, but it can be modified and/or extended to simulate any natural (or even forced) circulation reactor. The development of the code (in accordance with the amount of time it demanded from the project) is treated quite extensively in this thesis.

The code will be benchmarked using experimental data, but more importantly, the experimental data will be used to validate predictions made by STEALTH about the effects of feedwater-sparger positioning on the frequency and decay ratio of thermal-hydraulic instabilities. To realize the experimental comparison, the GENESIS facility has recently been modified to include ten more sparger positions, amounting to a total of thirteen possible feedwater inlets, spread out over the entire length of the downcomer.
Chapter 2

Relevant Physics

2.1 Thermodynamics

Before the equations that govern the flow are treated, some thermodynamics needs to be introduced. The energy brought into the system has to be related to changes in pressure, density and temperature (including phase). It is possible to do calculations with internal energy, but it is common, especially when dealing with phase transitions, to make use of enthalpy. The specific reason for this choice will be provided later on in this chapter.

The analysis begins with the second law of thermodynamics that equates the internal energy to internally reversible heat and work

\[ dU = \delta Q_{\text{int.}}^{\text{rev.}} - \delta W_{\text{int.}}^{\text{rev.}} \] (2.1)

\[ \delta Q_{\text{int.}}^{\text{rev.}} = TdS \] (2.2)

\[ \delta W_{\text{int.}}^{\text{rev.}} = pdV \] (2.3)

From this, together with the equations for the enthalpy and the Gibbs Free Energy;

\[ H = U + pV \] (2.4)

\[ G = U - ST + pV \] (2.5)

it immediately follows that

\[ TdS = dU + pdV \] (2.6)

\[ dH = dU + pdV + Vdp = TdS + Vdp \] (2.7)

\[ dG = dU - TdS - SdT + pdV + Vdp = Vdp - SdT \] (2.8)

From equations (2.6) to (2.8) thermodynamic partial derivatives with respect to temperature and pressure can be produced, known as Maxwell Relations:

\[ \left[ \frac{\partial G}{\partial T} \right]_p = -S \] (2.9)

\[ \left[ \frac{\partial G}{\partial p} \right]_T = V \] (2.10)

\[ \left[ \frac{\partial S}{\partial p} \right]_T = - \left[ \frac{\partial}{\partial p} \left[ \frac{\partial G}{\partial T} \right]_p \right]_T = - \left[ \frac{\partial}{\partial T} \left[ \frac{\partial G}{\partial p} \right]_T \right]_p = - \left[ \frac{\partial V}{\partial T} \right]_p \] (2.11)

\(^1\)A complete overview of the thermodynamics in this section can be found in Equilibrium Statistical Physics [5] and Fundamentals of Engineering Thermodynamics [6].
In order to relate enthalpy changes to changes in temperature and pressure, the infinitesimal changes in entropy and enthalpy are Taylor expanded to first order:

\[
dS = \left[ \frac{\partial S}{\partial T} \right]_p dT + \left[ \frac{\partial S}{\partial p} \right]_T dp
\]

\[
dH = \left[ \frac{\partial H}{\partial T} \right]_p dT + \left[ \frac{\partial H}{\partial p} \right]_T dp
\]

Equations (2.7), (2.6), (2.12) and (2.13) are combined to obtain

\[
T \left[ \frac{\partial S}{\partial T} \right]_p dT - T \left[ \frac{\partial V}{\partial p} \right]_T dp + V dp = \left[ \frac{\partial H}{\partial T} \right]_p dT + \left[ \frac{\partial H}{\partial p} \right]_T dp
\]

which can be rearranged to

\[
\left\{ T \left[ \frac{\partial V}{\partial T} \right]_p + \left[ \frac{\partial H}{\partial p} \right]_T - V \right\} dp = \left\{ T \left[ \frac{\partial S}{\partial T} \right]_p - \left[ \frac{\partial H}{\partial T} \right]_p \right\} dT
\]

Thus, for constant temperature the following must hold:

\[
\left[ \frac{\partial H}{\partial p} \right]_T = V - T \left[ \frac{\partial V}{\partial T} \right]_p
\]

Finally, equations (2.13) and (2.16) are combined and a relation that equates changes in enthalpy to changes in temperature and pressure is found:

\[
dH = \left[ \frac{\partial H}{\partial T} \right]_p dT + \left\{ V - T \left[ \frac{\partial V}{\partial T} \right]_p \right\} dp
\]

### 2.2 Equations Governing the Flow

#### 2.2.1 Navier Stokes

The flow is governed by the well-known Navier-Stokes equations; the continuity equation (2.18), which states the conservation of mass; the momentum balance equation (2.19), which states the conservation of momentum; and the (internal) energy balance equation (2.20), which states the conservation of energy:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0
\]

\[
\frac{\partial}{\partial t} (\rho \vec{v}) + \nabla \cdot (\rho \vec{v} \vec{v}) = -\nabla p + \nabla \cdot \vec{\tau} + \rho g
\]

\[
\frac{\partial}{\partial t} (\rho u^o) + \nabla \cdot (\rho u^o \vec{v}) = -\nabla \cdot \vec{q}^f + Q - \nabla \cdot \rho \vec{v} + \nabla \cdot (\vec{\tau} \cdot \vec{v}) + \vec{v} \cdot \rho g
\]

Since a one-dimensional system is concerned, an integration is carried out over the cross-sectional area. The strategy will be to integrate over a control-volume and then dividing out the dimension of length \(x\). Employing Gauss’s Divergence Theorem\(^2\) on the continuity equation leads to:

\[
\iint_V \frac{\partial \rho}{\partial t} dV + \iint_V \nabla \cdot (\rho \vec{v}) dV = V \frac{\partial \rho}{\partial t} + \iiint_A (\rho \vec{v}) \cdot dA = 0
\]

\(^2\)Gauss’s Divergence Theorem can be found in most mathematical books on calculus, for instance [7].
Realizing that in the one-dimensional system $\rho$ and $\vec{v}$ are cross-sectional averages, the averaging operator on $\rho$ will by convention be dropped and $\vec{v}$ will only have a component in the $x$-direction: $v_x$. The control-volume is ‘squeezed’ in the $x$-direction and its infinitesimal width $\Delta x$ is divided out:

\[
A \frac{\partial \rho}{\partial t} + \lim_{\Delta x \to 0} \frac{[A \rho v_x]_{x+\Delta x} - [A \rho v_x]_x}{\Delta x} = A \frac{\partial \rho}{\partial t} + \frac{\partial M}{\partial x} = 0 \tag{2.22}
\]

The same is applied to the momentum balance equation:

\[
A \frac{\partial (\rho v_x)}{\partial t} + \frac{\partial}{\partial x} \left( A \rho v_x^2 \right) = -A \frac{\partial p}{\partial x} - \int_{P_w} \tau_w dP_w + \rho g A \tag{2.23}
\]

where only friction due to the walls is assumed to be present. The definition of wall here includes any obstacle in the flow path that has a certain length in the flow direction; not just the outer perimeter of the tube. The integral over $\tau_w$ then runs along the cross-sectional perimeter of any obstacle. After inserting a relation for the wall friction$^3$ and the hydraulic diameter:

\[
\int_{P_w} \tau_w dP_w = \tau_w P_w = f \frac{M^2}{2D_h A \rho} \tag{2.24}
\]

\[
D_h = \frac{4A}{P_w} \tag{2.25}
\]

a one-dimensional momentum balance equation is obtained:

\[
\frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( \frac{M^2}{A \rho} \right) = -A \frac{\partial p}{\partial x} - \frac{P_w M^2}{8A^2 \rho} + \rho g A \tag{2.26}
\]

Lastly, the energy balance equation is reduced to one dimension, employing the same procedure as before. Because there exists no internal heat generation, and shear and gravitational effects on friction are negligible, equation (2.20) simplifies to:

\[
A \frac{\partial \rho u_o}{\partial t} + \frac{\partial A \rho v_x}{\partial x} \left( u_o + \frac{\rho}{\rho} \right) = q' \tag{2.27}
\]

It now becomes clear why the choice to work with enthalpy instead of internal energy is often made, as the part in brackets in the derivative to $x$ is readily identified as the enthalpy, and therefore much easier to keep track of numerically. After neglecting kinetic energy, the following one-dimensional enthalpy balance equation is obtained:

\[
A \frac{\partial \rho h}{\partial t} + \frac{\partial M h}{\partial x} = q' + A \frac{\partial p}{\partial t} \tag{2.28}
\]

in which a time-dependent pressure term is now present.

### 2.2.2 Constitutive Equations

To complete the system of equations the last unknowns need to be quantified. No model for the heat transfer is incorporated, i.e. the external power is transferred directly from the fuel rods into the coolant:

\[
q' = \frac{P_{\text{ext}}}{L_{\text{heated}}} \tag{2.29}
\]

$^3$This equation was proposed by Darcy. The friction factor $f$ here is the Darcy friction factor, not to be confused with the Fanning friction factor, which is four times smaller (additional reading can be found in [8]).
The kinematic viscosity is calculated according to a model\footnote{The coefficients $H_c$, $n_i$, $k_i$ and $l_i$ can be found in Appendix A.} introduced by Scalabrin et.al. [9]:

$$
\nu = H_c \sum_{i=1}^{6} n_i \left( \frac{T}{T_{crit}} \right)^{k_i} \left( \frac{\rho}{\rho_{crit}} \right)^{l_i}
$$

(2.30)

The system of equations is completed with an equation of state as a function of pressure and temperature:

$$
\rho = \rho(p,T)
$$

(2.31)

### 2.3 Mixture models

The system currently described is a single phase approximation. To start the two-phase analysis the equations derived in the previous sections are rewritten in terms of mixture properties. Since in the previous section effectively an integration over area was carried out, each mixture property is an average over the cross-sectional area. In fact, from this point forward all physical quantities should be considered averages over the cross-sectional area as a consequence of the dimensional reduction in the previous section; averaging operators have all been dropped.

Most parameters and mixture relations presented in this section are treated in Nuclear Systems I [8], in which a more thorough and in-depth background is provided of the physical relations presented here.

Several new parameters are needed to describe the two-phase system. The slip ratio, flow quality, void fraction, static quality, mixture density and mixture enthalpy are introduced respectively:

$$
S = \frac{v_g}{v_l}
$$

(2.32)

$$
\chi = \frac{M_g}{M}
$$

(2.33)

$$
\alpha = \frac{\chi_{st}}{\chi_{st} + (1 - \chi_{st}) \frac{\rho_g}{\rho_l}}
$$

(2.34)

$$
\chi_{st} = \frac{\alpha \rho_g}{\rho_m}
$$

(2.35)

$$
\rho_m = \alpha \rho_g + (1 - \alpha) \rho_l
$$

(2.36)

$$
\rho_m = \frac{1}{\rho_m} \left( \frac{h_g \alpha \rho_g + h_l (1 - \alpha) \rho_l}{\rho} \right)
$$

(2.37)

$$
\chi_{st} = \alpha \rho_g + (1 - \alpha) \rho_l
$$

(2.38)

The lower case $h$ indicates the switch to enthalpy per unit mass (a division by $\rho V$). The subscripts $l$ and $g$ denote the liquid and vapor phase respectively; (static) mixture properties are denoted by the subscript $m$.

Equation (2.17) is now rewritten in terms of two-phase quantities, using the above mixture relations. First, the single phase derivatives in (2.17) are written in terms of two-phase and
mixture properties.

\[
\left[ \frac{\partial V}{\partial T} \right]_p = \left[ \frac{\partial V_g}{\partial T} \right]_p + \left[ \frac{\partial V_l}{\partial T} \right]_p =
\]

\[
= \left[ \frac{\partial}{\partial \rho_g} \left( \chi_{st} \rho_m V \right) \right]_p \left[ \frac{\partial \rho_g}{\partial T} \right]_p + \left[ \frac{\partial}{\partial \rho_l} \left( \frac{(1 - \chi_{st}) \rho_m V}{\rho_l} \right) \right]_p \left[ \frac{\partial \rho_l}{\partial T} \right]_p
\]

\[
= \chi_{st} V \left\{ -\frac{\rho_m \frac{\partial \rho_m}{\partial T}}{\rho_g^2} \left[ \frac{\partial \rho_g}{\partial T} \right]_p \right\} + (1 - \chi_{st}) \left\{ -\frac{\rho_m \frac{\partial \rho_m}{\partial T}}{\rho_l^2} \right\} \left[ \frac{\partial \rho_l}{\partial T} \right]_p
\]

\[
= -\chi_{st} \rho_m V \alpha \left[ \frac{\partial \rho_g}{\partial T} \right]_p - \left(1 - \chi_{st}\right) \rho_m V (1 - \alpha) \left[ \frac{\partial \rho_l}{\partial T} \right]_p
\]

\[
\left[ \frac{\partial h_m}{\partial T} \right]_p = \chi_{st} \left[ \frac{\partial h_g}{\partial T} \right]_p + h_g \left[ \frac{\partial \chi_{st}}{\partial T} \right]_p + (1 - \chi_{st}) \left[ \frac{\partial h_l}{\partial T} \right]_p - h_l \left[ \frac{\partial \chi_{st}}{\partial T} \right]_p
\]

\[
= c_{p,m} + (h_g - h_l) \left[ \frac{\partial \chi_{st}}{\partial T} \right]_p = c_{p,m} + h_{\Delta} \left[ \frac{\partial \chi_{st}}{\partial T} \right]_p
\]

where the following definition of \(c_{p,m}\) as a mass-averaged quantity has been used:

\[
\begin{align*}
\frac{\partial h_m}{\partial T} & = \chi_{st} \left\langle \frac{T}{\rho V} \left[ \frac{\partial S}{\partial T} \right]_p \right\rangle_g + (1 - \chi_{st}) \left\langle \frac{T}{\rho V} \left[ \frac{\partial S}{\partial T} \right]_p \right\rangle_l \\
& = \chi_{st} \left[ \frac{\partial h_g}{\partial T} \right]_p + (1 - \chi_{st}) \left[ \frac{\partial h_l}{\partial T} \right]_p
\end{align*}
\]

An expression is needed for the partial derivative of \(\chi_{st}\) in the last term on the right-hand side of equation (2.40). The following relations are considered:

\[
\begin{align*}
h_m & = \chi_{st} u_g + (1 - \chi_{st}) u_l + \left( \frac{\chi_{st}}{\rho_m} + (1 - \chi_{st}) \frac{p}{\rho_l} \right) \\
& = u_m + p \left( \frac{\alpha}{\rho_m} + \frac{1 - \alpha}{\rho_m} \right) = u_m + \frac{p}{\rho_m} \\
& \Rightarrow \frac{\partial h_m}{\partial p} = \frac{1}{\rho_m} \\
\chi_{st} & = \frac{h_m - h_l}{h_{\Delta}} \\
& \Rightarrow \frac{\partial \chi_{st}}{\partial h_m} = \frac{1}{h_{\Delta}}
\end{align*}
\]

Using these derivatives, the following is deduced:

\[
\left[ \frac{\partial \chi_{st}}{\partial T} \right]_p dT = d\chi_{st} - \left[ \frac{\partial \chi_{st}}{\partial p} \right]_T dp = d\chi_{st} - \left[ \frac{\partial \chi_{st}}{\partial h_m} \frac{\partial h_m}{\partial p} \right]_T dp = d\chi_{st} - \frac{dp}{\rho_m h_{\Delta}}
\]

Combining equations (2.39), (2.40) and (2.44) with equation (2.17) results in

\[
dh_m = \left[ \frac{\partial h_m}{\partial T} \right]_p dT + \left\{ \frac{1}{\rho_m} - \frac{T}{\rho_m V} \left[ \frac{\partial V}{\partial T} \right]_p \right\} dp
\]

\[
= c_{p,m} dT + h_{\Delta} d\chi_{st} + \left( \frac{\chi_{st} \alpha T}{\rho_g^2} \left[ \frac{\partial \rho_g}{\partial T} \right]_p + \frac{(1 - \chi_{st}) (1 - \alpha) T}{\rho_l^2} \right) \left[ \frac{\partial \rho_l}{\partial T} \right]_p dp
\]

or, upon rearrangement

\[
\frac{dT}{c_{p,m}} = \left( dh_m - h_{\Delta} d\chi_{st} + \left( \frac{\chi_{st} \alpha T}{\rho_g^2} \left[ \frac{\partial \rho_g}{\partial T} \right]_p + \frac{(1 - \chi_{st}) (1 - \alpha) T}{\rho_l^2} \right) \left[ \frac{\partial \rho_l}{\partial T} \right]_p \right)
\]

This relation, in combination with an expression for the saturation temperature, can be used to determine the temperature and static quality of the mixture for known enthalpy and pressure.
In the previous section the equation of state was left undefined. Contrary to the presumption made in equation (2.31), the density in the two-phase mixture model is related to pressure, temperature and void fraction. To obtain such an equation of state, a first order Taylor expansion of the density is carried out in terms of these three quantities:

\[
d\rho_m = \left[ \frac{\partial \rho_m}{\partial T} \right]_{p,\alpha} dT + \left[ \frac{\partial \rho_m}{\partial p} \right]_{T,\alpha} dp + \left[ \frac{\partial \rho_m}{\partial \alpha} \right]_{p,T} d\alpha
\]

The derivatives are again rewritten in terms of two-phase and mixture properties:

\[
\left[ \frac{\partial \rho_m}{\partial T} \right]_{p,\alpha} = \alpha \left[ \frac{\partial \rho_g}{\partial T} \right]_p + (1 - \alpha) \left[ \frac{\partial \rho_l}{\partial T} \right]_p
\]

\[
\left[ \frac{\partial \rho_m}{\partial p} \right]_{T,\alpha} = \left[ \frac{\partial \rho_l}{\partial p} \right]_T + \alpha \left[ \frac{\partial \rho_g}{\partial p} \right]_T - \frac{\partial \rho_l}{\partial p} \right]_T
\]

\[
\left[ \frac{\partial \rho_m}{\partial \alpha} \right]_{p,T} = \rho_g - \rho_l
\]

Inserting each of these into (2.47), an equation of state is obtained:

\[
d\rho_m = \left\{ \left[ \frac{\partial \rho_f}{\partial T} \right]_p + \alpha \left( \left[ \frac{\partial \rho_g}{\partial T} \right]_p - \left[ \frac{\partial \rho_f}{\partial T} \right]_p \right) \right\} dT
\]

\[
+ \left\{ \left[ \frac{\partial \rho_f}{\partial p} \right]_T + \alpha \left( \left[ \frac{\partial \rho_g}{\partial p} \right]_T - \left[ \frac{\partial \rho_f}{\partial p} \right]_T \right) \right\} dp + (\rho_g - \rho_f) d\alpha
\]

Because for single phase flow, i.e. where no vapor is present, \( \rho_l \) should equal the subcooled \( \rho \); \( \rho_l \) is replaced by \( \rho_f \), which is defined as:

\[
\rho_f = \begin{cases} 
\rho (p, T) & \text{single phase flow} \\
\rho_l & \text{two-phase flow} 
\end{cases}
\]

where \( \rho_l \) and \( \rho_g \) refer to the saturation states

\[
\rho_l = \rho_l (T_{sat})
\]

\[
\rho_g = \rho_g (T_{sat})
\]

In addition to the equation of state (2.51), a relation for the saturation temperature is needed, which is just a function of pressure:

\[
T_{sat} = T_{sat} (p)
\]

The Navier Stokes equations in terms of the mixture properties read:

\[
A \frac{\partial \rho_m}{\partial t} + \frac{\partial M}{\partial x} = 0
\]

\[
\frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( \frac{M^2}{A \rho_m^2} \right) = -A \frac{\partial p}{\partial x} - f \frac{P_m M^2}{8 A^2 \rho_m^2} + \rho_m g A
\]

\[
A \frac{\partial \rho_m h_m}{\partial t} + \frac{\partial M h_m}{\partial x} = q' + A \frac{\partial p}{\partial t}
\]

\[\text{The two-phase equation of state given in Nuclear Systems I [8] assumes constant temperature and pressure, resulting in an expression employing just the last term on the right hand side of (2.51). It is important to note that the extension to equation (2.51) is essential when dealing with temperature fluctuations.}\]
in which dynamic mixture properties are introduced, denoted with the superscript $^+$. The dynamic density $\rho_m^+$ and dynamic mixture, or mixing-cup, enthalpy $h_m^+$ are defined as:

$$
\rho_m^+ = \frac{M^2/A^2}{\langle \ddot{\tilde{v}}_g^2\tilde{\alpha}_g + \ddot{\tilde{v}}_l^2 (1 - \tilde{\alpha}) \rho_f \rangle}
$$

$$
\approx \frac{\rho_f/(1 - \chi)^2}{\alpha \left( S_2 \rho_m^+ \rho_f - 1 \right) + 1}
$$

$$
h_m^+ = \chi h_g + (1 - \chi) h_f
$$

Since the phase velocities and the void fraction were by default defined to be an average over the cross-sectional area, the non-averaged values are displayed with a tilde. The difference between the static quantities $\rho_m$ and $h_m$, and the dynamic quantities $\rho_m^+$ and $h_m^+$, originates from the fact that the dynamic quantities are weighted with the phase velocities, as can been seen in equations (2.59) and (2.60). This is why dynamic quantities only appear in equations (2.56)-(2.58) in terms where the averages have included velocities. Since the system is one-dimensional, the average of the denominator of the first right hand side term in equation (2.59) can be approximated by taking the averages of each variable separately, leading to the final form of equation (2.59).

Because two-phase flow is being modeled, a friction multiplier has to be included in equation (2.57), in order to account for the increase in friction due to the void present. Friction multipliers are defined for liquid or vapor states. The latter is mainly used for condensing channels and the first for boiling channels, which is the case for coolant flow. Different relations for the liquid friction multiplier $\Phi_{lo}^2$ exist; the friction multiplier employed here is the same as Marcel used in his work [1]. It was found by Zhang et al. [10] for flow of refrigerants in small-diameter tubes and is given by the relation:

$$
\Phi_{lo}^2 = (1 - \chi)^2 + 2.87\chi^2 \left( \frac{p_{crit}}{p} \right) + 1.68\chi^{0.8} (1 - \chi)^{0.25} \left( \frac{p_{crit}}{p} \right)^{1.64}
$$

The friction multiplier itself is defined as

$$
\Phi_{lo}^2 = \frac{\left( \frac{df}{dz} \right)_{fric}^{TP}}{\left( \frac{df}{dz} \right)_{fric}^{lo}} = \frac{\rho_m^+ f}{\rho_f f_{lo}}
$$

from which it is immediately clear that it can be introduced into (2.57) as follows:

$$
f = \Phi_{lo}^2 f_{lo} \frac{\rho_m^+}{\rho_f}
$$

---

It may not be obvious here, but the flow quality $\chi$, not to be confused with the static quality $\chi_{st}$, is a quantity dependent on mass fluxes and thus is dynamic as well.

Condensation of coolant is present merely outside the primary loop of the system.

Some of these relations can be found in Nuclear Systems I [8].
The liquid only friction factor $f_{lo}$ is a function of the Reynolds number$^9$:

$$f_{lo} = \begin{cases} 
0 & \text{if } Re < 10^{-3} \\
\frac{64}{Re} & \text{if } 10^{-3} \leq Re < 2.1 \cdot 10^3 \\
0.316Re^{-0.25} & \text{if } 2.1 \cdot 10^3 \leq Re < 3 \cdot 10^4 \\
0.184Re^{-0.2} & \text{if } 3 \cdot 10^4 \leq Re < 4 \cdot 10^4 \\
\left(1 - 1.8\log\left(\frac{6.2}{Re} + \left(\frac{D_h}{3.7}\right)^{10/9}\right)\right)^2 & \text{if } Re \geq 4 \cdot 10^4
\end{cases}$$

laminar flow

(2.64)

turbulent flow

the Reynolds number being

$$Re = \frac{\rho_m |v| D_h}{\mu_f} = \frac{|M| D_h}{A \rho_f v}$$

Relation (2.62) is valid for a single-phase liquid mass flow equal to the two-phase mixture mass flow. This is why in equation (2.65) the mixture mass flow $M$ must be substituted, but the dynamic viscosity $\mu_f$ remains a fluid property. Inserting the friction factor into equation (2.57) leads to the following momentum balance equation:

$$\frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( \frac{M^2}{A \rho_m} \right) = -A \frac{\partial p}{\partial x} - \Phi_{lo} f_{lo} P_w \frac{M^2}{8 A^2 \rho_f} + \rho_m g A$$

(2.66)

2.3.1 Homogeneous Equilibrium Mixture model

At this point assumptions have to be made about the flow. First the Homogeneous Equilibrium Mixture model (HEM) is discussed, which assumes a thermodynamic equilibrium between the phases; $T_l = T_g$, and equal phase velocities, uniform within the cross-sectional area; $v_l = v_g$. In other words a perfect mixture of the phases is assumed. It follows directly from the mixture relations that in the HEM model $S = 1$, $\chi_{st} = \chi$, $\rho_m = \rho_m^+$ and $h_m = h_m^+$. Apart from these equalities the Navier Stokes equations (2.56)-(2.58) or any other mixture relations discussed so far don’t change.

2.3.2 Drift Flux model

The Drift Flux model (DF) differs from the HEM model only in the fact that it allows for the two phases to have different velocities; $v_l \neq v_g$. This means that $S \neq 1$ and that the dynamic quantities differ from their static cousins. Starting from a HEM point of view; the slip ratio needs to be defined, leading to the flow quality and the dynamic quantities $\rho_m^+$ and $h_m^+$.

To determine the slip ratio, a closer look is taken at the vapor velocity. The drift velocity $v_d$ is defined as the difference between the vapor velocity $v_g$ and the volumetric velocity $v$ (which is the ratio of volumetric flow rate $Q$ over cross-sectional area $A$); the vapor velocity multiplied by the void fraction equals the volumetric vapor velocity:

$$v_d = v_g - v$$

(2.67)

$$v_g = \alpha v_g = \alpha v + \alpha (v_g - v)$$

(2.68)

$^9$These relations for $f_{lo}$ are the most common ones for the liquid friction factor and can also be found in Nuclear Systems I [8]. The last relation (for high Reynold’s numbers) is known as the Haaland equation and can be found in [11].
When the volumetric vapor velocity is averaged over the cross-sectional area, the superficial velocity is obtained, which is defined as follows:

\[
\langle v_g \rangle = \langle \tilde{\alpha} v \rangle + \langle \tilde{\alpha} (v_g - v) \rangle \tag{2.69}
\]

\[
= C_o \alpha \langle v \rangle + \alpha V_g \tag{2.70}
\]

\[
C_o \equiv \frac{\langle \tilde{\alpha} v \rangle}{\alpha \langle v \rangle} \tag{2.71}
\]

\[
V_g \equiv \frac{\langle \tilde{\alpha} (v_g - v) \rangle}{\alpha} \tag{2.72}
\]

where the tildes again depict non-averaged values. \( C_o \) is called the concentration parameter and \( V_g \) the effective drift velocity. Physically, the effective drift velocity represents the rate at which vapor passes through a unit area parallel to the cross-sectional area, that is already traveling with the flow at the volumetric velocity \( v \), scaled with the void fraction \( \alpha \). From equation (2.70) a new relation for \( \alpha \) is obtained:

\[
\alpha = \frac{\langle v_g \rangle}{C_o \langle v \rangle + V_g} \tag{2.73}
\]

It is useful to rewrite this relation in terms of mass flow and flow quality:

\[
\alpha = \frac{\chi M/\rho_g A}{C_o \left( \frac{\chi}{\rho_l} + \frac{1-\chi}{\rho_l} \right) \frac{M}{A} + V_g} = \frac{1}{C_o \left( 1 + \frac{1-\chi}{\chi} \frac{\rho_g}{\rho_l} \right) + V_g \frac{\rho_g A}{\chi M}} \tag{2.74}
\]

Comparing this result with equation (2.34), a relation is obtained for the slip ratio:

\[
S = C_o + \frac{(C_o - 1) \chi \rho_l}{(1 - \chi) \rho_g} + \frac{V_g \rho_l A}{(1 - \chi) M} \tag{2.75}
\]

The first two terms on the right can be attributed to non-uniform void distribution and the last term on the right to local velocity differentials between the two phases. Both the concentration parameter \( C_o \) and the effective drift velocity \( V_g \) are functions of the flow regime. Zuber et al. [12] suggested \( C_o = 1.2 \) for bubbly and slug flow; \( C_o = 0 \) for near zero void fraction, and \( C_o = 1.0 \) for high void fractions. The drift velocity was given for bubbly and slug flow regimes as:

\[
V_g = (1 - \alpha)^n V_\infty ; \quad 0 < n < 3 \tag{2.76}
\]

in which \( V_\infty \) is the bubble rise terminal velocity in the liquid. This terminal velocity, along with the parameter \( n \) and some statements about the concentration parameter \( C_o \), can be found in appendix B for different flow regimes.

Equation (2.75) can not be used directly, however, since the slip ratio is dependent on the flow quality and vice versa. It is therefore combined with a relation for the flow quality:

\[
\chi = \frac{1}{1 + \frac{\chi_s \rho_l}{\chi_s \rho_g} S} \tag{2.77}
\]

which can be deduced from equations (2.34) and (2.35). Eliminating \( \chi \) from equations (2.75) and (2.77) leads to:

\[
S = \frac{C_o + \frac{V_g \rho_l A}{M}}{1 - \frac{\chi_s (C_o - 1) \rho_l}{1 - \chi_s} - \frac{V_g \rho_l \chi_s A}{M (1 - \chi_s)}} \tag{2.78}
\]

Once the slip ratio is known, equation (2.77) will give the proper flow quality, and the dynamic density and enthalpy can be obtained from equations (2.59) and (2.60) respectively.
Chapter 3

Concerning the flow in GENESIS

3.1 Additional friction

This chapter deals with additional limitations and/or amendments of the equations derived in chapter 2, because of the physical and geometrical constitution of the GENESIS facility. The first consideration is regarding the distribution of friction, which largely determines the outcome of steady-state solutions of the system. In section 2.2.1 a one-dimensional momentum balance equation was derived and in section 2.2.2 the addition of friction multipliers was discussed. The GENESIS facility, however, is obviously not comprised of just a pipe, but has bends, corners, fuel rods and other elements obstructing the flow. Smooth transitions from, for instance, rectangular to circular piping and obstruction of the flow by fuel rods are covered in the determination of the effective hydraulic diameter (2.25) and the increase in wall perimeter (section 2.2.1). The spacers, (sharp) bends, corners and valves present in the GENESIS facility do need an explicit representation in the momentum balance equation, in the form of a pressure loss term. If the pressure losses due to these obstructions are defined as consecutive perturbations \( \Delta p_i \mathcal{H}(x_i) \), in which \( \mathcal{H} \) is the Heaviside step function and \( x_i \) the position of the obstruction in the system, added to a global pressure \( p \); the integral over the cross-sectional area of the gradient of \( p \), as was done for equation 2.23 in section 2.2.1, becomes:

\[
\int_A \nabla \left( p + \sum_i \Delta p_i \mathcal{H}(x_i) \right) \, dA = A \frac{\partial}{\partial x} \left( p + \sum_i \Delta p_i \mathcal{H}(x_i) \right) = A \frac{\partial p}{\partial x} + A \sum_i \Delta p_i \delta(x - x_i) \tag{3.1}
\]

The pressure loss term \( \Delta p_i \) is in [Pa] and is described as:

\[
\Delta p_i = C_v \left( \frac{A_i}{A} \right)^2 \frac{M^2}{2A^2 \rho_m^2} = \Phi^2_{obs} K_i \frac{M^2}{2A^2 \rho_f} \tag{3.2}
\]

The factor \( K \) was empirically found and includes the squared ratio of the projected frontal area of the obstruction over the unobstructed cross-sectional area. Relation (3.2) also includes a two-phase friction multiplier. For instantaneous obstructions of the flow, the assumption is usually made that the two-phase friction factor is equal to the friction factor for liquid single phase flow ([8]): \( f_{TP} = f_{lo} \). Reviewing equation (2.62), the friction factor can then be given as:

\[
\Phi^2_{obs} = \frac{\left( \frac{dp}{dx} \right)^{TP}}{\left( \frac{dp}{dx} \right)^{lo}} = \frac{\rho_f}{\rho_m} \tag{3.3}
\]
from which it follows that the density in equation (3.2) is again changed from a two-phase to a liquid only property. For homogeneous flow and thermodynamic equilibrium (HEM), employing equations (2.34) and (2.36), the friction factor for instantaneous flow obstructions reduces to:

$$\Phi_{obs}^2 = 1 + \left( \frac{\rho_f}{\rho_g} - 1 \right) \chi$$

(3.4)

For low slip ratios, this friction multiplier is in fact also a good approximation in the DF model. With the inclusion of all flow obstructions, the one-dimensional momentum balance equation becomes:

$$\frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( \frac{M^2}{A\rho_m} \right) = -A \frac{\partial p}{\partial x} - \sum_i \Phi_{obs}^2 K_i^2 M^2 A \rho_f \delta (x - x_i) - \Phi_{lo}^2 f_{lo} \frac{P_w M^2}{8A^2 \rho_f} + \rho g A$$

(3.5)

### 3.2 Pressure vessel and feedwater inlet

The GENESIS facility is kept at a constant pressure of about 11.4 bar by means of two pressure valves positioned on the top of the pressure vessel. Inside the vessel there will normally be a separation of phases present under thermodynamic equilibrium. This means that during a pressure build-up, steam is exited through the valves out of the vessel, towards the condensor and the secondary loop, whereas the (saturated) liquid coolant will stay in the primary loop and flow into the downcomer\(^1\). This pressure release somehow has to be incorporated in the one-dimensional model. During the development of this part of the code, the conservation of the physical outcome was regarded more important than keeping true to the exact physical mechanisms of the processes involved. In other words; care was taken not to violate the solution by disturbing key parameters. The strategy that was chosen for the pressure release of the system will now be discussed.

To maintain a constant pressure it is clear that mass has to be exited as the system is heating up. To this end a mass flow is defined that exits the system at a certain point\(^2\) in the (one-dimensional) pressure vessel, proportional to the excess pressure:

$$M_{out} = F (p - P_{sys})$$

(3.6)

\(F\) is a proportionality factor, \(P_{sys}\) is the minimum pressure that activates the valve (11.4 bar) and \(p\) is the actual pressure inside the vessel. This relation will be worked out further in chapter 4. Also the energy that accompanies the outflow of mass and the simultaneous discharge of excess steam from the pressure vessel will be discussed in chapter 4, because their treatment is of a more numerical nature.

Apart from an outflow of mass, there is also an inflow of mass at the feedwater inlet. This is numerically a lot easier to handle, especially since the massflow and temperature of the feedwater are input parameters; explicitly set before the experiment. The implications the outflow and inflow of mass have on the set of equations will be discussed with the discretization scheme in chapter 4.

---

\(^1\)Details on the geometry and dimensions of the facility can be found in Appendix C. Careful considerations have been made by Marcel [1] in the design of the vessel to ensure that no carry under of vapor occurs.

\(^2\)In fact, in the code several nodes are defined to allow for outflow of mass.
Chapter 4

Numerical Scheme

4.1 Discretization

In all the equations in the following section the HEM model is assumed. Because the extension to the DF model from HEM is rather straightforward, and the implications have been dealt with in section 2.3.2, the discretization of the DF model equations will not be discussed. In section 4.2.3 the extension to the DF model will be treated in general terms.

For the discretization of the equations a first order implicit upwind scheme is chosen. A one-dimensional grid is formulated on which the physical quantities are defined. This grid, along with the spatial definition of the physical quantities, is shown in figure 4.1.

![Figure 4.1: Discretization grid](image)

A specific place and time can now be assigned to all of the quantities\(^1\). From figure 4.1 it is clear that all quantities are defined inside the nodes of the grid; except for the mass flow, which is defined halfway between the nodes. Also, quantities directly related to flow obstructions are defined at the actual position of the obstruction in the grid, using a linear approximation between the nearest neighbours. Employing the one-dimensional grid above, the entire system of equations is discretized. With the inclusion of mass in- and outflow\(^2\), as discussed in section 3.2, the continuity equation becomes:

\[
A \frac{\rho_j^{n+1} - \rho_j^n}{\Delta t} = - \frac{M_j^{n+1/2} - M_j^{n-1/2} + M_{out}^{n+1} - M_{in}^n}{\Delta x}
\]  

(4.1)

\(^1\)To keep the indices clear and legible, from this point forward all mixture subscripts are dropped. It should be kept in mind that, when extending to the DF model, the distinction between dynamic and static quantities has to be made again.

\(^2\)The inflow and outflow of mass are only defined for specific nodes in the system; respectively at the feedwater inlet and in (part of) the pressure vessel.
Before the momentum balance equation can be discretized, it needs to be linearized. Note that the in- and outflow of mass don’t need linearization, because their values are determined independently of the momentum balance equation. The linearization is done by splitting the accelerational and frictional terms into their current and their previous time value, depicted as \( n + 1 \) and \( n \) respectively. Frictional forces are by definition counter-directional to the flow. In order to secure this characteristic, the absolute value is taken of the previous value of the flow in the frictional terms. Note that the accelerational terms can not be guaranteed to be positive, because of the impossibility of incorporating an absolute value of the current time value of the mass flow \( M^{n+1} \) in the solution scheme (in anticipation of section 4.3). Again including mass in- and outflow, equation (3.5) then becomes:

\[
\frac{M^{n+1}_{j+\frac{1}{2}} - M^n_{j+\frac{1}{2}}}{\Delta t} \Delta x + \frac{M^{n+1}_{j+1}M^n_{j+1}}{A\rho^{n+1}_{j+1}} - \frac{M^n_{j+1}M^n_{j}}{A\rho^n_j} + \frac{(M^{out}_{j+1})^2 - (M^{in}_{j})^2}{A\rho^{n+1}_j} = -A \left( \rho^{n+1}_{j+1} - \rho^{n+1}_j \right) - \Phi_{sp}^{n+1} K_i \frac{M^{n+1}_j |M^n_j|}{2A\rho^{n+1}_j} \Delta x - \Phi_{loj}^{n+1} \frac{P_u M^{n+1}_{j+\frac{1}{2}} |M^n_{j+\frac{1}{2}}|}{8A^2 \rho f^{n+1}_{j+\frac{1}{2}}} + \rho^{n+1}_{j+\frac{1}{2}} g \Delta x \quad (4.2)
\]

Since the GENESIS facility is not of uniform cross-sectional area and the one-dimensional spacial grid is not equi-distant, i.e. \( A \) and \( \Delta x \) may vary in space, there should be definitive spacial indices on them. These are left out, however, for better legibility. The integration over \( x \) canceled the Dirac delta function and ensured the definition of the obstructions at their exact position \( x_i \). In the accelerational terms the mass flows are subscripted with indices ’inside’ a node of the grid, although they are not defined there. Physically the best estimate of the mass flow at undefined places, is given by the nearest value of the mass flow counter-directional to the flow, i.e. its ’upwind’ neighbour. For this reason the indices in question are not definitive, but subject to flow reversals. If the flow is in the positive, respectively negative \( x \)-direction, the index \( j \) becomes \( j - \frac{1}{2} \), respectively \( j + \frac{1}{2} \), and so on. From this point on this upwind scheme should always be assumed whenever quantities are subscripted with a non-definitive index, i.e. an index of a position where the quantity is not defined.

Next, the enthalpy balance equation is discretized. Mass flow in and out are included, the enthalpy entered (\( h_{in} \)) is the feedwater enthalpy and the enthalpy exited (\( h_{out} \)) has yet to be defined:

\[
A \rho^{n+1}_j h^{n+1}_j - \rho^n_j h^n_j \frac{\Delta t}{\Delta x} + \frac{M^{n+1}_{j+\frac{1}{2}}h^{n+1}_{j+\frac{1}{2}} - M^n_{j+\frac{1}{2}}h^n_{j+\frac{1}{2}}}{\Delta x} + \frac{M^{out}_{j+1}h^{out}_{j+1} - M^{in}_{j}h^{in}_{j}}{\Delta x} = q^{n+1}_j + A \frac{\rho^{n+1}_j - \rho^n_j}{\Delta t} \quad (4.3)
\]

Bearing in mind that the partial derivatives in any Taylor expansion are defined at the point from which the approximation is made, equation (4.4) for the temperature and quality becomes:

\[
c_{p,n} \chi_{st,j} T^{n+1}_j - T^n_j = \left( h^{n+1}_j - h^n_j \right) - h\Delta \left( \chi_{st,j} T^{n+1}_j - \chi_{st,j} T^n_j \right) - T^n_j \left[ \chi_{st,j} \frac{\partial \rho_j^n}{\partial T} \frac{\partial \rho_j^n}{\partial T} \right]_j + \left( \frac{1 - \chi_{st,j}^n}{(\rho_j^n)^2} \right) \frac{\partial \rho_j^n}{\partial T} \frac{\partial \rho_j^n}{\partial T} \left( \rho_j^n \right)^n \left( p_j^{n+1} - p_j^n \right) \quad (4.4)
\]
Finally, the equation of state is discretized:

$$\rho_j^{n+1} = \rho_j^n + \alpha_j^n \left( \frac{\partial p_f}{\partial T} \right)_j^{n} + \alpha_j^n \left( \frac{\partial p_g}{\partial T} \right)_j^{n} \left(T_j^{n+1} - T_j^n\right)$$

$$+ \left( \frac{\partial p_f}{\partial p} \right)_j^{n} + \alpha_j^n \left( \frac{\partial p_g}{\partial p} \right)_j^{n} \left(p_j^{n+1} - p_j^n\right)$$

$$+ \left( \rho_j^n - \rho_j^n \right) \left( \alpha_j^{n+1} - \alpha_j^n \right) \quad (4.5)$$

The discretization of the constitutive equations is straightforward and will not be discussed.

### 4.2 Algorithm

The equations governing the flow are inter-dependent: to be able to solve the entire system, some attention must be paid to the solution scheme. Use is made of a specific algorithm, which is iterated until a convergence criterium is reached. The solution scheme provides for a good conservation of mass and energy; the conservation of momentum is not subject to scrutiny. The details concerning this algorithm are discussed in this section.

The enthalpy is determined from the enthalpy balance equation. However, equation (2.58) cannot be calculated implicitly, since the density, mass flow and pressure of the next time step c.q. iteration step are all still unknowns. For the case of density this is fixed by deducting the discretized continuity equation (4.1) from the discretized enthalpy balance equation:

$$A h_j^{n+1} \frac{\rho_j^{n+1} - \rho_j^n}{\Delta t} + h_j^{n+1} \frac{M_j^{n+1} - M_j^{n-1} + M_{out}^{n+1} - M_{inj}}{\Delta x} = 0$$

$$+ A \rho_j^n \frac{h_j^{n+1} - h_j^n}{\Delta t} + M_{out}^{n+1} \left( h_{out}^{n+1} - h_j^{n+1} \right) - M_{inj} \left( h_{inj} - h_j^{n+1} \right) \Delta x$$

$$= q_j^{n+1} + A \frac{p_j^{n+1} - p_j^n}{\Delta t} \quad (4.6)$$

The double subscripts in the fourth left hand term refer to regular or backward (in brackets) flow and are evidence to the influence the upwind scheme has on the choice of neighbour; the outcome of deducting the continuity equation depends on the direction of the flow. From this point forward the double index will be omitted as regular (forward) flow will be assumed in the indexing. The unknown value of the density has been extracted, but the problem persists for the mass flow and pressure. For this reason the algorithm is iterated in order to converge towards a general solution of all balance equations in terms of quantities that closely resemble the true solution at the particular time step. This leads to the following discretized enthalpy balance equation:\footnote{The superscript $k$ defines the iteration value. It should be understood that $k \equiv n + 1, k$ except for the very first $k$ which equals $n, k.$}

$$A \rho_j^n \frac{h_j^{k+1} - h_j^n}{\Delta t} + M_{j-\frac{1}{2}}^k \frac{h_j^{k+1} - h_{j-\frac{1}{2}}^{k+1} + h_j^{k+1} M_{inj} - M_{out}^k}{\Delta x}$$

$$+ M_{out}^k h_{out}^{k+1} - M_{inj} h_{inj} \Delta x = q_j^{n+1} + A \frac{p_j^{n+1} - p_j^n}{\Delta t} \quad (4.7)$$
In the GENESIS facility a thermodynamic equilibrium exists between liquid and vapor in the pressure vessel. When the pressure inside the vessel exceeds the system pressure, vapor is released through the valve and a new equilibrium is reached. For the simulation of this process the outflow of energy (enthalpy) is decoupled from the mass flow. The enthalpy directly associated with the mass outflow is defined to equal the enthalpy in the node: \( h_{\text{out}} \equiv h_m \), from which it directly follows that the outflow of mass drops entirely from the enthalpy balance equation. This condition is not sufficient, however, to ensure that all void has left the system before the coolant enters the downcomer, a constraint known as ‘no carry-under’. The outflow of mass is governed by the excess of pressure in the system, but in no way does it guarantee the exit of all vapor from the pressure vessel. An additional drain of energy (enthalpy) is therefore needed. The excess enthalpy in the vessel amounts to \( h_{\Delta} \chi_{st} \rho_{m} V_{\text{vessel}} \). This excess enthalpy is continuously drained from the coolant in the vessel. A ‘relaxation constraint’ \( \lambda \) is added to ensure a smooth exit of the vapor and to avoid large gradients inside the vessel. It is a function that gradually goes to 1 in the first few nodes inside the vessel and is 1 for the remainder of the vessel. The additional drain of enthalpy from the vessel can be put in numerical terms as follows:

\[
\lambda_j A h_{\Delta} \frac{\chi_{st} k_j^k \Delta x}{\Delta t} \tag{4.8}
\]

which leads to the final form of the one-dimensional discretized enthalpy balance equation:

\[
A \rho_j^k \frac{h_j^{k+1} - h_j^n}{\Delta t} + \frac{M_{j-\frac{1}{2}} h_{j+\frac{1}{2}}^{k+1} - M_{j-\frac{1}{2}} h_{j-\frac{1}{2}}^{k+1}}{\Delta x} + M_{in,j} \frac{\left(h_j^{k+1} - h_{in,j}\right)}{\Delta x} = q_j^{n+1} + A \frac{p_j^k - p_j^n}{\Delta t} - \lambda_j A h_{\Delta} \frac{\chi_{st} k_j^k \Delta x}{\Delta t} \tag{4.9}
\]

The next step in the algorithm is the determination of temperature and quality. Equation (4.4) is however not employed in the algorithm, for a specific reason. Early experience with STEALTH proved this step in the algorithm to be error-prone during strong transients. Due to its dependence on previous values of temperature, an error made would stand uncorrected during following iterations. This is a characteristic common to some of the other equations as well, but the combination of determining temperature and quality simultaneously proved to be numerically more unstable. For this reason the thermodynamic properties package NIST [13] is referenced to provide temperature and (static) quality at the new iteration value \( T_{k+1} \) and \( \chi_{st}^{k+1} \) from known values of enthalpy and pressure \( (h^{k+1} \text{ and } p^k) \). NIST makes use of the "Helmholtz energy model" equation of state for R-134a, proposed by Tillner-Roth et al. [14]. The void fraction \( \alpha^{k+1} \) can then be determined from equation (2.34).

The NIST package can also provide densities for given values of enthalpy and pressure, and so the equation of state (2.51) could be eliminated as well. However, an explicit representation of the change in density is actually needed in the mass conservation scheme, as will become clear in the next section. The equation of state is thus regularly incorporated in the algorithm.

\footnote{Equation 4.4 was treated in this thesis because it can be useful for fluids not included in NIST, or if for some other reason the NIST package can not be referenced.}
and considering the iteration scheme, it is discretized as follows:

\[
\rho_j^{k+1} = \rho_j^k + \left\{ \frac{\partial \rho_f}{\partial T} \right|_j^k + \alpha_j^k \left( \frac{\partial \rho_g}{\partial T} \right|_j^k - \frac{\partial \rho_f}{\partial T} \right|_j^k \right\} (T_j^{k+1} - T_j^k) + 1 \left\{ \frac{\partial \rho_f}{\partial p} \right|_j^k + \alpha_j^k \left( \frac{\partial \rho_g}{\partial p} \right|_j^k - \frac{\partial \rho_f}{\partial p} \right|_j^k \right\} (p_j^k - p_j^{k-1}) + (\rho_{sj}^k - \rho_{sf}^k) (\alpha_j^{k+1} - \alpha_j^k)
\]  

(4.10)

Special notice should be taken of the iteration values on \( p \), which, because its value has not yet been determined, is one less than on \( T \) and \( \alpha \).

### 4.2. ALGORITHM

The momentum balance equation can now be carried out implicitly, apart from the unknown pressure. Before the momentum balance equation is discussed, however, some attention is paid to the determination of pressure, for which a procedure called pressure correction is introduced\(^5\). The idea is to adjust the pressure in such a way that it corrects the mass flow in order to preserve mass. To do this the difference in mass flow is analysed as it is calculated with an implicit and an explicit pressure. Neglecting the frictional and accelerational terms, the subtraction of the two momentum balance equations leads to:

\[
M^{k+1} (p^{k+1}) - M^* (p^k) \quad \Rightarrow \quad \frac{M_j^{k+1}}{\Delta t} = -A \frac{\Delta p_j^{k+1} - \Delta p_j^k}{\Delta x}
\]  

(4.11)

in which \( M^* \) is a prediction for the mass flow based on the previous iteration value of the pressure\(^6\), \( M' = M^{k+1} - M^* \) and \( p' = p^{k+1} - p^k \). Next, the continuity equation (4.1) is needed, in which the equation of state (4.10) is employed for the change in density:

\[
A \frac{\Delta t}{\Delta x} \left\{ \frac{\partial \rho_f}{\partial T} \right|_j^k + \alpha_j^k \left( \frac{\partial \rho_g}{\partial T} \right|_j^k - \frac{\partial \rho_f}{\partial T} \right|_j^k \right\} (T_j^{k+1} - T_j^k) + A \frac{\Delta t}{\Delta x} \left\{ \frac{\partial \rho_f}{\partial p} \right|_j^k + \alpha_j^k \left( \frac{\partial \rho_g}{\partial p} \right|_j^k - \frac{\partial \rho_f}{\partial p} \right|_j^k \right\} (p_j^{k+1} - p_j^k) + A \frac{\Delta t}{\Delta x} (\rho_{sj}^n - \rho_{sf}^k) (\alpha_j^{k+1} - \alpha_j^k) = -A \frac{M_j^{k+1} - M_{j-\frac{1}{2}}^{k+1} + M_{outj}^{k+1} - M_{inj}}{\Delta x}
\]  

(4.12)

In this equation \( M^{k+1} \) and \( p^{k+1} \) are now replaced with \( M' + M^* \) and \( p' + p^k \) respectively and expression (3.6) is substituted for the outflow of mass, with the new iteration value \( p^{k+1} \) for the pressure. This leads to a final expression for the pressure correction:

\[
A \frac{\Delta t}{\Delta x} (p_j^k - p_j^{k-1}) + A \frac{\Delta t}{\Delta x} (p_j^{k+1} - p_j^k) + F p_j^k + A \frac{\Delta t}{\Delta x} \left\{ \frac{\partial \rho_f}{\partial p} \right|_j^k + \alpha_j^k \left( \frac{\partial \rho_g}{\partial p} \right|_j^k - \frac{\partial \rho_f}{\partial p} \right|_j^k \right\} (p_j^k - p_{sys}) = M_{j-\frac{1}{2}}^{k+1} - M_{j+\frac{1}{2}}^k + M_{inj} - F (p_j^k - P_{sys})
\]

\[
\frac{A \Delta x}{\Delta t} \left\{ \frac{\partial \rho_f}{\partial T} \right|_j^k + \alpha_j^k \left( \frac{\partial \rho_g}{\partial T} \right|_j^k - \frac{\partial \rho_f}{\partial T} \right|_j^k \right\} (T_j^{k+1} - T_j^k) = -A \frac{\Delta x}{\Delta t} (\rho_{sj}^k - \rho_{sf}^k) (\alpha_j^{k+1} - \alpha_j^k)
\]  

(4.13)

\(^5\)The pressure correction method as derived by manipulation of the continuity and momentum balance equations was introduced by Zijlema et al. [15; 16], and extended to compressible flow by Bijl et al.[17].

\(^6\)This 'predictor-step' is carried out for each iteration; \(* \equiv k + 1, *\).
From this equation the pressure correction $p'$ can be calculated, after which with equation (3.6) the mass outflow $M_{\text{out}}$, and with equation (4.11) also the mass flow correction $M'$ can be obtained. The discretized momentum balance equation for the pressure correction scheme is written in terms of the prediction of the mass flow $M^*$:

$$\frac{M^*_{j+\frac{1}{2}} - M^k_{j+\frac{1}{2}}}{\Delta t} \Delta x + \frac{M^*_{j+1}M^k_{j+\frac{1}{2}}}{A\rho^k_{j+\frac{1}{2}}} - \frac{M^*_{j}M^k_{j}}{A\rho^k_{j+\frac{1}{2}}} + \frac{(M_{\text{out}}^k)^2 - (M_{\text{in}}^k)^2}{A\rho^k_{j+\frac{1}{2}}} =$$

$$- A \left( p^k_{j+\frac{1}{2}} - p^k_{j} \right) - \Phi^{k+1}_{sp} \frac{k}{2} M^*_{j} |\Delta x| + \Phi^{k+1}_{lo} \frac{P_{w} M^*_{j+\frac{1}{2}}}{8A^2 \rho^k_{j+\frac{1}{2}} |\Delta x|} + \rho^k_{j+\frac{1}{2}} g A \Delta x$$

(4.14)

4.2.2 Overview of solution algorithm

The algorithm described in the previous section will be iterated until convergence is reached. The convergence criterium is satisfied when the new iterated value for the pressure $p^k_{j+1}$ is within a specified limit of the previous iteration value $p^k$, everywhere in the system. Here follows an overview of the solution algorithm:

1. Find the physical quantities\footnote{Because these quantities all depend on pressure and/or temperature, they are re-evaluated each iteration.} $h_{\Delta}, \rho_t, \rho_g, \frac{\partial \rho_f}{\partial T}, \frac{\partial \rho_g}{\partial T}, \frac{\partial \rho_f}{\partial p},$ and $\frac{\partial \rho_g}{\partial p}$

2. Use equation (4.9) to compute $h^{k+1}$

3. Find the temperature $T^{k+1}$ and static quality $\chi_{st}^{k+1}$ from NIST using $h^{k+1}$ and $p^k$ and derive $\alpha^{k+1}$ with equation (2.34)

4. Compute $\rho^{k+1}$ using equation (4.10)

5. Compute $M^*$ using equation (4.14)

6. Use equation (4.13) to find $p^{k+1}$ and with equations (3.6) and (4.11) find $M_{\text{out}}$ and $M^{k+1}$ respectively

7. Iterate steps 1 to 6 until the convergence criterium is satisfied

8. Store each variable to the new time value.

4.2.3 Drift Flux model

For the DF model the algorithm is a little different. The first four steps of the overview in the previous section remain the same, even as far as the enthalpy balance equation is concerned. Naturally, the enthalpy itself is the physical property calculated from the enthalpy balance equation. To be able to perform this calculation implicitly with the inclusion of the
dynamical enlthapry, one would need a representation of the dynamical enlthapry in terms of the
static enlthapry, or the other way around. This is not at all straightforward and for this reason
the dynamical enlthapry is approximated by using the static enlthapry; i.e. the enlthapry balance
equation from the HEM model remains unchanged in the algorithm for the DF model. This
difficulty is not present in the momentum balance equation, because the dynamical density can
be calculated before the momentum balance equation is carried out. The momentum balance
equation used in the program is therefore indeed the dynamical equation \((3.5)\).

So the first four steps in the previously presented overview of the algorithm remain the
same, but after step four the slip ratio, the flow quality (which is no longer equal to the
static quality) and the dynamical density are calculated. The model from Zuber et al. is used
to estimate the parameters \(C_o\) and \(V_g\). The slip ratio and flow quality are obtained from
equations \((2.75)\) and \((2.77)\), and the dynamical density can be calculated using equation \((2.59)\).
The latter will be used in the momentum balance equation. Steps 5 to 8 in the algorithm are
again the same as for HEM.

### 4.3 Solving the system of equations

The discretized balance equations each form a linear system of equations (of size \(N\); the total
number of nodes), which can be written in matrix form as:

\[
A \phi = s
\]

In this expression \(\phi\) is a vector carrying the new value \(k + 1\) of the variable to be calculated,
which can be either \(h\), \(M^*\) or \(p'\), \(s\) is a solution vector in which all the terms are gathered that
are independent of \(\phi\) and \(A\) is a cyclic tri-diagonal matrix carrying all the terms dependent on
\(\phi\). \(A\) is cyclic because the code is describing a closed loop, so that the first node is dependent
on the last node and vice-versa; and it is tri-diagonal because each node can be dependent on
either neighbour, due to the upwind scheme. To solve the system \((4.15)\), use is made of the
Sherman-Morrison (SM) method for cyclic tri-diagonal matrices\(^\text{8}\).

The SM method describes \(A\) as a non-cyclic tri-diagonal matrix \(A'\) with a perturbation
\(u \otimes v\), as follows: \(A \phi = (A' + u \otimes v) \phi = s\). If the elements of \(A\) are defined as

\[
A = \begin{bmatrix}
a_1 & c_1 & \ldots & \beta \\
b_2 & a_2 & \ldots & \\
\vdots & \vdots & \ddots & \\
0 & \ldots & a_{N-1} & c_{N-1} \\
\alpha & \ldots & b_N & a_N
\end{bmatrix},
\]

then \(A'\), \(u\) and \(v\) are defined as

\[
A' = \begin{bmatrix}
a_1 - \gamma & c_1 & \ldots & 0 \\
b_2 & a_2 & \ldots & \\
\vdots & \vdots & \ddots & \\
0 & \ldots & a_{N-1} & c_{N-1} \\
0 & \ldots & b_N & a_N - \frac{\alpha \beta}{7}
\end{bmatrix},
\]

\[
u = \begin{bmatrix}
\gamma \\
0 \\
\vdots \\
0 \\
\alpha
\end{bmatrix},
\]

\[
v = \begin{bmatrix}
1 \\
0 \\
\vdots \\
0 \\
\beta \\
\gamma
\end{bmatrix}.
\]

\(^\text{8}\)A very nice description of the Sherman-Morrison method can be found in Numerical Recipes [18].
The parameter $\gamma$ is independent and can be set to any value. If for the vectors $y$ and $z$ the following holds:

$$\mathbf{A}'^\prime y = \mathbf{s}, \quad \mathbf{A}'^\prime z = \mathbf{u}$$  \hspace{1cm} (4.18)

then the solution to system (4.15) is given by:

$$\phi = y - \left[ \frac{\mathbf{v} \cdot y}{1 + \mathbf{v} \cdot z} \right] z$$  \hspace{1cm} (4.19)

The proof of this claim will now be provided. Considering the following expression:

$$\left( \mathbf{A}'^\prime + \mathbf{u} \otimes \mathbf{v} \right)^{-1} = \left( 1 + \mathbf{A}'^{-1} \cdot \mathbf{u} \otimes \mathbf{v} \right)^{-1} \cdot \mathbf{A}'^{-1}$$

$$= \left( 1 - \mathbf{A}'^{-1} \cdot \mathbf{u} \otimes \mathbf{v} + \mathbf{A}'^{-1} \cdot \mathbf{u} \otimes \mathbf{v} \cdot \mathbf{A}'^{-1} \cdot \mathbf{u} \otimes \mathbf{v} - \ldots \right) \cdot \mathbf{A}'^{-1}$$

$$= \mathbf{A}'^{-1} - \frac{\mathbf{A}'^{-1} \cdot \mathbf{u} \otimes \mathbf{v} \cdot \mathbf{A}'^{-1}}{1 + \mathbf{v} \cdot \mathbf{A}'^{-1}}$$

in which, exploiting the associativity of outer and inner products, $\lambda \equiv \mathbf{v} \cdot \mathbf{A}'^{-1} \cdot \mathbf{u}$; we can immediately write:

$$\phi = \left( \mathbf{A}'^\prime + \mathbf{u} \otimes \mathbf{v} \right)^{-1} \cdot \mathbf{s}$$

$$= \mathbf{A}'^{-1} \cdot \mathbf{s} - \left[ \frac{\mathbf{A}'^{-1} \cdot \mathbf{u} \otimes \mathbf{v} \cdot \mathbf{A}'^{-1}}{1 + \mathbf{v} \cdot \mathbf{A}'^{-1}} \right] \cdot \mathbf{s}$$

$$= \mathbf{y} - \left[ \frac{\mathbf{z} \cdot \mathbf{v} \cdot \mathbf{A}'^{-1}}{1 + \mathbf{v} \cdot \mathbf{z}} \right] \mathbf{z}$$

with which the acclaimed solution (4.19) has been recovered. QED

For solving the non-cyclic tri-diagonal systems (4.18) an algorithm is used that was introduced by Holmes [19]. Introducing matrix $\mathbf{B}$, which is the tri-diagonal part of $\mathbf{A}$, i.e. $\mathbf{A}$ with $\alpha = 0$ and $\beta = 0$, and the same definition of elements as in (4.16), the following parameters are defined:

$$w_1 = a_1, \quad v_1 = \frac{c_1}{w_1}, \quad \text{and} \quad z_1 = \frac{s_1}{w_1}$$

in which $s_1$ is the first element of the solution vector $\mathbf{s}$. After this an iteration follows:

For $i = 2, 3, \ldots, N$

$$w_i = a_i - b_i w_{i-1}$$

$$v_i = \frac{c_i}{w_i}$$

$$z_i = \frac{s_i - b_i z_{i-1}}{w_i}$$

in which $N$ is again the size of $\mathbf{s}$. The elements of $\mathbf{\phi}$ are now found as follows:

$$\phi_N = z_N$$

For $i = N - 1, N - 2, \ldots, 1$

$$\phi_i = z_i - v_i \phi_{i+1}$$

All other matrix calculations needed to solve the system of equations are very straightforward.
Chapter 5

Results

5.1 Convergence

The STEALTH-code was first subjected to a convergence test. The program was run long enough for a steady state solution to be reached, with different values for the time step $\Delta t$; all other parameters were kept constant. The same was done for different grid spacings $\Delta x$. The most important physical quantities were normalized against the value obtained from the run with the smallest time step or grid spacing. The resulting plots are shown in figures 5.1 and 5.2.

![Figure 5.1: Convergence in $\Delta t$](image1)

![Figure 5.2: Convergence in $\Delta x$](image2)

From figure 5.1 it is clear that already at a $\Delta t$ of 0.3 seconds the system shows good convergence; decreasing the time step even further does not add much to the accuracy of the results. A timestep of 0.2 seconds was chosen as default and with this value the calculations resulting in figure 5.2 were done. This figure paints a less desirable picture. Even though the scale on the $y$ axis is small, for decreasing $\Delta x$ the system is not converging to a steady-state solution. Figure 5.2 is also a bit ‘jumpier’ as figure 5.1, most likely owing to the fact that, contrary to the time step, the grid spacing is a more discrete parameter; the important parts in the facility (core, riser, pressure vessel, downcomer) are separately divided into (an integer number of) nodes, so the grid spacing can differ a little in each part. A non-uniform grid spacing should not alter the exact solution, but it is a known fact in CFD\(^1\) that large deviations in grid spacing

---

\(^1\)Computational Fluid Dynamics
can lead to numerical instability and fluctuating solutions. Considering the large increase in computational demand for an increasing number of nodes, the convergence in $\Delta x$ could not be tested further and a value of 2 centimeters was chosen as default for the grid spacing.

## 5.2 Benchmarking STEALTH

As a check on its performance the program keeps track of its own mass and energy conservation. Without going into detail, mass conservation errors were below 2% during transients and typically below 0.1% during steady-state; energy conservation errors were below 1% during transients and typically below 0.1% during steady-state. These errors apply only to the calculations performed, which reached a maximum of 15 minutes in terms of 'reactor time'. A more extensive analysis of conservation of mass and energy was not deemed necessary.

Before a look is taken at the steady-state results, a few remarks about Zuber’s model for the slip ratio are in order. As pointed out in section 2.3.2, this model promotes a change in parameters for different regimes\(^2\). To quantify the model the slip ratio and flow quality resulting from the different regimes are plotted in figure 5.3. It is clear that the 'large bubbles'-regime overpredicts the slip ratio and flow quality for static qualities greater than about 0.1. Even for very small static qualities the slip ratio is significantly higher than for the ‘churn flow’-model, but there the resulting flow qualities from each regime differ very little. The switch from one regime to the next as the system crosses the 'border-line' static quality can result in large sudden changes in slip ratio and flow quality. The latter has a considerable effect on the two-phase friction distribution, as it is present in the two-phase friction multipliers. Because over-predicting the slip ratio suppresses the mass flow, due to a too high estimate of the two-phase friction, care should be taken in evaluating the point at which the model switches from 'large bubbles' to 'churn flow'. To avoid large differentials in flow quality, a static quality of 0.005 is defined to separate the regimes (although for this 'border-line' quality the 'large bubbles'-regime might as well have been eliminated altogether).

\[ \begin{align*}
\text{Slip ratio} & \quad 0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0 \\
\chi_{st} & \quad 0.0 & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 \\
\end{align*} \]

\[ \begin{align*}
\text{Largebubbles} & \quad \text{Churnflow} \\
\end{align*} \]

Figure 5.3: Slip ratio

\(^2\)Review appendix B for a complete overview of Zuber’s model.
5.2.1 Power-to-flow map

The STEALTH-code was compared with mass flow data at different powers, taken from Marcel [1]. Using his power- and mass flow-scaling ratios for the ESBWR, a power-to-flow map was created, shown in figure 5.4. The comparison is made with experimental values obtained from measurements by Marcel (GENESIS); values calculated with the ATHLET-code, as done by Marcel; and values calculated with the new STEALTH-code; both for the HEM and the DF model. The nominal point was calculated with the TRACG system code [20]. For very low powers both models from STEALTH produce results quite similar to experiment (although a bit steeper), but as the power is raised they can be seen to overpredict the mass flow. When the power is raised further, the HEM model shows a large decrease in mass flow. This is caused by the fact that for high powers (or better: for high void fractions), flow qualities increase faster than void fractions, illustrated in figure 5.5. This means that the driving force hardly changes for high powers while the two-phase friction keeps increasing with power. The DF model shows much more similar behaviour to the GENESIS data, because of the representation of the dynamic density in the momentum balance equation: if the slip ratio increases, the dynamic density increases. This devaluates the frictional term in equation (2.57) as compared to the gravitational term, that is dependent on the static density. So the effect of increasing friction due to increasing flow quality is in the DF model compensated by the increasing dynamic density, which leads to the higher mass flows for high powers.

![Power to flow maps for different models](Figure 5.4)

Figure 5.4: Power to flow maps for different models

However, the overprediction of the mass flow is in the DF model still present. This could be due to an under-estimation of the (two-phase) friction. The friction distribution in the system is essential to the outcome of the steady-state mass flow. Unfortunately, the input for ATHLET could not be retrieved, and the limited capabilities for measuring pressure drops in the facility (the available dp-meters are depicted in figure C.1) make it hard to compare the friction over all parts of the system. The valves in GENESIS, positioned right before the core inlet and at the end of the riser, are an important part of the total friction distribution
and are equipped with dp-meters. Matching the pressure drops over these two elements with measurement data\(^3\) was however not enough to equalize the mass flows.

It is likely that a more complicated model for the determination of two-phase flow regimes and the subsequent friction leads to a different estimate of the mass flow in STEALTH. The model for the slip ratio was in fact quite modest, as the very slight difference between the two in figure 5.5 proves. Under-estimating the slip ratio implies under-estimating the flow quality and therefore under-estimating the two-phase friction, while the driving force (the void fraction) remains unscathed. Considering the impact of incorporating the dynamic density in the model, the inclusion of the dynamic enthalpy might also have quite an impact on the flow.

There are other phenomena that STEALTH does not take into account, of which subcooled boiling is an important example. The GENESIS facility is not equipped to measure it, but one can safely assume subcooled boiling to be present. Small bubbles forming on the surface of the fuel rods are swept into the subcooled flow and both increase friction and decrease average density below the boiling boundary. For low power the added driving force is expected to increase the flow, whereas for high powers the added friction would subdue this effect and lower the mass flow, making for a 'flatter' power-to-flow map. Another effect not taken into account is 'carry under': saturated liquid containing bubbles while flowing into the downcomer. STEALTH is programmed not to allow any void to be present beyond the pressure vessel, but because Marcel paid specific attention in the design of the pressure vessel to avoiding carry under, it is not expected to be present in the GENESIS facility either.

Marcel claims that the data was taken at a constant subcooling number \(N_{\text{sub}}\) of about 0.9 and phase change number \(N_{\text{pch}}\) of about 5.2, but this would result in a straight line in a power-to-flow map\(^4\). The input parameters for STEALTH resulted in subcooling numbers of between 0.89 and 0.91 for the different data-points, and the phase change number ranged

\(^3\)The resulting \(K\)-factors can be found in appendix C.
\(^4\)The definition of the subcooling, phase change and other dimensionless numbers can be found at the end of the nomenclature.
from 1.0 for low powers to about 6.4 for high powers (for the DF model). At nominal power a phase change number of about 4.3 was observed. Its ratio with the phase change number of 5.2 mentioned earlier equals that of STEALTH’s high estimate of the flow over the nominal value for the flow in GENESIS. The remainder of this chapter, and in fact this thesis, will deal only with the results from the DF model, as its behaviour is most similar to the experiment.

### 5.2.2 Further comparison with GENESIS data

As a final benchmark, a comparison was made with experimental data\(^5\) concerning quality profile, void fractions, subcooling and temperature profile. The most relevant data\(^6\) are shown in table 5.1. Looking at the core exit temperature for the GENESIS data, in relation to the pressure at the core inlet, there is a discrepancy to be noted. A pressure of 12.45 bar corresponds to a saturation temperature of about 47.8 °C (from NIST); lower pressures correspond to lower saturation temperatures. For the core exit temperature to reach 49 °C, superheated boiling would have to occur. It is possible that superheated vapor is present at the core exit, but it’s hard to believe that it would lead to measurements of more than a degree above boiling temperature.

<table>
<thead>
<tr>
<th></th>
<th>Core inlet</th>
<th>Core exit</th>
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<tbody>
<tr>
<td></td>
<td>pressure</td>
<td>mass flow</td>
</tr>
<tr>
<td></td>
<td>bar</td>
<td>l/s</td>
</tr>
<tr>
<td>GENESIS</td>
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</tr>
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</tr>
<tr>
<td>STEALTH</td>
<td>1.23</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Table 5.1: Comparison of numerical with experimental data

The pressure in the pressure vessel in GENESIS was a bit higher for the displayed data than nominal: 11.55 instead of 11.40 bar. This means that the pressure in the entire system was a bit higher and so also the saturation temperature. One would thus expect the nominal subcooling number in GENESIS to be a bit lower for a given feedwater temperature, and the nominal void fraction to be a bit higher than in table 5.1. The void fraction for GENESIS is in fact quite low in comparison to the flow quality; the slip ratio appears to be about 2.70, which is very high.

First the core inlet temperature was matched with the GENESIS data and then the subcooling number\(^7\); both at the nominal pressure of 11.40 bar. This lead to the first two entries for STEALTH in table 5.1. Finally the pressure in the pressure vessel was increased to 11.55 bar, leading to the final entry for STEALTH in table 5.1. The temperature comparison is made with the data from measurements just inside the core; approximately 2 cm after the start of the heated section\(^8\) (this measurement point will from now on be refered to as ’T1’).

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5Measurements performed by Van Wessem [21].
6Values for the flow quality are not directly available from GENESIS, but calculated using the analysis in appendix E.
7The subcooling number for the GENESIS data was calculated for saturation conditions at 12.45 bar; not for those at 49 °C.
8The reason for this choice is that ever since a bypass of the heat exchanger was installed in the facility, the thermometer below the core inlet is situated at a dead end of the flow and gives unreliable results, possibly due to imperfect mixing of the coolant and the feedwater.
The slightly higher core inlet temperature for the final data-set is the result of the warmer saturated coolant flow in the downcomer, due to the higher saturation temperature.

As was observed before when the power-to-flow map was discussed, the mass flows are higher in STEALTH. Also, the void fractions obtained from STEALTH are higher than those measured in GENESIS, even for matching pressure and subcooling number. A higher estimate of the saturation temperature in STEALTH would lead to a lower quality and void fraction, but since the impact of increasing flow quality on the void fraction decreases rapidly (as was illustrated in figure 5.5), this would not lead to significantly higher void fractions. The over-estimated void fraction leads to a lower hydro-static pressure which is the reason why the pressure at the core inlet is lower for STEALTH, even when the pressure in the vessel is the same.

The goal of this thesis, as formulated in chapter 1, is to investigate the effect of feedwater sparger positioning on the stability of natural circulation reactors. This analysis will be done at a constant point in the stability map, i.e. at constant phase change and subcooling number (4.3 and 0.9 respectively). Although the mass flow is of importance to the stability of the system itself, it only affects the magnitude of the decay ratios and not the trend as a function of feedwater sparger position. The imperfect data-match with GENESIS is therefore not considered to significantly upset the stability analysis in the next section. This, added to the fact that without additional measurement data the comparison is hard to put into more detail; the benchmarking was carried no further.

### 5.3 Stability analysis

At this point, the influence of feedwater sparger positioning was analysed, at nominal power (the settings that led to the first entry for STEALTH in table 5.1). However, for a more detailed analysis the system was destabilized by increasing the riser outlet friction. The goal of the destabilization is to increase decay ratios and to improve the prospect of finding trends in the stability of the system. To increase the riser outlet friction in the GENESIS facility, the riser valve was partly turned shut, until the decay ratios were about five-fold. For STEALTH, the friction factor for the riser valve was obtained empirically by matching the pressure drop over the riser valve with that from the GENESIS facility. The resulting decay ratios from STEALTH, however, were only about twice as high. This most likely has to do with the presumably incorrect friction distribution in STEALTH and the applied two-phase flow model, as discussed in the previous section. To increase decay ratios up to the experimental level, the stability analysis was done with a much higher friction factor for the riser valve. The data discussed in this section will be from this 'extra-destabilized' case; displayed in table 5.2. The table shows the distance of the sparger from the core inlet, the transit time from sparger to core inlet, the time lag from the cross-correlation of the mass flow with the core inlet temperature, the resonance frequency, the decay ratio, and the phase difference. The distances shown in table 5.2 correspond exactly to those in the GENESIS facility, except for the ones denoted with an asterisk. The distance of 359 mm corresponds to the geometrical core inlet, c.q. the end of the downcomer. The transit times are calculated by STEALTH as

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9 The $K$-factors for the riser outlet valve can be found in appendix C.
10 This terminology is a bit misleading. When referring to the core inlet, what is actually meant is the start of the heated section.
5.3. STABILITY ANALYSIS

<table>
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<th>Distance (mm)</th>
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<th>Time lag (s)</th>
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Table 5.2: Numerical data for different sparger positions for the destabilized case

The resonance frequencies and the decay ratios are obtained from the auto-correlation of the mass flow. The phase difference is defined as the resonance frequency from the auto-correlation of the mass flow multiplied by the timelag from the cross-correlation with the core inlet temperature\(^{11}\). To perturb the system a random sequence of power peaks of 3 kW (on top of the nominal power), with a duration of 1 second was used\(^{12}\). For a full description of the determination of decay ratios and resonance frequencies the reader is referred to appendix E, in which also the evaluation of their uncertainties is treated. The uncertainties apply only to the fit of the (auto- and cross-) correlation and are given in table D.3. For the STEALTH data these are the only uncertainties present, as the program will reproduce the exact same results for specific settings. Because of the order of magnitude of the given uncertainties, they are not significant in the plots provided in this chapter and were therefore left out.

Before the correlation data is discussed, what needs to be noted is the more or less constant difference between the transit times and the time lags. This difference is of the same order as $\Delta t$: about 0.2 seconds. The explanation is rather straightforward and (as expected) of a numerical nature. The perturbations in pressure due to the inflicted perturbations in power are too small to disturb the convergence criterium\(^{13}\). As can be reviewed in section 4.2, the mass flow during a time-step is calculated after the temperature. So if the convergence criterium is already satisfied after the first iteration, which is often the case for steady-state flow, the

\(^{11}\)The factor of $2\pi$ is considered clear from the context and is thus consistently omitted.

\(^{12}\)The sequence is random in time in order to suppress low-frequency appearances in the correlation coupled to the period of the sequence.

\(^{13}\)One could argue in favor of a stricter criterium, but this has major consequences for the calculation speed, as the calculation of the non-perturbed time-steps would readily conflict with this stricter criterium as well.
temperature is not re-calculated within that same time-step and is thus effectively delayed by $\Delta t$ in relation to the mass flow. More precisely: the temperature at the feedwater inlet can only react to the mass flow during the following iteration, which, if the convergence criterium was satisfied, carries a new time value. One would therefore expect the time lags and thus the phase differences to be a bit lower than displayed in table 5.2, increasingly so for smaller phase differences.

5.3.1 The influence of temperature feedback

An interesting fact from table 5.2 is the behaviour of the resonance frequency. It decreases for increasing phase difference, repeating itself after a phase difference of $2\pi$. The trend is displayed in figure 5.6 and is in fact directly related to the temperature feedback from the feedwater sparger disturbing the mass flow. But before this feedback can be discussed, the system’s response to the power perturbation is investigated. The power, the core inlet mass flow and temperature, and the core exit and riser exit void fraction are plotted in figure 5.7. The plots are rescaled to fit the window, as it is just the responses that are of interest.

![Figure 5.6: Frequency as a function of phase difference](image)

As the power is perturbed the mass flow decreases; partly because the expansion of the extra boiling blocks the flow below the boiling boundary (a very slight effect); and partly because the increase of void causes an increase of friction in the core (the mass flow reacts quicker to the increase of friction than to the added driving force). Two subsequent peaks are observed in the core exit void, occuring with a time difference of just over a second. The first peak corresponds to the direct impact of the power perturbation on the core exit quality; the second peak corresponds to the increase in core exit quality due to the decrease in mass flow, that caused the boiling boundary to drop. When the power is back to nominal and the added void is leaving the core, the mass flow can be seen to rise, reaching a maximum just when the core void is almost back to nominal and the added void has not quite reached the riser valve. So the mass flow shows a maximum already before the added void has left the system. As the added void passes the riser valve the mass flow drops again, reaching a minimum when most of the added void has left the riser and there is a local drop in void entering the riser due to the higher mass flow before. The second maximum in mass flow occurs again in between the
second maxima of the core exit void and the riser exit void. This characteristic is observed for all different sparger positions and is independent of the duration of the power peak. The peaks in core exit void are separated one riser transit time from the subsequent peak in riser exit void. If the maximum of the mass flow lies in between them, it is half a riser transit time separated from each peak. The reaction time of the core exit void to the mass flow is related to the core transit time, so the minimum of the mass flow is separated from the peak of the core exit void by one core transit time. If the minimum of the mass flow occurs in between the peaks of riser exit void and core exit void, these two peaks would be separated by twice the core transit time. The time lag between the two mass flow maxima can thus be estimated by once the riser transit time plus twice the core transit time, as illustrated in figure 5.7.

The interference of the temperature feedback on the mass flow can now be treated. Note that the response depicted in figure 5.7 is for a sparger located far from the core inlet, so that the mass flow perturbation has almost decayed when the perturbation in temperature reaches the core inlet. When the transit time from sparger to core inlet becomes smaller, the perturbation in coolant temperature starts interfering with the mass flow in the scenario described above, which can alter its resonance frequency. The most extreme cases of interference will be treated here to explain the mechanism, depicted in figures 5.8 and 5.9.

Looking at figure 5.8, there is a minimum in coolant temperature close to where the mass flow reaches a minimum as well. At the same time the core exit void is rising due to this lower mass flow, but the effect is largely subdued by the lower coolant temperature. The core exit void would then normally drop again (see figure 5.7), but rather stays above nominal because of the rise in coolant temperature that follows, which was caused by the first maximum in mass flow. The (second) maximum in the core exit void is thus delayed, which naturally delays the

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14 The response of the coolant temperature at the core exit is a shift of 1.61 seconds to the right. Since local perturbations in mass flow travel with the speed of sound they can be considered instantaneous.
Figure 5.8: Response perturbed by temperature (distance = 3849 mm)

(second) maximum in the riser exit void. As the second maximum in the mass flow tends to occur in between these maxima, it is effectively delayed as well, causing the auto-correlation to produce a lower resonance frequency. The effect is quite clear from figure 5.8.

In stead of 'stretching’ the maxima, each can effectively be split into two maxima as well; shown in figure 5.9. The mechanism is very much the same as described in the previous paragraph, but the interference of the coolant temperature comes later, so that the core exit void was already dropping. This creates an extra local minimum and maximum in core exit void, riser exit void and mass flow. The first local maximum corresponds to the original (undisturbed) resonance frequency of the system.

In STEALTH, twice the core transit time and once the riser transit time equals 7.22 seconds, corresponding to a frequency of about 0.139 Hz; not coincidentally exactly the highest resonance frequency found in table 5.2. The disturbance from the coolant temperature

Figure 5.9: Response perturbed by temperature (distance = 4849 mm)
5.3. STABILITY ANALYSIS

described in the previous paragraphs is the source of the resonance frequency behaviour as displayed in figure 5.6. One can conclude from this paragraph that the influence of feedwater sparger positioning on the stability is already apparent.

5.3.2 Decay ratios

The decay ratios from table 5.2 are plotted in figure 5.10, against their distance to the core inlet and against their phase difference. Figure 5.10 very clearly shows the influence of feedwater sparger positioning on the decay ratio. As predicted by Zboray et al. [3], the decay ratio is low for feedwater inlets close to the core inlet, but more importantly, it is also very low for feedwater inlets at positions corresponding to phase differences of $2k\pi$. At positions corresponding to phase differences of $(2k - 1)\pi$, the interference of the out-of-phase feedback of the core inlet temperature is very pronounced, as decay ratios are raised by a factor of up to 6. As the feedwater sparger is moved further away from the core inlet, the effect becomes less profound. This makes sense since the perturbation in coolant temperature needs time to travel from the feedwater sparger to the core inlet. For phase differences greater than $2\pi$ the feedback from the feedwater inlet cannot affect the first period of the mass flow perturbation, having thus a less prominent (secondary) effect on its decay ratio. For even greater phase differences the decay ratio is expected to converge to its non-perturbed value (a state for which the coolant temperature would be truly constant). Comparing the decay ratios in figure 5.10 to this 'infinite-distance' value of about 0.2, one can conclude that the temperature feedback of the coolant can have both a stabilizing and a destabilizing effect, the latter being the most prominent.

To show the correlation between the phase difference and the distance from the feedwater sparger to the core inlet, the two are plotted in figure 5.11. The phase difference is plotted as a function of the resonance frequency, as it was defined in table 5.2 (blue); and as a function of the riser transit time plus twice the core transit time, as discussed in section 5.3.1 (red). The straight red line shows that the time lags from the cross correlation correspond nicely to the transit times from feedwater inlet to core inlet and that the velocity of the coolant through the downcomer is very constant over the entire length of the downcomer. The first data point is off the line, because the coolant velocity is smaller inside the (geometrical) core, which means the wavelength of the perturbation is shorter inside the core. This makes for a steeper gradient.
for distances below 359 mm and shifts the remainder of the data in figure 5.11 upwards. The blue data-points clearly show the influence of the system’s resonance frequency on the phase difference, as shown in figure 5.6.

5.3.3 Experimental data

The same analysis was performed on experimental data from GENESIS\textsuperscript{15}, to be found in table D.2 in appendix D. Since the temperature measurement in front of the core inlet is unreliable, as was explained in section 5.2, the cross-correlation of the experimental data is performed with the mass flow and the temperature at ‘T1’. The decay ratios from the different feedwater inlet positions are plotted in figure 5.12, in which the STEALTH data is also depicted, and figure 5.13. Appendix D gives the uncertainties in resonance frequency and decay ratio for GENESIS. The uncertainties in the appendix apply to the fit of the correlation, but for GENESIS there is an uncertainty involved in the measurement data as well. To be conclusive about these uncertainties, a large number of measurements would have to be performed. The lack of measurements prevents the ascertainment of a sensible standard deviation and one should at least be cautious not to make rash conclusions. Nevertheless, the trends displayed by figures 5.12 and 5.13 are very clear and certainly look significant\textsuperscript{16}.

Looking at figure 5.13, the phase difference seems to be off, in the sense that the minimum decay ratio does not occur at a phase difference of $2\pi$. Also, the period of the oscillation is quite different for the numerical and experimental data; clear from figure 5.12. The phase difference is a function of the frequency of the auto-correlation of the mass flow and the time lag of the cross-correlation with the core inlet temperature. The latter depends on the distance from the feedwater sparger to the core inlet and the coolant velocity through the downcomer. In the destabilized system, the experimental volumetric flow was about 0.47 l/s and the numerical volumetric flow, due to the ‘extra-high’ riser outlet friction, was about

\textsuperscript{15}Measurements performed by Van Wessem [21].

\textsuperscript{16}It should be noted that a second run of measurements with GENESIS has been performed for the non-destabilized system that produced data nearly identical to the experimental data treated in this thesis. It has however not been included in the current analysis.
5.3. STABILITY ANALYSIS

0.27 l/s. Since the geometries are the same, the ratio of volumetric flows equals the ratio of velocities. The resonance frequency divided by the volumetric flow is then proportional to the phase difference and should therefore have the same ratio between numerical and experimental data as the reciprocal ratio of the wavelengths of the oscillation:

\[ \phi = 2\pi f_{\text{res}} \frac{df_w}{v_{dc}} \propto \frac{f_{\text{res}}}{v_{dc}} \]  

(5.1)

\[ \left[ \frac{f_{\text{res}}}{v_{dc}} \right]_{\text{GENESIS}} \approx \left[ \frac{f_{\text{res}}}{v_{dc}} \right]_{\text{STEALTH}} \approx \frac{0.129}{0.47} / \frac{0.139}{0.27} = 0.53 \]  

(5.2)

\[ \frac{\lambda_{\text{STEALTH}}}{\lambda_{\text{GENESIS}}} \approx \frac{4}{7.5} = 0.53 \]  

(5.3)

The wavelengths of the oscillation are approximated from figure 5.12. It is a very crude approximation, but the difference in velocity and resonance frequency appear to be a satisfactory explanation for the deviating wavelength in STEALTH.

Figure 5.14 shows the correspondence between the time lag from the cross-correlation and the distance from the feedwater sparger to the core inlet for the experimental data. The plot
shows no data-points deviating from the straight line; like the first data-point in figure 5.11. This was to be expected because all sparger distances in GENESIS are situated in the downcomer section and so no velocity-gradients are present between the data-points. The phase difference crosses zero at a positive sparger distance, which either suggests that the transit time from the geometrical core inlet to the heated section is very small, or that the time lags from the cross-correlation were consistently too low. The negative time lag of the first data-point in table D.2 and the fact that the minimum decay ratio for GENESIS occurs at a phase difference of less than $2\pi$ support the second assumption. If this is true it is not caused by an incorrect cross-correlation, since the experimental data for the feedwater sparger closest to the core inlet actually shows the temperature to react before the mass flow. It would thus either be a problem with the data acquisition or there is in fact some unexplained phenomenon going on in the core section.

There exists an optimal position for the feedwater sparger, for which the phase difference is $2\pi$. From equation (5.1) one can deduce that this distance is determined by the coolant velocity through the downcomer and the resonance frequency of the mass flow. The latter is related to the transit times through core and riser sections, c.q. to the velocities in core and riser sections. The massflow is the same throughout the system, so for equal densities in the riser, i.e. equal void fractions, the velocities should remain of the same proportion to each other. By varying the subcooling in order to maintain the same core exit quality, this optimal feedwater sparger position could be kept constant during the start-up of a reactor. In practice, if the difference between the phase change number and the subcooling number is

![Figure 5.15: Transfer functions for the different feedwater spargers](image)
kept constant during start-up, the core exit quality should remain the same\textsuperscript{17} and the optimal feedwater sparger position in a reactor becomes a static feature rather than a dynamic one.

The data was also subjected to a Fourier analysis, in order to gain insight in the system’s transfer functions\textsuperscript{18}. The results for STEALTH for the different feedwater sparger positions can be found in figure 5.15. The numbers in the upper right corners correspond to the distance of the feedwater sparger from the core inlet; the unit on the $x$-axes is Hertz. Note that the scales on the $y$-axes are set to the same limit. The representation of frequencies in the input signal (the power perturbation) can be found in appendix F. It is clear that there are a lot of strong low frequency resonances (below 0.1 Hz) in the system. These are related to slow variations of the system around its steady-state solution. Above about 0.2 Hz the Fourier spectrum tends to zero, and in between 0.1 and 0.2 Hz the thermal-hydraulic resonances are found that relate directly to the subject of this thesis. For sparger distances of up to about 4 meters (or 1 period of oscillation) the peaks in figure 5.15 correspond very nicely to the resonance frequencies stated in table 5.2. As the phase difference approaches $2\pi$ at around 3.5 meters, all resonances (including the low frequency ones) can be seen to be suppressed. For sparger distances greater than 4 meters the thermal-hydraulic resonances fade away as the spectrum becomes more chaotic.

5.4 The non-destabilized system

The effect of feedwater sparger positioning was also investigated in the non-destabilized system; both for STEALTH and GENESIS. The resulting data is given in table D.1 in appendix D and the decay ratios are given in figure 5.16. The results from STEALTH clearly show the same trend as in the destabilized system, but for smaller decay ratios. A second minimum in decay ratio is again observed at a phase difference of (close to) $2\pi$. The shorter period of oscillation is due to the higher coolant velocity in the non-destabilized system. In the decay ratios from GENESIS no trend can be observed.
Chapter 6

Conclusions

6.1 Concluding remarks and recommendations

The STEALTH-code was developed in order to investigate the effects of feedwater sparger positioning on the thermal-hydraulic stability of natural circulation reactors. The power-to-flow map produced by STEALTH proved quite inaccurate in terms of magnitude, although the extension from HEM to the Drift Flux model showed great improvement of steady-state solutions throughout the entire range of powers. In section 5.2 the consequences of a low estimate of the slip ratio were discussed and 'flattening' of the profile due to subcooled boiling in GENESIS was mentioned. The applied flow model is of great influence to the power-to-flow profile as well. The imperfect data-match was not considered to impede on the stability analysis this thesis focuses on and therefore no further bench-marking was done.

Even though the program produced quantitatively inaccurate results, qualitatively it performed well. It proved to be very useful for the analysis of different thermal-hydraulic phenomena inside a reactor, due to the large amount of data-output that is impossible to match experimentally, and the absence of measurement noise. It was also able to show undisputably the effect feedwater sparger positioning has on the stability of the system.

The question that remains is if the effect of feedwater sparger positioning is significant in industrial reactors, or if it is smothered by natural chaos and more prominent feedbacks of a different kind. The results from GENESIS for the non-destabilized system aren't promising. The wavelength of the perturbation is of importance here; very short wavelengths are much more subject to (turbulent) diffusion. The coolant velocity through the downcomer should preferably be high and the thermal-hydraulic resonance frequency of the reactor small. That means long transit times through core and riser and a relatively small cross-sectional area for the downcomer. STEALTH might prove a valuable tool for the prediction of thermal-hydraulic stability in natural circulation reactors of different geometries.

6.2 Future work

The STEALTH-code has proved to be very promising as a useful tool in calculating thermal-hydraulic phenomena in natural circulation reactors, but for use in research for which quantitative results are of main interest, the STEALTH-code would certainly need more extensive benchmarking. A correct representation of the friction distribution will prove essential, along with an accurate estimate of the flow quality. The model can still be extended to include a
dynamic representation of the enthalpy, as discussed in section 4.2.3. It would be interesting to see what impact a more detailed model to establish flow regimes and slip ratio would have on the prediction of friction and mass flow in STEALTH and the development of steady-state outcomes. The numerical stability and convergence of STEALTH should also be examined. Mass and energy conservation should always be checked, but the errors they introduce are no direct source of numerical instability.

Recommended is to extend the experimental facility with additional measurement capability; especially dp-meters and perhaps more positions along the core for void fraction measurements (although this is difficult in a practical sense). Experimental data is indispensable for a true validation of the STEALTH-code. Once the friction distribution and mass flow in STEALTH are matched empirically with the dp-measurements, a more detailed description of the flow regime could be incorporated in the code. Additions like a model for the heat transfer from fuel rod to coolant, subcooled boiling, and a (turbulent) diffusion model should be rather straightforward to implement and certainly interesting to look at. Void reactivity feedback could quite easily be accounted for as well. It is also recommended to look into different convergence criteria for the algorithm, other than just monitoring the pressure, and to eliminate the delay in coolant temperature perturbation, as discussed in section 5.3, perhaps by repeating the algorithm once by default. An examination of the presence of numerical diffusion for different grids might prove interesting as well (numerical diffusion was the major setback in using the MONA-code during the investigation by Zboray et al. [3]).

After the inclusion of void reactivity feedback in STEALTH one could investigate the separate contributions of void reactivity feedback and coolant temperature feedback to the total stability of the reactor. It should be possible with STEALTH to produce stability maps for each feedback separately, by either switching off the void reactivity feedback or defining a constant coolant temperature beyond the feedwater sparger.
Bibliography


Appendices
Appendix A

Parameters for the Scalabrin Viscosity Model

The Scalabrin model for the kinematic viscosity of R-134A is

\[ \nu = H_c \sum_{i=1}^{6} n_i \left( \frac{T}{T_{crit}} \right)^{k_i} \left( \frac{\rho}{\rho_{crit}} \right)^{l_i} \]

The coefficients are given in Table A.1:

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</table>

Table A.1: Scalabrin parameters for the viscosity
Appendix B

Parameters for the Zuber slip ratio

The slip ratio is given by the relation
\[ S = C_o + \frac{(C_o - 1) \chi \rho_l}{(1 - \chi) \rho_g} + \frac{V_g \rho_l A}{(1 - \chi) M} \]
for which
\[ V_g = (1 - \alpha)^n V_{\infty} \]
The coefficients \( n \) and \( V_g \) are given in Table B.1:

<table>
<thead>
<tr>
<th>Flow Regime</th>
<th>( n )</th>
<th>( V_\infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small bubbles (d&lt;0.5 cm)</td>
<td>3</td>
<td>( \frac{g(\rho_l-\rho_g)}{18\mu_l}d^2 )</td>
</tr>
<tr>
<td>Large bubbles (d&lt;2 cm)</td>
<td>1.5</td>
<td>1.53 ( \left[ \frac{\sigma g(\rho_l-\rho_g)}{\rho_l^2} \right]^{\frac{1}{4}} )</td>
</tr>
<tr>
<td>Churn flow</td>
<td>0</td>
<td>1.53 ( \left[ \frac{\sigma g(\rho_l-\rho_g)}{\rho_l^2} \right]^{\frac{1}{4}} )</td>
</tr>
<tr>
<td>Slug flow (in tube of diameter ( D ))</td>
<td>0</td>
<td>0.35 ( \sqrt{\frac{g(\rho_l-\rho_g)}{\rho_l D}} )</td>
</tr>
</tbody>
</table>

Table B.1: Zuber parameters for the slip ratio

The flow regimes and concentration parameter are defined as follows:

<table>
<thead>
<tr>
<th>Void fraction</th>
<th>Flow Regime</th>
<th>( C_o )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; ( \alpha ) ≤ 0.1</td>
<td>Large bubbles</td>
<td>0</td>
</tr>
<tr>
<td>0.1 &lt; ( \alpha ) ≤ 1</td>
<td>Churn flow</td>
<td>1</td>
</tr>
</tbody>
</table>

Table B.2: Determination of flow regime
Appendix C

The GENESIS facility

A schematic view of the GENESIS facility is depicted in the figure below. It was obtained from [22].

![Schematic view of the GENESIS facility](image)

Figure C.1: The GENESIS facility

In table C.1 additional information is given about the friction distribution in STEALTH. The friction factors for bends, inactive feedwater inlets (subscript inlets), and the active feedwater inlet (subscript \( fw \)) were obtained from [23]. The friction factor for the spacers was taken from [1] and those for the core inlet and the riser outlet valve were calibrated by comparing numerical
results with pressure drop measurements from GENESIS. For $K_{value}$ the non-destabilized and the destabilized values are depicted. Theoretically, a $K$-factor of 200 corresponds to a ball valve that is turned shut over an angle of about 60°.

$$
\begin{array}{|c|c|c|c|c|}
\hline
K_{bend} & K_{sp} & K_{core,in} & K_{inlets} & K_{fw} & K_{value} \\
\hline
1.30 & 0.69 & 5.20 & 0.40 & 1.30 & 1.00 & 200 \\
\hline
\end{array}
$$

Table C.1: Friction factors for STEALTH

The dimensions and technical details of the GENESIS facility are given in table C.2. Further details can be found in [1].

$$
\begin{array}{|c|c|c|c|}
\hline
\text{Parameter} & \text{Symbol} & \text{Unit} & \text{Unit} \\
\hline
\text{Nominal power} & P & kW & 25.4 \\
\text{Nominal subcooling number} & N_{sub} & - & 0.90 \\
\text{Nominal phase change number} & N_{pich} & - & 5.20 \\
\text{Nominal pressure} & p & bar & 11.4 \\
\text{Fuel rods} & \text{number} & - & 25 \\
& (heated) length & L_{fr} & m & 1.41 \\
& diameter & D_{fr} & cm & 0.60 \\
\text{Core} & \text{length} & L_{fr} & m & 1.91 \\
& diameter & D_{c} & cm & 3.60 \\
\text{Riser} & \text{length} & L_{c} & m & 5.90 \\
& diameter & D_{r} & cm & 4.20 \\
\text{Downcomer} & \text{length} & L_{r} & m & 10.44 \\
& diameter & D_{dc} & cm & 2.29 \\
\text{Pressure vessel} & \text{length} & L_{vessel} & m & 0.68 \\
& diameter & D_{vessel} & cm & 30.60 \\
\hline
\end{array}
$$

Table C.2: Technical details for the GENESIS facility
### Stability analysis data

The same table that was given in section 5.3 for the destabilized case ($K_{\text{valve}} = 200$) is given here for the non-destabilized case ($K_{\text{valve}} = 1$).

<table>
<thead>
<tr>
<th>Distance (mm)</th>
<th>Transit time (s)</th>
<th>Time lag (s)</th>
<th>$f_{\text{res}}$ (Hz)</th>
<th>DR</th>
<th>$\phi/2\pi$ (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0*</td>
<td>0</td>
<td>-</td>
<td>0.139</td>
<td>0.013</td>
<td>0.11</td>
</tr>
<tr>
<td>359*</td>
<td>0.61</td>
<td>0.84</td>
<td>0.135</td>
<td>0.013</td>
<td>0.11</td>
</tr>
<tr>
<td>564</td>
<td>0.73</td>
<td>0.92</td>
<td>0.162</td>
<td>0.054</td>
<td>0.15</td>
</tr>
<tr>
<td>1169</td>
<td>1.17</td>
<td>1.37</td>
<td>0.162</td>
<td>0.075</td>
<td>0.22</td>
</tr>
<tr>
<td>1834</td>
<td>1.66</td>
<td>1.94</td>
<td>0.159</td>
<td>0.087</td>
<td>0.31</td>
</tr>
<tr>
<td>2369</td>
<td>2.06</td>
<td>2.32</td>
<td>0.155</td>
<td>0.094</td>
<td>0.36</td>
</tr>
<tr>
<td>2869</td>
<td>2.42</td>
<td>2.64</td>
<td>0.151</td>
<td>0.099</td>
<td>0.40</td>
</tr>
<tr>
<td>3379</td>
<td>2.78</td>
<td>2.96</td>
<td>0.148</td>
<td>0.096</td>
<td>0.44</td>
</tr>
<tr>
<td>3849*</td>
<td>2.92</td>
<td>3.33</td>
<td>0.135</td>
<td>0.080</td>
<td>0.45</td>
</tr>
<tr>
<td>4379</td>
<td>3.50</td>
<td>3.75</td>
<td>0.141</td>
<td>0.076</td>
<td>0.53</td>
</tr>
<tr>
<td>4849*</td>
<td>3.58</td>
<td>4.06</td>
<td>0.128</td>
<td>0.060</td>
<td>0.52</td>
</tr>
<tr>
<td>5378</td>
<td>4.23</td>
<td>4.41</td>
<td>0.137</td>
<td>0.052</td>
<td>0.60</td>
</tr>
<tr>
<td>6383</td>
<td>4.95</td>
<td>5.07</td>
<td>0.138</td>
<td>0.033</td>
<td>0.70</td>
</tr>
<tr>
<td>7343</td>
<td>5.65</td>
<td>5.82</td>
<td>0.147</td>
<td>0.029</td>
<td>0.85</td>
</tr>
<tr>
<td>8318</td>
<td>6.36</td>
<td>6.47</td>
<td>0.157</td>
<td>0.039</td>
<td>1.02</td>
</tr>
<tr>
<td>9304</td>
<td>7.07</td>
<td>7.25</td>
<td>0.160</td>
<td>0.052</td>
<td>1.16</td>
</tr>
<tr>
<td>10280</td>
<td>7.78</td>
<td>7.90</td>
<td>0.156</td>
<td>0.058</td>
<td>1.24</td>
</tr>
</tbody>
</table>

Table D.1: Numerical data for different sparger positions for the nominal case

The experimental data is given in table D.2 on the next page. The uncertainties in resonance frequency and decay ratio for both GENESIS and STEALTH are summed up in table D.3. Note the order of magnitude in the column headings.
### Table D.2: Numerical data for different sparger positions for the GENESIS facility

<table>
<thead>
<tr>
<th>Distance (mm)</th>
<th>Time lag (s)</th>
<th>$f_{res}$ (Hz)</th>
<th>DR</th>
<th>$\phi/2\pi$ (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>564</td>
<td>-0.10</td>
<td>0.129</td>
<td>0.259</td>
<td>-0.01</td>
</tr>
<tr>
<td>1169</td>
<td>0.78</td>
<td>0.127</td>
<td>0.468</td>
<td>0.10</td>
</tr>
<tr>
<td>1834</td>
<td>1.03</td>
<td>0.121</td>
<td>0.627</td>
<td>0.12</td>
</tr>
<tr>
<td>2369</td>
<td>1.56</td>
<td>0.116</td>
<td>0.618</td>
<td>0.18</td>
</tr>
<tr>
<td>2869</td>
<td>2.13</td>
<td>0.110</td>
<td>0.556</td>
<td>0.24</td>
</tr>
<tr>
<td>3379</td>
<td>2.61</td>
<td>0.108</td>
<td>0.560</td>
<td>0.28</td>
</tr>
<tr>
<td>4379</td>
<td>3.60</td>
<td>0.103</td>
<td>0.408</td>
<td>0.37</td>
</tr>
<tr>
<td>5378</td>
<td>4.81</td>
<td>0.103</td>
<td>0.260</td>
<td>0.49</td>
</tr>
<tr>
<td>6383</td>
<td>5.72</td>
<td>0.102</td>
<td>0.154</td>
<td>0.58</td>
</tr>
<tr>
<td>7343</td>
<td>6.71</td>
<td>0.105</td>
<td>0.248</td>
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</tr>
<tr>
<td>8318</td>
<td>7.76</td>
<td>0.109</td>
<td>0.113</td>
<td>0.85</td>
</tr>
<tr>
<td>9304</td>
<td>8.53</td>
<td>0.108</td>
<td>0.414</td>
<td>0.92</td>
</tr>
<tr>
<td>10280</td>
<td>9.41</td>
<td>0.114</td>
<td>0.296</td>
<td>1.08</td>
</tr>
</tbody>
</table>

### Table D.3: Uncertainties for STEALTH and GENESIS data

<table>
<thead>
<tr>
<th>Distance (mm)</th>
<th>$u(f_{res}) \cdot 10^{-3}$ (Hz)</th>
<th>$u(DR) \cdot 10^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GENESIS</td>
<td>STEALTH</td>
</tr>
<tr>
<td></td>
<td>dest.</td>
<td>non-dest.</td>
</tr>
<tr>
<td>0*</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>359*</td>
<td>4.2</td>
<td>0.25</td>
</tr>
<tr>
<td>564</td>
<td>0.30</td>
<td>2.7</td>
</tr>
<tr>
<td>1169</td>
<td>0.29</td>
<td>3.0</td>
</tr>
<tr>
<td>1834</td>
<td>0.52</td>
<td>2.1</td>
</tr>
<tr>
<td>2369</td>
<td>0.58</td>
<td>1.5</td>
</tr>
<tr>
<td>2869</td>
<td>0.18</td>
<td>1.2</td>
</tr>
<tr>
<td>3379</td>
<td>0.10</td>
<td>1.1</td>
</tr>
<tr>
<td>3849*</td>
<td>0.11</td>
<td>0.89</td>
</tr>
<tr>
<td>4379</td>
<td>0.11</td>
<td>4.8</td>
</tr>
<tr>
<td>4849*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5378</td>
<td>0.65</td>
<td>1.2</td>
</tr>
<tr>
<td>6383</td>
<td>0.65</td>
<td>2.8</td>
</tr>
<tr>
<td>7343</td>
<td>0.11</td>
<td>5.3</td>
</tr>
<tr>
<td>8318</td>
<td>0.63</td>
<td>5.7</td>
</tr>
<tr>
<td>9304</td>
<td>0.19</td>
<td>4.5</td>
</tr>
<tr>
<td>10280</td>
<td>0.25</td>
<td>3.8</td>
</tr>
</tbody>
</table>

Table D.3: Uncertainties for STEALTH and GENESIS data
Appendix E

Determination of decay ratios and core exit quality

E.1 Decay ratio and resonance frequency

For the determination of the decay ratios the following procedure has been employed. The analysis is done with MatLab. The time signals of mass flow and temperature are first detrended, after which the auto correlation is performed. It is then normalized to 1 and the right half of the correlation is selected for analysis. Of the resulting signal only a pre-defined first part is considered, typically about 3 times the initial guess for the period, which is then fitted using the following model function:

\[ y = \beta_1 e^{\beta_4 x} \cos (\beta_5 (x - \beta_2)) + \beta_3 e^{\beta_6 x} \]

For the \( \beta \)-vector an emperically sound first guess is used. The \textit{nlinfit} function in Matlab then fits the coefficients of this non-linear regression function using least squares estimation, returning a new estimate for \( \beta \). This procedure is repeated once more, producing the ultimate estimate for the \( \beta \)-vector. The decay ratio is by definition equal to the decay of the cosine in the model. Since the correlation was normalized to 1 and the period of the cosine is \( \frac{2\pi}{\beta_5} \), it can be calculated as:

\[ DR = e^{2\pi \beta_4 / \beta_5} \]

with an uncertainty of

\[ u(DR) = \sqrt{\left( \frac{\partial \beta_4}{\partial DR} d\beta_4 \right)^2 + \left( \frac{\partial \beta_5}{\partial DR} d\beta_5 \right)^2} \]

in which \( d\beta \) is half the width of the 95% confidence interval of the corresponding \( \beta \). The resonance frequency is the reciprocal of the period of the fit:

\[ f_{\text{res}} = \frac{\beta_5}{2\pi} \]

with an uncertainty of twice the standard deviation:

\[ u(f_{\text{res}}) = \frac{1}{2\pi} d\beta_5 \]
E.2 Core exit quality

The steady-state position (height) of the boiling boundary in the core is determined as follows:

\[ z_{bb} = \frac{M}{A\rho_l} \left( h_l - h_{in} \right) \frac{q'}{A\rho_l} \]  

(E.1)

in which the first fraction is the single phase coolant velocity in the core and the latter part is the time it takes for the coolant to reach the saturation enthalpy. The core exit quality is then determined by the evaporation enthalpy and the energy added to the coolant after it has reached the saturation temperature:

\[ \chi = q' \frac{L_{heated}}{Mh_\Delta} = P \frac{1 - M}{P} \frac{h_l - h_{in}}{Mh_\Delta} \]

\[ = \frac{N_{pch} - N_{sub}}{N_\rho} \]

(E.2)

The core exit quality can thus be written in terms of the dimensionless numbers \( N_{pch}, N_{sub} \) and \( N_\rho \).
Figure F.1 shows the representation of frequencies in the power perturbation, from which it is clear that the relevant frequencies (up to 0.4 Hz) are sufficiently present in the input signal of the transfer function of the system.

Figure F.1: Discrete Fourier transform of the perturbation in power
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_p$</td>
<td>specific heat (constant pressure)</td>
<td>$J , kg^{-1} , K^{-1}$</td>
</tr>
<tr>
<td>$f$</td>
<td>friction factor</td>
<td></td>
</tr>
<tr>
<td>$f$</td>
<td>frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration</td>
<td>$m , s^{-2}$</td>
</tr>
<tr>
<td>$h$</td>
<td>specific enthalpy</td>
<td>$J , kg^{-1}$</td>
</tr>
<tr>
<td>$k_i$</td>
<td>Scalabrin viscosity parameter</td>
<td></td>
</tr>
<tr>
<td>$l_i$</td>
<td>Scalabrin viscosity parameter</td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>Zuber slip ratio parameter</td>
<td></td>
</tr>
<tr>
<td>$n_i$</td>
<td>Scalabrin viscosity parameter</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>pressure</td>
<td>$kg , m^{-1} , s^{-2}$</td>
</tr>
<tr>
<td>$q''$</td>
<td>heat flux</td>
<td>$J , m^{-2} , s^{-1}$</td>
</tr>
<tr>
<td>$q'$</td>
<td>one-dimensional heat flux</td>
<td>$J , m^{-1} , s^{-1}$</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
<td>s</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>time step</td>
<td>s</td>
</tr>
<tr>
<td>$u$</td>
<td>specific internal energy</td>
<td>$J , kg^{-1}$</td>
</tr>
<tr>
<td>$v$</td>
<td>velocity</td>
<td>$m , s^{-1}$</td>
</tr>
<tr>
<td>$v$</td>
<td>volumetric velocity</td>
<td>$m , s^{-1}$</td>
</tr>
<tr>
<td>$x$</td>
<td>coordinate in one-dimensional space</td>
<td>m</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>grid size</td>
<td>m</td>
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<tr>
<td>$z_{bb}$</td>
<td>height of boiling boundary</td>
<td>m</td>
</tr>
<tr>
<td>$A$</td>
<td>cross-sectional area</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$C_o$</td>
<td>concentration parameter</td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td>diameter</td>
<td>m</td>
</tr>
<tr>
<td>$DR$</td>
<td>decay ratio</td>
<td></td>
</tr>
<tr>
<td>$F$</td>
<td>proportionality factor</td>
<td></td>
</tr>
<tr>
<td>$G$</td>
<td>Gibb’s free energy</td>
<td>J</td>
</tr>
<tr>
<td>$H$</td>
<td>enthalpy</td>
<td>J</td>
</tr>
<tr>
<td>$H_c$</td>
<td>Scalabrin viscosity parameter</td>
<td>$kg , m^{-1} , s^{-1}$</td>
</tr>
<tr>
<td>$K$</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>$M$</td>
<td>mass flow</td>
<td>$kg , s^{-1}$</td>
</tr>
<tr>
<td>$N$</td>
<td>total number of nodes</td>
<td></td>
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<td>perimeter</td>
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<td>J</td>
</tr>
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<td>J</td>
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<td>entropy</td>
<td>$J , K^{-1}$</td>
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<tr>
<td>slip ratio</td>
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<td></td>
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<tr>
<td>Symbol</td>
<td>Description</td>
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</tr>
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<td>------------------------</td>
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<td>temperature</td>
<td>$K$</td>
</tr>
<tr>
<td>$U$</td>
<td>internal energy</td>
<td>$J$</td>
</tr>
<tr>
<td>$V$</td>
<td>volume</td>
<td>$m^3$</td>
</tr>
<tr>
<td>$V_g$</td>
<td>effective drift velocity</td>
<td>$m \cdot s^{-1}$</td>
</tr>
<tr>
<td>$V_\infty$</td>
<td>bubble rise terminal velocity</td>
<td>$m \cdot s^{-1}$</td>
</tr>
<tr>
<td>$W$</td>
<td>work</td>
<td>$J$</td>
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</tbody>
</table>

**Greek**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>void fraction</td>
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<td>$\beta$</td>
<td>volumetric flow ratio</td>
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<td>$\epsilon$</td>
<td>wall roughness</td>
<td>$m$</td>
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<td>$\lambda$</td>
<td>relaxation constraint</td>
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<td>$kg^2 \cdot m^{-4} \cdot s^{-1}$</td>
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<td>kinematic viscosity</td>
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<td>$kg \cdot m^{-1} \cdot s^{-2}$</td>
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<tr>
<td>$\phi$</td>
<td>phase difference</td>
<td>$rad$</td>
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<tr>
<td>$\chi$</td>
<td>flow quality</td>
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<tr>
<td>$\chi_{st}$</td>
<td>static quality</td>
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</tr>
<tr>
<td>$\Phi^2$</td>
<td>two-phase friction multiplier</td>
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**subscripts**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$c$</td>
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<td>inside the pressure vessel</td>
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<td>Symbol</td>
<td>Description</td>
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<td>( w )</td>
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<tr>
<td>( \Delta )</td>
<td>evaporation</td>
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</tbody>
</table>

superscripts

- \( n \) time step
- \( o \) stagnation
- \( TP \) two-phase
- \( + \) dynamic
- \( * \) initial guess
- \( \prime \) correction

math

- \([ \quad ]_p \) at constant pressure
- \([ \quad ]_T \) at constant temperature
- \([ \quad ]_\alpha \) at constant void fraction
- \(< > \) average over area
- \(~\) non-averaged

Dimensionless numbers

\[
Re = \frac{\rho \|\vec{v}\| D_h}{\mu}
\]

\[
N_\rho = \frac{\rho_l - \rho_a}{\rho_g}
\]

\[
N_{\text{sub}} = \frac{(h_L - h_{\text{in}}) N_\rho}{h_\Delta}
\]

\[
N_{\text{pch}} = \frac{P N_\rho}{M h_\Delta}
\]
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