Modelling fuel behaviour in a reactor park using fuel cycle kinetics.

Master Thesis Report

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Modelling fuel behaviour in a reactor park using fuel cycle kinetics

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Abstract

In this thesis the theory of fuel cycle kinetics is re-examined. The fuel cycle kinetics theory is a powerful tool to describe the time-dependent fuel behaviour of large populations of nuclear reactors. The fuel cycle kinetics theory is based on the point kinetics theory and the principles of a reactor park. The point kinetics theory is a simplification of the space-, energy- and time-dependent diffusion balance equation to only a time-dependent equation. A reactor park is the description of the interconnections between a population of nuclear reactors with various designs. In the fuel cycle kinetics theory the point kinetics theory is used as a model to simplify space-energy- and time-dependent burn-up equations of the reactors in a reactor park to a set of only time-dependent equations, one for every reactor type.

The fuel cycle kinetics theory is verified by means of a number of test cases. In the first test case the same symbiotic system is used as was used by Maudlin. There is no difference between the two obtained results. The second test case is that of only Fast Breeder Reactor, FBR, deployment. Here the result of the fuel cycle kinetics equation is checked against the result obtained from TRITON. TRITON is a module of the SCALE code system that is used for depletion analysis of 3-D reactor models. With the use of the pseudo-initial condition the results of the fuel cycle kinetics and TRITON calculations are almost identical. The pseudo-initial condition is a correction on the initial condition to adjust for neglecting the time dependency of the parameters in the fuel cycle kinetics equations. In the third case a symbiotic system of FBRs and Pressurised Water Reactors, PWRs, is researched. There is only a small difference in the asymptotic growth between the fuel cycle kinetics results and the TRITON results. In the last test case the same system of FBRs and PWRs is used to investigate two demanded asymptotic growths obtained from the upper and lower boundary of the expected growth of nuclear reactors in the upcoming years. Two iteration steps on the coupling matrices in the fuel cycle kinetics equations were needed to reach these asymptotic growths. The results obtained from the last iteration in the fuel cycle kinetics equations are almost identical to the results obtained from the TRITON calculations with the coupling obtained from the iteration.

The test cases demonstrate that the expectations of the theory of being computationally cheap and accurate in predicting the fuel behaviour of nuclear reactor populations over longer time spans are correct.
# Contents

1 Introduction .................................................. 3  
  1.1 Fuel Cycle Kinetics ........................................ 4  
  1.2 Objective .................................................. 4  
  1.3 Outline of this Thesis ...................................... 4  

2 Point Kinetics ............................................... 7  
  2.1 Factorisation and Weighting ................................. 7  
  2.2 Point Kinetics Equations .................................. 8  

3 Reactor Park .................................................. 13  
  3.1 Reactor Types ............................................... 13  
  3.1.1 Nuclear Reactions ....................................... 14  
  3.1.2 Light Water Reactor .................................... 14  
  3.1.3 Fast Breeder Reactor .................................... 14  
  3.1.4 Reactor Composition .................................... 15  
  3.2 Single Reactor .............................................. 15  
  3.3 Fuel Accumulation .......................................... 17  
  3.3.1 External Fuel Cycle .................................... 17  
  3.3.2 Temporary Storage Pool ................................. 18  
  3.3.3 Fuel Accumulation ....................................... 18  
  3.4 Reactor Accumulation ....................................... 19  
  3.4.1 Factorisation and Weighting ............................. 19  
  3.4.2 Reactor Accumulation .................................... 20  
  3.4.3 Asymptotic Growth Rate ................................. 21  

4 Fuel cycle kinetics .......................................... 23  
  4.1 Factorisation and Weighting ................................. 23  
  4.2 Parameters .................................................. 24  
  4.2.1 Generation Time ......................................... 24  
  4.2.2 Reactivity ................................................ 25  
  4.2.3 Delayed Critical Mass Fraction ......................... 25  
  4.2.4 Delayed Critical Mass Source ......................... 26  
  4.2.5 Discharge ................................................. 27  
  4.2.6 Charge .................................................... 27  
  4.2.7 Core Coupling ............................................ 28
Nomenclature

\(A\) General matrix
\(\beta\) Delayed fraction
\(\chi\) Emission spectrum
\(\Delta t\) Fuel cycle period
\(\delta\) Core discharge
\(\epsilon\) Reprocessing loss
\(\gamma\) Growth rate
\(\kappa\) Discharge rate
\(\Lambda\) Generation time
\(\lambda\) Microscopic reaction rate/Radioactive decay constant
\(\nu\) Yield
\(\Phi\) Flux
\(\pi\) Production rate
\(\psi\) Average core composition
\(\rho\) Reactivity
\(\Sigma\) Macroscopic cross section
\(\tau\) Fuel cycle lag time
\(\theta\) Blanket fraction
\(C\) Production and Loss matrix
\(C_{T'B\rightarrow T'}\) Coupling matrix from blanket \(T'\) to core \(T\)
\(C_{T'C\rightarrow T}\) Coupling matrix from core \(T'\) to core \(T\)
\(D\) Discharge matrix
\begin{align*}
F & \quad \text{Fission source operator} \\
H & \quad \text{Matrix with the difference between the discharge composition of the core and the residing composition} \\
L & \quad \text{Loss matrix} \\
M & \quad \text{Migration and Loss operator} \\
P & \quad \text{Production matrix} \\
\varepsilon_{T',B\to T} & \quad \text{Coupling coefficient of blanket } T' \text{ to core } T \\
\varepsilon_{T',C\to T} & \quad \text{Coupling coefficient of core } T' \text{ to core } T \\
\xi & \quad \text{Charge rate} \\
\zeta & \quad \text{Reduced precursor} \\
B & \quad \text{Number of blankets} \\
b & \quad \text{Breeding bonus} \\
C & \quad \text{Precursor} \\
c & \quad \text{Reduced precursor} \\
E & \quad \text{Energy} \\
L & \quad \text{Loss rate} \\
LHS & \quad \text{Left-hand side} \\
N & \quad \text{Amount of atoms} \\
n & \quad \text{Number of batches in the core} \\
P & \quad \text{Production rate} \\
R & \quad \text{Number of reactors} \\
RHS & \quad \text{Right-hand side} \\
S & \quad \text{Source} \\
s & \quad \text{Reduced source} \\
t & \quad \text{Time} \\
v & \quad \text{Velocity} \\
w & \quad \text{Weights} \\
w & \quad \text{Absolute worth} \\
\textbf{Subscripts} \\
0 & \quad \text{Initial condition}
\end{align*}
<table>
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<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>(a)</td>
<td>Absorption</td>
</tr>
<tr>
<td>(B)</td>
<td>Blanket</td>
</tr>
<tr>
<td>(C)</td>
<td>Core</td>
</tr>
<tr>
<td>(c)</td>
<td>Capture</td>
</tr>
<tr>
<td>(d)</td>
<td>Delayed</td>
</tr>
<tr>
<td>(F)</td>
<td>Fast Breeder Reactor</td>
</tr>
<tr>
<td>(f)</td>
<td>Fission</td>
</tr>
<tr>
<td>(i)</td>
<td>Isotope index</td>
</tr>
<tr>
<td>(k)</td>
<td>Delay group</td>
</tr>
<tr>
<td>(L)</td>
<td>Light Water Reactor</td>
</tr>
<tr>
<td>(P)</td>
<td>Pressurised Water Reactor</td>
</tr>
<tr>
<td>(p)</td>
<td>Prompt</td>
</tr>
<tr>
<td>(R)</td>
<td>Reactor index</td>
</tr>
<tr>
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<td>Radioactive decay</td>
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<tr>
<td>(s)</td>
<td>Scatter</td>
</tr>
<tr>
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<tr>
<td>(t)</td>
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</tr>
<tr>
<td>(x)</td>
<td>External fuel cycle</td>
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</tbody>
</table>

**Superscripts**

<table>
<thead>
<tr>
<th>Superscript</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0)</td>
<td>Temporary storage pool</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Reactivity</td>
</tr>
<tr>
<td>(\rightarrow)</td>
<td>Vector</td>
</tr>
<tr>
<td>(\wedge)</td>
<td>Accumulation</td>
</tr>
<tr>
<td>(cm)</td>
<td>Critical mass</td>
</tr>
</tbody>
</table>

vii
List of Figures

1.1 Prediction of the maximum and minimum energy production by nuclear reactors [2]. 3
3.1 Scheme of a reactor park. .......................... 13
3.2 Core composition and blanket composition averaged per cycle after every cycle in
time for an FBR .......................... 16
3.3 Accumulations of isotopes and FBRs in an FBR deployment. .......................... 19
3.4 Reactor accumulation determined with use of $\gamma$. .......................... 21
4.1 Scheme of the coupling coefficients. .......................... 28
4.2 Initial and Pseudo-initial condition. .......................... 31
4.3 Scheme of the coupling coefficients in a symbiotic deployment of FBRs and LWRs. 34
5.1 Results for a symbiotic deployment of FBRs and LWRs obtained by Maudlin and
by solving fuel cycle kinetics equation with the parameters from Maudlin. .......................... 37
5.2 The growth of reactors in an FBR deployment. .......................... 39
5.3 Axial and radial slice of a block model of a fast breeder reactor. .......................... 39
5.4 Error between the number of FBRs between the TRITON calculation and the
Fuel Cycle Kinetics with and without use of the pseudo-initial condition. .......................... 40
5.5 The growth of reactors in a symbiotic deployment of FBRs and PWRs. .......................... 42
5.6 Errors in the numbers of FBRs and PWRs between the TRITON calculation and
the Fuel Cycle Kinetics with and without use of the Pseudo-Initial Condition. .......................... 43
5.7 Close-up of the errors in the numbers of FBRs and PWRs between the TRITON
calculation and the Fuel Cycle Kinetics with and without use of the Pseudo-Initial Condition. .......................... 44
5.8 Behaviour of the system changing the set-up. .......................... 45
5.9 The upper and lower asymptotic growth of the energy demand by a symbiotic
system of FBRs and PWRs determined with the fuel cycle kinetics equations. .......................... 48
5.10 The upper and lower asymptotic growth of the energy demand by a symbiotic
system of FBRs and PWRs determined with TRITON. .......................... 49
5.11 A symbiotic system of FBRs and PWRs with an asymptotic growth $\gamma_\infty = 4.23\%$
determined by fuel cycle kinetics with the equilibrium compositions and critical
mass weights from the TRITON calculation of section 5.4. .......................... 50
5.12 The upper and lower asymptotic growth of the energy demand by a symbiotic
system of FBRs and PWRs determined with the fuel cycle kinetics equations. .......................... 52
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.13</td>
<td>The upper and lower asymptotic growth of the energy demand by a symbiotic system of FBRs and PWRs determined with TRITON.</td>
<td>53</td>
</tr>
<tr>
<td>5.14</td>
<td>Errors in the numbers of FBRs and PWRs between the TRITON calculation and the Fuel Cycle Kinetics.</td>
<td>54</td>
</tr>
<tr>
<td>5.15</td>
<td>The upper and lower asymptotic growth of the energy demand by a symbiotic system of FBRs and PWRs determined with the fuel cycle kinetics equations.</td>
<td>55</td>
</tr>
<tr>
<td>5.16</td>
<td>Errors in the numbers of FBRs and PWRs between the Fuel Cycle Kinetics solutions.</td>
<td>56</td>
</tr>
<tr>
<td>6.1</td>
<td>Scheme of a reactor park with different designed FBRs and LWRs.</td>
<td>61</td>
</tr>
<tr>
<td>A.1</td>
<td>Prediction of the maximum and minimum energy production by nuclear reactors</td>
<td>65</td>
</tr>
</tbody>
</table>
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>Parameters for an FBR in symbiotic deployment from [3].</td>
<td>36</td>
</tr>
<tr>
<td>5.2</td>
<td>Parameters for an LWR in symbiotic deployment from [3].</td>
<td>36</td>
</tr>
<tr>
<td>5.3</td>
<td>Values for an FBR deployment.</td>
<td>38</td>
</tr>
<tr>
<td>5.4</td>
<td>Asymptotic parameters of an FBR in an FBR deployment.</td>
<td>38</td>
</tr>
<tr>
<td>5.5</td>
<td>Values for an FBR in a symbiotic deployment of FBRs and PWRs.</td>
<td>41</td>
</tr>
<tr>
<td>5.6</td>
<td>Values for a PWR in a symbiotic deployment of FBRs and PWRs.</td>
<td>41</td>
</tr>
<tr>
<td>5.7</td>
<td>Asymptotic parameters of an FBR in a symbiotic deployment.</td>
<td>41</td>
</tr>
<tr>
<td>5.8</td>
<td>Asymptotic parameters of a PWR in a symbiotic deployment.</td>
<td>41</td>
</tr>
<tr>
<td>5.9</td>
<td>Values for the starting set-up of an FBR in a symbiotic deployment of FBRs and PWRs for figure 5.8d.</td>
<td>46</td>
</tr>
<tr>
<td>5.10</td>
<td>Values for the starting set-up of a PWR in a symbiotic deployment of FBRs and PWRs for figure 5.8d.</td>
<td>46</td>
</tr>
<tr>
<td>5.11</td>
<td>Values for the upper boundary of the energy demand for an FBR in a symbiotic deployment of FBRs and PWRs.</td>
<td>47</td>
</tr>
<tr>
<td>5.12</td>
<td>Values for the upper boundary of the energy demand for a PWR in a symbiotic deployment of FBRs and PWRs.</td>
<td>47</td>
</tr>
<tr>
<td>5.13</td>
<td>Asymptotic parameters for the upper boundary of the energy demand for an FBR in a symbiotic deployment.</td>
<td>48</td>
</tr>
<tr>
<td>5.14</td>
<td>Asymptotic parameters for the upper boundary of the energy demand for a PWR in a symbiotic deployment.</td>
<td>48</td>
</tr>
<tr>
<td>5.15</td>
<td>Values for the lower boundary of the energy demand for an FBR in a symbiotic deployment of FBRs and PWRs.</td>
<td>48</td>
</tr>
<tr>
<td>5.16</td>
<td>Values for the lower boundary of the energy demand for a PWR in a symbiotic deployment of FBRs and PWRs.</td>
<td>48</td>
</tr>
<tr>
<td>5.17</td>
<td>Asymptotic parameters for the lower boundary of the energy demand for an FBR in a symbiotic deployment.</td>
<td>49</td>
</tr>
<tr>
<td>5.18</td>
<td>Asymptotic parameters for the lower boundary of the energy demand for a PWR in a symbiotic deployment.</td>
<td>49</td>
</tr>
<tr>
<td>5.19</td>
<td>Values for the upper boundary of the energy demand for an FBR in a symbiotic deployment of FBRs and PWRs.</td>
<td>51</td>
</tr>
<tr>
<td>5.20</td>
<td>Values for the upper boundary of the energy demand for a PWR in a symbiotic deployment of FBRs and PWRs.</td>
<td>51</td>
</tr>
<tr>
<td>5.21</td>
<td>Asymptotic parameters for the upper boundary of the energy demand for an FBR in a symbiotic deployment.</td>
<td>51</td>
</tr>
</tbody>
</table>
5.22 Asymptotic parameters for the upper boundary of the energy demand for a PWR in a symbiotic deployment. ................................................................. 51
5.23 Values for the lower boundary of the energy demand for an FBR in a symbiotic deployment of FBRs and PWRs. ................................................................. 52
5.24 Values for the lower boundary of the energy demand for a PWR in a symbiotic deployment of FBRs and PWRs. ................................................................. 52
5.25 Asymptotic parameters for the lower boundary of the energy demand for an FBR in a symbiotic deployment. ................................................................. 52
5.26 Asymptotic parameters for the lower boundary of the energy demand for a PWR in a symbiotic deployment. ................................................................. 52

A.1 Current Nuclear Programs ......................................................... 66
A.2 Nations Planning Nuclear ....................................................... 67
A.3 Potential Entrants ................................................................. 67
A.4 World Total ................................................................. 68
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CHAPTER 1

Introduction

With the global economic crisis in the past it is time to look forward. The way how the global economy will recover is of great influence on the energy prospects for the upcoming years. It will be the paths that governments lay down on subjects such as climate change and energy security that will determine the direction of energy production in the future. With the world electricity demand growing the most strongly of all final forms of energy [1], and the goal of reduced global CO₂ emissions, countries have to change their energy policies. Due to this a global shift to nuclear power, renewables and low-carbon technologies is expected. The energy strategy that a country chooses is of great influence on the growth of nuclear power. Figure 1.1 shows the amount of nuclear power now operating and the upper and lower predictions for the upcoming century. With this growing demand for nuclear power new innovations are needed.

![Figure 1.1](image.png)

**Figure 1.1:** Prediction of the maximum and minimum energy production by nuclear reactors [2].
1.1 Fuel Cycle Kinetics

In this work a neglected theory that describes the time-dependent fuel behaviour of nuclear reactors is re-examined. This theory is called fuel cycle kinetics. The field of fuel cycle kinetics was first researched in the 1970s by Paul Joseph Maudlin [3]. During his Ph.D. he developed the theory of fuel cycle kinetics.

The fuel cycle kinetics theory projects the behaviour of a population of reactors due to their designs and fuel cycles in a short amount of time and in a computationally cheap way. This is done with an analogy to the point kinetics theory and the use of the principle of a reactor park. The point kinetics theory is a simplification of the space-, energy- and time-dependent diffusion balance equation into only a time-dependent equation. A reactor park can be defined as an interconnected population of reactors of various types with different fuel cycles. In the fuel cycle kinetics theory, the point kinetics theory is used to simplify the space-, energy- and time-dependent burn-up equations of reactors into only time-dependent equations. This creates a set of equations, the fuel cycle kinetics equations. Each equation describes the fuel behaviour of one type of reactor and the interconnection between the reactor and other types of reactors in the reactor park.

1.2 Objective

The goal of this thesis is to create a working fuel cycle kinetics model and test its use in predicting reactor park behaviour. In the model the input parameters of the fuel cycle kinetics equations are created and these equations are solved. The solutions of the fuel cycle kinetics are then checked by TRITON [4] calculations on the same systems.

1.3 Outline of this Thesis

The path to the fuel cycle kinetics laid out in this thesis starts with the derivation of the point kinetics equations, chapter 2. The point kinetics equations are a widely used simplification of the diffusion balance equations for the space-, energy and time-dependent neutron flux. These diffusion balance equations can be approximated by only time-dependent equations, the point kinetics equations.

In chapter 3 a method is introduced that explains how, from the burn-up equations of a single reactor, the fuel behaviour of multiple reactors can be described. By implementing the concept of the external fuel cycle and a temporary storage pool an interconnection between reactors is created and the description of a reactor park is born. In the concept of a reactor park the reactors do not have to be situated at the same geographical location, they only have to be linked by their external fuel cycles.

With the use of the concept of a reactor park and the point kinetics theory, a theory can be derived for describing the time-dependent fuel behaviour of populations of nuclear reactors, called the fuel cycle kinetics theory. This is done in chapter 4. The fuel cycle kinetics equations are a set of simple ODEs which consist of parameters that describe the production, loss and transfer of critical mass between the multiple nuclear reactors of the same and different types situated in a reactor park.

With the fuel cycle kinetics equations multiple test cases will be used to check the theory. The first test case checks the working of the fuel cycle kinetics equations with data at hand from Maudlin’s thesis. The other test cases that are performed are the deployment of only FBRs and
a symbiotic system of FBRs and PWRs. For the symbiotic system two cases are done, one to verify the results obtained by comparison to TRITON calculations on such a system and one on fulfilling a pre-determined asymptotic growth of nuclear reactors, which is then checked by TRITON calculations. All results are shown and explained in chapter 5.

In the final chapter, chapter 6, the theory and results will be discussed and recommendations are given on how to enhance the accuracy and make the work more realistic in the future. Also recommendations are given on how to extend the theory to use it for other purposes.
CHAPTER 2

Point Kinetics

In this chapter the point kinetics equations will be derived. They are the underlying theory for the fuel cycle kinetics which will be derived in chapter 4. The point kinetics equations are a simplification of the space-, energy- and time-dependent diffusion equations to only time-dependent equations. Only the general outline of the theory will be given here, a more detailed description can be found in [5].

2.1 Factorisation and Weighting

The first step towards the point kinetics equations is the factorisation of the neutron flux, \( \phi(\vec{r}, E, t) \), into a time-dependent amplitude, \( p(t) \), and a space-, energy-, and time-dependent shape function, \( \psi(\vec{r}, E, t) \).

\[
\phi(\vec{r}, E, t) = p(t) \cdot \psi(\vec{r}, E, t) \tag{2.1}
\]

This factorisation is done because the flux variation in time has changes that are primarily in the flux amplitude and there are only small changes in the shape function. To make this factorisation unique an extra equation is needed. This equation is used to shift the major time-dependence into the amplitude function by keeping the integral, over space and energy, of the shape function constant in time. Before integration a weight function, \( w(\vec{r}, E) \) is added. The constraint condition on the time-dependence of the shape function is given by imposing

\[
\int_V \int_0^\infty w(\vec{r}, E) \psi(\vec{r}, E, t) \frac{v(E)}{v_0(E)} dEdV = constant \tag{2.2}
\]

The weight function should be chosen in such a way that it minimizes the error that results from inaccuracies in the shape function. In point kinetics the solution is sensitive to an error in the reactivity. From perturbation theory [5] it follows that using the initial adjoint flux, \( \Phi_0^* \), as the weighting function reduces these inaccuracies. For this reason the weighting function in point kinetics is

\[
\vec{w}(\vec{r}, E) = \Phi_0^*(\vec{r}, E) \tag{2.3}
\]
2.2. POINT KINETICS EQUATIONS

With the initial adjoint flux as the weighting function the constraint condition of the shape function for the point kinetics becomes

\[
\int_V \int_0^\infty \Phi_0(\vec{r},E)\phi(\vec{r},E,t) \frac{dE}{v(E)} dV = K_0 \tag{2.4}
\]

where \(K_0\) is a constant.

2.2 Point Kinetics Equations

The starting point for the derivation of the point kinetics equation is the time-dependent diffusion equation.

\[
\frac{1}{v} \frac{\partial \Phi}{\partial t} = (F_p - \mathcal{M}) \Phi + S_d \tag{2.5}
\]

where \(F_p\) is the prompt fission operator, which describes the production of prompt neutrons, where prompt neutron are the neutrons that are emitted instantly after a fission event. The migration operator \(\mathcal{M}\) describes the loss of neutrons due to absorption and leakage. Further \(v\) is the speed, \(\Phi\) the flux and \(S_d\) the delayed neutron source. The delayed neutron source describes the production of delayed neutrons. There is no independent source, \(S\), in the diffusion equation because the derivation is done for a critical reactor. The total fission operator \(F\) can be written in a prompt part and a delayed part

\[
F = F_p + F_d \tag{2.6}
\]

which can be written out in the following integrals

\[
\frac{1}{v} \frac{\partial \Phi}{\partial t} = \int_{E'} \chi_p(E') \int \nu_p(E') \Sigma_f(\vec{r},E',t) \phi(\vec{r},E',t) dE' + \int_{E'} \chi_d(E') \int \nu_d(E') \Sigma_f(\vec{r},E',t) \phi(\vec{r},E',t) dE' \tag{2.7}
\]

with \(\chi\) being the emission spectrum, \(\nu\) the yield, number of neutrons per fission, and \(\Sigma\) the macroscopic cross section. The operator \(F_d\) does not describe the delayed neutron source. This is done by \(S_d\). The term \(\mathcal{E}\Phi\) is called the quasi-stationary delayed neutron source and describes the delayed neutrons that would be produced in a stationary reactor with fission cross sections and neutron flux as they exist at time ‘t’. The migration operator in diffusion theory is given by

\[
\mathcal{M} \Phi = -\nabla \cdot D(\vec{r},E',t) \nabla \phi(\vec{r},E',t) + \Sigma_i(\vec{r},E',t) \phi(\vec{r},E',t) - \int_{E'} \Sigma_s(\vec{r},E' \rightarrow E,t) \phi(\vec{r},E',t) dE' \tag{2.8}
\]

with \(D\) being the neutron diffusion coefficient. The first term on the RHS of equation (2.8) gives the rate of neutron loss due to leakage. The second and third terms on the RHS of equation (2.8) give the rate of neutron loss caused by absorption.

Replacing the prompt fission operator in equation (2.5) with the fission operator minus the delayed fission operator gives

\[
\frac{1}{v} \frac{\partial \Phi}{\partial t} = (F - \mathcal{M} - F_d) \Phi + S_d \tag{2.9}
\]
The weighting is applied to this equation by taking the inner product with the weighting function, the initial adjoint flux $\Phi_0^*$, which gives

$$\frac{\partial}{\partial t} \left\{ \Phi_0^*, \frac{1}{v} \Phi \right\} = \left\{ \Phi_0^*, (F - M - F_d) \Phi \right\} \quad (2.10)$$

The first term on the LHS of equation (2.10) becomes with use of the factorisation from equation (2.1) the following

$$\frac{\partial}{\partial t} \left\{ \Phi_0^*, \frac{1}{v} \Phi \right\} = \frac{dp}{dt} \left\{ \Phi_0^*, \frac{1}{v} \Phi \right\} + p(t) \frac{\partial}{\partial t} \left\{ \Phi_0^*, \frac{1}{v} \Phi \right\} \quad (2.11)$$

With use of the constraint from equation (2.4) the second term on the RHS of equation (2.11) can be seen as equal to zero. Now applying the factorisation to the rest of equation (2.10) gives

$$\left\{ \Phi_0^*, \frac{1}{v} \Phi \right\} \frac{dp(t)}{dt} = \left\{ \Phi_0^*, (F - M - F_d) \Psi \right\} p(t) + \left\{ \Phi_0^*, S_d \right\} \quad (2.12)$$

From perturbation theory [5], it follows that equation (2.12) needs to be divided by the inner product of the weight function with the product of the fission operator and the flux shape function.

$$F(t) = \left\{ \Phi_0^*, F \Psi \right\} \quad (2.13)$$

which gives the following

$$\left\{ \Phi_0^*, \frac{1}{v} \Psi \right\} \frac{dp(t)}{dt} = \frac{\Phi_0^*, (F - M) \Psi}{\Phi_0^*, F \Psi} p(t) - \frac{\Phi_0^*, F_d \Psi}{\Phi_0^*, F \Psi} p(t) + \frac{\Phi_0^*, S_d}{\Phi_0^*, F \Psi} \quad (2.14)$$

From this the parameters can be defined that create the point kinetics equations. From the LHS of equation (2.14) the generation time can be extracted

$$\Lambda(t) = \left\{ \Phi_0^*, \frac{1}{\psi} \right\}$$

The generation time is the average time between the birth of a single neutron and the birth of a single neutron in the next generation. The first term on the RHS of equation (2.14) is the reactivity.

$$\rho(t) = \frac{\Phi_0^*, (F - M) \Psi}{\Phi_0^*, F \Psi} \quad (2.15)$$

The reactivity is a measure of the difference of the core neutron multiplication from unity. The second term on the RHS of equation (2.14) is the delayed neutron fraction.

$$\beta(t) = \frac{\Phi_0^*, F_d \Psi}{\Phi_0^*, F \Psi} \quad (2.16)$$
The delayed neutron fraction is the fraction of fission neutrons that are delayed due to decay. The last term on the RHS of equation (2.14) is the delayed source.

\[ s_d(t) = \frac{\{\Phi_0', S_d\}}{\{\Phi_0', F\Psi\}} \]  

(2.18)

Taking this all together gives

\[ \Lambda(t) \frac{dp(t)}{dt} = \left( p(t) - \beta(t) \right) p(t) + s_d(t) \]  

(2.19)

The delayed source describes the delayed neutrons that are coming from the decay of precursors. Precursors are fission products that after a fission event undergo radioactive decay and emit a neutron. These neutrons are called the delayed neutrons. Typically these precursors can be split into six delayed groups, \( k \), which depend on the half-lives of the precursors.

With the use of these precursor groups equation (2.17) can also be written in terms of them.

\[ \beta(t) = \sum_k \beta_k(t) \]  

(2.20)

with

\[ \beta_k(t) = \frac{\{\Phi_0', F_{dk}\Psi\}}{\{\Phi_0', \Psi\}} \]  

(2.21)

The delayed neutron source can be described in terms of these six delayed groups as follows

\[ \vec{S}_d = \sum_k \chi_{dk}(E) \lambda_k C_k(\vec{r}, E, t) \]  

(2.22)

With \( \lambda_k \) the radioactive decay constant for group \( k \) and \( C_k \) the precursors for group \( k \). Inserting this to equation (2.18) gives

\[ s_d(t) = \frac{\{\Phi_0', \vec{S}_d\}}{\{\Phi_0', F\Psi\}} = \frac{F_0}{\Phi_0', \Psi} \sum_k \lambda_k \{\Phi_0', C_{dk} \} \]  

(2.23)

\[ = \frac{F_0}{\Phi_0', \Psi} \sum_k \lambda_k \zeta_k \]  

\[ = \frac{\Lambda(t)}{\Lambda_0} \sum_k \lambda_k \zeta_k \]

The time-independent quantity

\[ F_0 = \{\Phi_0', F_0 \Psi_0\} \]  

(2.24)

is introduced in equation (2.23). The reason for this will be explained with the derivation of the balance equation for the reduced precursors, \( \zeta \). The reduced precursors in equation (2.23) can be described by

\[ \zeta_k(t) = \frac{1}{F_0} \{\Phi_0', C_{dk} \} \]  

(2.25)

Delft University of Technology 10

J.A.B. van Rhijn
The balance equation for these reduced precursors can be described by

\[
\frac{d}{dt}\left\{ \Phi^*_{i0}, \chi_{ak} C_k \right\} = -\lambda_k \left\{ \Phi^*_{i0}, \chi_{ak} C_k \right\} + \left\{ \Phi^*_{i0}, F_{ak} \Psi \right\} p(t) \quad (2.26)
\]

Dividing by equation (2.24) and with equations (2.25) and (2.21) gives

\[
\frac{d\zeta_k(t)}{dt} = -\lambda_k \zeta_k(t) + \frac{F(t)}{F_0} \beta_k(t) p(t) \quad (2.27)
\]

The reason for the use of the constant \( F_0 \) is that only a constant can be taken into the derivative on the LHS of equation (2.27). Now taking equations (2.19), (2.23) and (2.27) together gives the point kinetics equations

\[
\frac{dp(t)}{dt} = \left( \frac{\rho(t) - \beta(t)}{\Lambda(t)} \right) p(t) + \frac{1}{\Lambda_0} \sum_k \lambda_k \zeta_k(t) \quad (2.28a)
\]

\[
\frac{d\zeta_k(t)}{dt} = -\lambda_k(t) \zeta_k(t) + \frac{F(t)}{F_0} \beta_k(t) p(t) \quad (2.28b)
\]
Reactor Park

Figure 3.1: Scheme of a reactor park.

A reactor park consists of multiple reactor types with various designs which are interconnected by an external fuel cycle, see figure 3.1. The reactors are all at different points in their cycles and thus have different compositions. The reactors do not have to be at the same geographical location to be part of a reactor park, they only have to be linked to the external fuel cycle. The fuel behaviour of a reactor park can be described by the accumulation of burn-up equations of single reactors for every type.

3.1 Reactor Types

Throughout this thesis two general types of reactor will be used for describing a reactor park, the light water reactor, LWR, and the fast breeder reactor, FBR. Both types will use mixed oxide fuel, MOX, which is UO$_2$ + PuO$_2$. The LWR that is used is a pressurised water reactor, PWR.
3.1. Nuclear Reactions

Three types of reactions are of importance in this thesis, capture, fission and radioactive decay. They look as follows

\[ AX + ^1n \rightarrow A+1X \] (3.1a)
\[ AX + ^1n \rightarrow \text{fission products} \] (3.1b)
\[ AX \rightarrow A-1X + \beta \] (3.1c)

In the capture reaction, equation (3.1a), a neutron is captured by a nucleus and together they form a new nucleus. In fission, equation (3.1b), a neutron is absorbed and after this the new compound nucleus undergoes fission. Other reactions due to absorption like \((n,2n), (n,p)\) etc are not included because the occurrence is not significant. In radioactive decay, equation (3.1c), a nucleus emits a beta-particle and a new nucleus remains. Other types of decay, gamma- and alpha-decay, do occur but the amount of reactions are negligible small.

With the reactions described in equations (3.1a), (3.1b) and (3.1c) the change of fuel in a reactor can be described.

3.1.2 Light Water Reactor

An LWR is a thermal reactor. This means that it operates using neutrons that are moderated to lower energies from the fission spectrum, these moderated neutrons are called thermal neutrons. Thermal neutrons have on average a kinetic energy of 0.025 eV [6] at room temperature. The LWR considered is this thesis has a core that contains only MOX fuel. In this thesis only the growth of plutonium is considered, so only PuO\(_2\) with a \(^{238}\text{U}\) feed is used. For this reason in the LWR the following reactions are of importance [7].

\[ ^{238}\text{U} + ^1n \rightarrow ^{239}\text{U} \] (3.2a)
\[ ^{239}\text{U} \rightarrow ^{239}\text{Np} + \beta \] (3.2b)
\[ ^{239}\text{Np} \rightarrow ^{239}\text{Pu} + \beta \] (3.2c)
\[ ^{239}\text{Pu} + ^1n \rightarrow \text{fission products} \] (3.2d)
\[ ^{239}\text{Pu} + ^1n \rightarrow ^{240}\text{Pu} \] (3.2e)
\[ ^{240}\text{Pu} + ^1n \rightarrow ^{241}\text{Pu} \] (3.2f)
\[ ^{241}\text{Pu} + ^1n \rightarrow \text{fission products} \] (3.2g)
\[ ^{241}\text{Pu} + ^1n \rightarrow ^{242}\text{Pu} \] (3.2h)

The first three reactions are the capture of a neutron by \(^{238}\text{U}\) which then undergoes double \(\beta\)-decay into \(^{239}\text{Pu}\). Further what can be seen is that only \(^{239}\text{Pu}\) and \(^{241}\text{Pu}\) fission in the thermal spectrum, in the other plutonium isotopes fission occurs but these amounts are very small. The other changes are caused by capture. Further only the plutonium isotopes from \(^{239}\text{Pu}\) to \(^{242}\text{Pu}\) are taken into account, because the concentration of the lower and higher plutonium isotopes is very small compared to these isotopes. Other reactions occur next to beta-decay in the isotopes \(^{239}\text{U}\) and \(^{239}\text{Np}\) but this amount is negligible small compared to the amount of beta-decay reactions.

3.1.3 Fast Breeder Reactor

An FBR is a fast reactor that breeds fuel. It is a fast reactor because it uses neutrons that are in the fast spectrum, which means that the majority of the neutrons have an energy > 100 keV.
Some neutrons though, will be in the thermal and epithermal range. [6]. The FBR consists of a core and a blanket. Initially the blanket is filled with $^{238}$U and the core with MOX fuel, the same as in the LWR only PuO$_2$ with a $^{238}$U feed is used. In the blanket the following reactions occur

$$^{238}\text{U} + \gamma \rightarrow ^{239}\text{U} \quad (3.3a)$$
$$^{239}\text{U} \rightarrow ^{239}\text{Np} + \beta \quad (3.3b)$$
$$^{239}\text{Np} \rightarrow ^{239}\text{Pu} + \beta \quad (3.3c)$$
$$^{238}\text{U} + \gamma \rightarrow \text{fission products} \quad (3.3d)$$
$$^{239}\text{Pu} + \gamma \rightarrow \text{fission products} \quad (3.3e)$$
$$^{239}\text{Pu} + \gamma \rightarrow ^{240}\text{Pu} \quad (3.3f)$$
$$^{240}\text{Pu} + \gamma \rightarrow \text{fission products} \quad (3.3g)$$
$$^{240}\text{Pu} + \gamma \rightarrow ^{241}\text{Pu} \quad (3.3h)$$
$$^{241}\text{Pu} + \gamma \rightarrow \text{fission products} \quad (3.3i)$$
$$^{241}\text{Pu} + \gamma \rightarrow ^{242}\text{Pu} \quad (3.3j)$$
$$^{242}\text{Pu} + \gamma \rightarrow \text{fission products} \quad (3.3k)$$

Again the first three reactions are the capture of a neutron by $^{238}$U which then undergoes double $\beta$-decay into $^{239}$Pu. Different compared to the LWR is that in the FBR also the fission of $^{238}$U and all the plutonium isotopes occurs, see equations (3.3d), (3.3e), (3.3g), (3.3i) and (3.3k). In an LWR these fissions also occur but these reactions are not of significant quantities. Also capture occurs for all plutonium isotopes, which is also the case in the LWR.

In the core the same reactions occur. The reactions do not change, only the relative amounts to each other change. The reason for this is that there is far less $^{238}$U in the core and that all the plutonium isotopes are initially already present in the MOX fuel that fills the core.

An extra feature of the FBR is that because the neutrons are in the fast spectrum, it is not only able to fission all plutonium isotopes but also minor actinides. This feature makes it possible that an FBR can be used as a waste burner for long-lived isotopes and also burn more polluted plutonium than a thermal reactor could.

### 3.1.4 Reactor Composition

The reactions mentioned in sections 3.1.2 and 3.1.3 change the composition of the fuel in a reactor over time, but after a period of time the fuel reaches an equilibrium. This is the case for both types of reactor. The reason for this is that the higher isotopes depend on the production of the first, $^{239}$Pu. If $^{239}$Pu reached an equilibrium, due to the fact that there is as much production of it as loss (fission, capture into the next isotope and a small fraction of decay) then $^{240}$Pu reaches equilibrium for the same reasons as $^{239}$Pu. It will then be followed by $^{241}$Pu and $^{242}$Pu. See figure 3.2 for an illustration of an FBR case.

### 3.2 Single Reactor

For describing the fuel behaviour of a reactor park, the fuel behaviour of the individual reactors has to be known. In general the change of fuel in a reactor is given by the fuel balance equation (3.4) From the equation we can see that the change in the amount, $N_i$, of an isotope $i$ depends
3.2. SINGLE REACTOR

REACTOR PARK

Figure 3.2: Core composition and blanket composition averaged per cycle after every cycle in time for an FBR

on a production term \( P(\vec{r}, t) \), a loss term \( L(\vec{r}, t) \), a discharge term \( \kappa(\vec{r}, t) \) and a charge term \( \xi(\vec{r}, t) \).

\[
\frac{dN_i(\vec{r}, t)}{dt} = P_i(\vec{r}, t) - L_i(\vec{r}, t) - \kappa_i(\vec{r}, t) + \xi_i(\vec{r}, t) \]  

(3.4)

If we integrate equation (3.4) spatially over \( \vec{r} \) and average over the fuel cycle period, \( \Delta t \), we get

\[
\frac{dN_i(t)}{dt} = P_i(t) - L_i(t) - \kappa_i(t) + \xi_i(t) \]  

(3.5)

The change of \( N_i \) in equation (3.5) is now given per cycle, \( \Delta t \), by the production and loss of an isotope, the amount that is being discharged from a reactor and the amount that is being charged. Each term can be determined as follow,

\[
P_i(t) = \lambda_{ci-1}N_{i-1}(t) \]  

(3.6a)

\[
L_i(t) = [\lambda_{ai} + \lambda_{ri}]N_i(t) \]  

(3.6b)

\[
\kappa_i(t) = \left[ \frac{1}{n\Delta t} - \frac{1}{2\lambda_{ai} - \lambda_{ri}} \right] N_i(t) + \lambda_{ci-1}N_{i-1}(t) \]  

(3.6c)

\[
\xi_i(t) = \left( 1 - \epsilon_i \right) \kappa_i(t - \tau) + S_i(t) \]  

(3.6d)

In equation (3.6a) we see that the production term depends on the capture microscopic reaction rate of isotope \( i-1 \), \( \lambda_{ci-1} \), multiplied with the time dependent number of atoms of isotope \( i-1 \), \( N_{i-1}(t) \). The loss term in equation (3.6b) depends on the absorption microscopic reaction rate, \( \lambda_{ai} \), and the radioactive decay constant, \( \lambda_{ri} \), multiplied with the time dependent number atoms of isotope \( i \), \( N_i(t) \). The discharge is dependent on the fuel management program. For an \( n \)-batch loading scheme with a fuel cycle period of \( \Delta t \) the discharge is as in equation (3.6c) [8]. This is a Taylor expansion of the batch discharge over the residing time. The charge term in equation (3.6d) is expressed in terms of the discharge term with the reprocessing loss, \( \epsilon_i \), taken into account. There is also an independent source term, \( S_i(t) \), to allow for external sources of pre-existing material like waste or to provide extra fuel to maintain reactor operation when needed. The independent source has a given constant fuel composition.
Equation (3.5) can also be written in vector notation, which will be used during the rest of this report.

\[
\frac{d\vec{N}(t)}{dt} = \vec{P}(t) - \vec{L}(t) - \vec{\kappa}(t) + \vec{\xi}(t)
\]  

(3.7)

### 3.3 Fuel Accumulation

To describe the fuel behaviour of a reactor park the fuel behaviours of the single reactors have to be taken together. This can be described by taking the sum of the burn up equations of each reactor, equation (3.5), of the same type in the park. Mathematically the accumulation of fuel isotopes can be described as follows

\[
\hat{N}_{Ti} = \sum_R N_{TRi}(t), R = 1, 2, ..., R_T(t)
\]

(3.8)

where \(\wedge\) represents the sign for accumulation, \(T\) is the index for the reactor type and \(R\) the index for each single reactor that is taken into account in the accumulation. The sum for the accumulation is only taken for reactors of the same type. \(R_T(t)\) is the total number of reactors of the same type \(T\) in a park at time \(t\). This accumulation can be used in equation (3.5) to describe the changes of accumulated fuel in a set of reactors of type \(T\) in a reactor park. This gives

\[
\frac{d\hat{N}_{Ti}(t)}{dt} = \hat{P}_{Ti}(t) - \hat{L}_{Ti}(t) - \hat{\kappa}_{Ti}(t) + \hat{\xi}_{Ti}(t)
\]

(3.9)

which gives the rate of change of isotopes in a reactor park. The separate terms from equation (3.9) look as follows

\[
\hat{P}_{Ti}(t) = \sum_R \lambda_{Ti,i-1} N_{TRi-1}(t)
\]

(3.10a)

\[
\hat{L}_{Ti}(t) = \sum_R [\lambda_{Ti,i} + \lambda_{Ti,i}] N_{TRi}(t)
\]

(3.10b)

\[
\hat{\kappa}_{Ti}(t) = \sum_R \kappa_{TRi}(t)
\]

(3.10c)

\[
\hat{\xi}_{Ti}(t) = \sum_R \xi_{TRi}(t)
\]

(3.10d)

Again this can also be described by vector notation

\[
\frac{d\vec{N}_{T}(t)}{dt} = \vec{P}_{T}(t) - \vec{L}_{T}(t) - \vec{\kappa}_{T}(t) + \vec{\xi}_{T}(t)
\]

(3.11)

### 3.3.1 External Fuel Cycle

The external fuel cycle is a general term for those parts of the nuclear fuel cycle that take place outside the reactors. Every burn-up equation is coupled to the external fuel cycle by its discharge and charge terms. This is schematically shown in figure 3.1 where single quotation mark, double quotation marks, etc., indicate that a reactor is at a different time in a cycle and therefore has a different isotopic composition. Here a closed fuel cycle is used, which means that fuel being
discharged from a reactor is reprocessed and then charged into the next reactor. The amount of fuel that is in the external fuel cycle can be described by

\[ \frac{dN_{x(t)}}{dt} = \sum_{T'} K_{T'i}(t) - (1 - \epsilon_i) \sum_{T'} K_{T'i}(t - \tau) \]  

(3.12)

where the index \( x \) tells that it is about the change in accumulated fuel of the external fuel cycle. The time that this reprocessing takes and the time it takes before being charged back into a new reactor is called the fuel cycle lag time \( \tau \). During the reprocessing also a small loss \( \epsilon \) occurs. These losses are caused by the chemical treatment which is used during reprocessing and also by natural decay. The accumulation of fuel in the external fuel cycle does not play a role in the total fuel accumulation, because effectively the fuel is only in the external fuel cycle during reprocessing and after this charged back into the next reactor. What happens when there is a difference between the amount of fuel that is being discharged and being charged into the next reactor is explained in the next section.

### 3.3.2 Temporary Storage Pool

To create a continuous model for the accumulating fuel, all the fuel has to be accounted for all the time. This means that also the fuel has to be included that is not being charged back immediately after reprocessing. This fuel is accumulating outside the reactors waiting until the moment that there is enough fuel to charge a new reactor. This is done by creating a temporary storage pool. The storage pool can be described by

\[ \frac{dN_0(t)}{dt} = (1 - \epsilon_i) \sum_{T'} K_{T'i}(t - \tau) - \sum_{T'} \xi_{T'i}(t) \]  

(3.13)

with the index 0 indicating that its about the fuel that is in the temporary storage pool. The change of fuel isotopes in the storage pool is the sum of the accumulating discharge, with reprocessing loss, minus the charge into the reactors. The summations are for all reactor types. For every type of reactor there exists a storage pool.

### 3.3.3 Fuel Accumulation

With use of the last sections the fuel behaviour in a reactor park can be described. This is done by taking all the fuel into account in and outside the reactors in a reactor park. This is done for every reactor type by adding the fuel present in every reactor, the core and the blanket, plus the fuel that is present in the temporary storage pool and external fuel cycle.

\[ \hat{N}_{T_i}(t) = \hat{N}_{TC_i}(t) + \hat{N}_{TB_i}(t) + \hat{N}_0(t) + \hat{N}_{x_i}(t) \]  

(3.14)

with the index \( C \) for core and the index \( B \) for blanket.

For the plutonium isotopes of an FBR this looks like figure 3.3a.
3.4 Reactor Accumulation

From the accumulation of fuel in a reactor park, detailed in the previous section, the number of reactors in a reactor park can be determined. This can be done by making use of a factorisation and weighting of the isotopes.

3.4.1 Factorisation and Weighting

In a similar way as flux factorisation is done in point kinetics, see equation (2.1), we can factor the accumulating fuel to a reactor amplitude, \( R(t) \), and the average core composition, \( \psi(t) \), which gives

\[
\vec{N}_T(t) = R_t(t) \vec{\psi}_T(t)
\] (3.15)

Again a constraint is needed to make the factorisation unique. This constraint is used to shift the major time dependence into the amplitude function. Similar to the constraint for the point kinetics, equation (2.4), the constraint has the following form

\[
\left\{ \vec{w}_T, \vec{\psi}_T(t) \right\} = K_T
\] (3.16)

where \( K_T \) again is a constant.

The weighting used are the critical mass weights. The reason for the use of critical mass weights is that critical mass weights tends to cancel out the contribution of the changes in core composition to the growth rate, \( \gamma(t) \). This effect is researched in [9]. This cancellation is stated as follows

\[
\left\{ \vec{w}_T, \frac{d\vec{\psi}_T(t)}{dt} \right\} = 0
\] (3.17)
In determining the critical mass weight two more constraints, one for the reactor fuel density and one for criticality, are stated.

\[
\sum_i \psi_{Ti}(t) = \text{constant fuel density} \quad \text{(3.18a)}
\]
\[
\sum_i w_i^\rho \psi_{Ti}(t) = \text{constant reactivity} \quad \text{(3.18b)}
\]

were the reactivity weights, \(w_i^\rho\) are determined by the difference between the microscopic fission and absorption cross sections, \(\nu \sigma_f - \sigma_a\). The reactivity weights are for the isotopes \(^{238}\text{U}\) and the plutonium isotopes \(^{239}\text{Pu}\) to \(^{242}\text{Pu}\).

With use of these reactivity weights the critical mass weights can be described as follows [10]

\[
w_{cm}^{Ti} = \left[ \frac{w_{T_i}^\rho - w_{T238}^\rho}{w_{T239}^\rho - w_{T238}^\rho} \right] \quad \text{(3.19)}
\]

where \(i = 239, 240, 241, 242\) for the plutonium isotopes. Taken this all together gives the following constraint

\[
\sum_i w_{cm}^{Ti} \psi_{Ti}(t) = \left\{ w_{cm}^T, \psi_{Ti}(t) \right\} = \text{constant critical mass} = K_{cm}^T \quad \text{(3.20)}
\]

Which gives that the average critical mass in one reactor is a constant, \(K_{cm}^T\).

### 3.4.2 Reactor Accumulation

With the accumulation of the fuel in a reactor park and the critical mass weights the number of reactors can be determined. This can be done by determining the amount of critical mass of the accumulating fuel, figure 3.3a, with the critical mass weights. This critical mass divided by the critical mass of one reactor gives the number of reactors at a given time.

\[
R_T(t) = \left\{ \frac{w_{cm}^T}{K_{cm}^T}, \left( N_{TC}(t) + N_{TB}(t) \right) \right\} \quad \text{(3.21)}
\]

In figure 3.3b it can been seen that the number of reactors grows with a logarithmic slope called the growth rate. In the figure the vertical axis has a logarithmic scale and the beginning of the blue line is exaggerated to give a more clear illustration. In real results the curve is much smaller, see the results for the first few cycles of figure 3.3b. The growth rate for a reactor type \(T\) is given by \(\gamma_T\). The growth rate can be determined by taking the amount of critical mass at the end of a cycle divided by the amount of critical mass at the beginning of a cycle.

\[
\gamma_T(t) = \left\{ \frac{w_{cm}^T}{w_T}, \left( N_{TC}(t) + N_{TB}(t) \right) \right\} \quad \text{(3.22)}
\]

Due to the fact that the isotope composition of a reactor goes to an equilibrium, which we saw in figure 3.2a and 3.2b, also the growth rate per cycle goes to an equilibrium which can be seen in figure 3.4a.
3.4.3 Asymptotic Growth Rate

The growth of reactors over long time scales can be determined with the use of growth rate per cycle and an initial number of reactors. The accumulation of reactors of type $T$ in a park can be calculated according to equation (3.23).

$$R_T(t + 1) = R_T(t) + \gamma_T(t) \cdot R_T(t)$$  (3.23)

The accumulation of reactors looks in general as the blue line in figure 3.4b. From the point where equilibrium is reached the asymptotic growth rate $\gamma_\infty$ can be determined. This asymptotic growth rate is also the growth rate that will be determined using the fuel cycle kinetics equation (4.60). It can be obtained by solving equation (3.24) for the point where figure 3.4b reaches equilibrium.

$$\frac{dR_T(t)}{dt} = \gamma_T(t)R_T(t)$$  (3.24)

Which leads to

$$R_T(t) = R_{T\infty} e^{\gamma_\infty t}$$  (3.25)

with $R_{T\infty}$ as the initial condition if you had purely asymptotic growth, see figure 3.4b. With this as the initial condition the asymptotic growth rate, $\gamma_\infty$ can be determined.
Fuel cycle kinetics

In this chapter the fuel cycle kinetics equation will be derived. The fuel cycle kinetics equations describe the time-dependent fuel behaviour of a population of nuclear reactors. For the derivation the critical mass weights and the concept of a reactor park will be used.

4.1 Factorisation and Weighting

The derivation starts with applying factorisation and weighting. First the weighting is applied by taking the inner product of the critical mass weights with equation (3.11) in the same way as is done in equation (3.16).

\[
\left\{ \frac{\text{d} N_T(t)}{\text{d} t}, \vec{w}^{\text{cm}} \hat{R}_T(t) \rightarrow N_T(t) \right\} = \left\{ \frac{\text{d} N_T(t)}{\text{d} t}, \vec{w}^{\text{cm}} \rightarrow \big[ P - L \big] \hat{N}_T(t) \right\} - \left\{ \frac{\text{d} N_T(t)}{\text{d} t}, \vec{w}^{\text{cm}} \rightarrow \hat{\kappa}_T(t) \right\} + \left\{ \frac{\text{d} N_T(t)}{\text{d} t}, \vec{w}^{\text{cm}} \rightarrow \hat{\xi}_T(t) \right\}
\]

Where \( P \) and \( L \) are the production and loss matrices as described in [11]. With the use of the factorisation from equation (3.15), equation (4.1) can be written as

\[
\left\{ \frac{\text{d} \vec{w}^{\text{cm}} \hat{R}_T(t) \rightarrow \vec{\psi}_T(t)}{\text{d} t} \right\} = \left\{ \frac{\text{d} \vec{w}^{\text{cm}} \hat{R}_T(t) \rightarrow \vec{\psi}_T(t)}{\text{d} t}, [ P - L ] \hat{R}_T(t) \vec{\psi}_T(t) \right\} - \left\{ \frac{\text{d} \vec{w}^{\text{cm}} \hat{\kappa}_T(t)}{\text{d} t}, \vec{w}^{\text{cm}} \rightarrow \hat{\kappa}_T(t) \right\} + \left\{ \frac{\text{d} \vec{w}^{\text{cm}} \hat{\xi}_T(t)}{\text{d} t}, \vec{w}^{\text{cm}} \rightarrow \hat{\xi}_T(t) \right\}
\]

(4.2)

Similarly to equation (2.11) and with use of equation (3.17) equation (4.2) becomes

\[
\left\{ \frac{\text{d} \vec{w}^{\text{cm}} \hat{R}_T(t) \rightarrow \vec{\psi}_T(t)}{\text{d} t} \right\} = \left\{ \frac{\text{d} \vec{w}^{\text{cm}} \hat{R}_T(t) \rightarrow \vec{\psi}_T(t)}{\text{d} t}, [ P - L ] \hat{R}_T(t) \vec{\psi}_T(t) \right\} - \left\{ \frac{\text{d} \vec{w}^{\text{cm}} \hat{\kappa}_T(t)}{\text{d} t}, \vec{w}^{\text{cm}} \rightarrow \hat{\kappa}_T(t) \right\} + \left\{ \frac{\text{d} \vec{w}^{\text{cm}} \hat{\xi}_T(t)}{\text{d} t}, \vec{w}^{\text{cm}} \rightarrow \hat{\xi}_T(t) \right\}
\]

(4.3)

where now the time dependence is shifted into the reactor amplitude.
In analogy to the point kinetics the production matrix can be split into a prompt and a delayed part, see equation (2.6). The production matrix can be written as follows

\[ P = P_p + P_d \] (4.4)

Putting this in equation (4.3) gives

\[
\begin{align*}
\left\{ \frac{-\omega_{cm} \rightarrow \omega_T, \psi_T(t)}{w_T, P \rightarrow \psi_T(t)} \right\} \frac{dR_T(t)}{dt} &= \left\{ \frac{-\omega_{cm} \rightarrow \omega_T, [P - L - P_d] \rightarrow \psi_T(t)}{w_T, P \rightarrow \psi_T(t)} \right\} R_T(t) + \left\{ \frac{-\omega_{cm} \rightarrow \omega_T, \hat{S}_{Td}(t)}{w_T, P \rightarrow \psi_T(t)} \right\} \\
- \left\{ \frac{-\omega_{cm} \rightarrow \omega_T, \kappa_T(t)}{w_T, P \rightarrow \psi_T(t)} \right\} + \left\{ \frac{-\omega_{cm} \rightarrow \omega_T, \xi_T(t)}{w_T, P \rightarrow \psi_T(t)} \right\}
\end{align*}
\] (4.5)

where \( \hat{S}_{Td}(t) \) is the accumulating delayed source. This will be explained more thoroughly in section 4.2.4

### 4.2 Parameters

Equation (4.5) is normalised by the gross critical mass production in the average park reactor core,

\[ P_T = \left\{ \frac{-\omega_{cm} \rightarrow \omega_T, P \rightarrow \psi_T(t)}{w_T, P \rightarrow \psi_T(t)} \right\} \] (4.6)

Applying this normalisation term transforms equation (4.5) into

\[
\begin{align*}
\left\{ \frac{-\omega_{cm} \rightarrow \omega_T, \psi_T(t)}{w_T, P \rightarrow \psi_T(t)} \right\} \frac{dR_T(t)}{dt} &= \left\{ \frac{-\omega_{cm} \rightarrow \omega_T, [P - L - P_d] \rightarrow \psi_T(t)}{w_T, P \rightarrow \psi_T(t)} \right\} R_T(t) + \left\{ \frac{-\omega_{cm} \rightarrow \omega_T, \hat{S}_{Td}(t)}{w_T, P \rightarrow \psi_T(t)} \right\} \\
- \left\{ \frac{-\omega_{cm} \rightarrow \omega_T, \kappa_T(t)}{w_T, P \rightarrow \psi_T(t)} \right\} + \left\{ \frac{-\omega_{cm} \rightarrow \omega_T, \xi_T(t)}{w_T, P \rightarrow \psi_T(t)} \right\}
\end{align*}
\] (4.7)

From this equation the fuel cycle kinetics parameters can be determined.

#### 4.2.1 Generation Time

Beginning with the LHS of equation (4.7)

\[
\begin{align*}
\left\{ \frac{-\omega_{cm} \rightarrow \omega_T, \psi_T(t)}{w_T, P \rightarrow \psi_T(t)} \right\} \frac{dR_T(t)}{dt} &= \Lambda_T(t) \frac{dR_T(t)}{dt}
\end{align*}
\] (4.8)
where $\Lambda_T(t)$ is the generation time. So

$$\Lambda_T(t) = \begin{cases} \rightarrow \omega_{cm} \rightarrow w_T, \psi_T(t) \\ \rightarrow \omega_{cm} \rightarrow w_T, P \psi_T(t) \end{cases}$$

(4.9)

The generation time is the time it takes for an average core to reproduce its own critical mass.

### 4.2.2 Reactivity

If the delayed production matrix is left out then the first term on the RHS of equation (4.7) is

$$\begin{aligned} \left\{ \rightarrow \omega_{cm} \rightarrow \psi_T(t) \right\} & \rightarrow P - L \rightarrow \psi_T(t) \\ \left\{ \rightarrow \omega_{cm} \rightarrow \psi_T(t) \right\} & \rightarrow P \rightarrow \psi_T(t) \end{aligned}$$

$$R_T(t) = \rho_T(t) R_T(t)$$

(4.10)

where $\rho_T(t)$ is the reactivity. So

$$\rho_T(t) = \begin{cases} \rightarrow \omega_{cm} \rightarrow \psi_T(t) \\ \rightarrow \omega_{cm} \rightarrow \psi_T(t) \end{cases}$$

(4.11)

The normalisation applied with equation (4.6) is done for similarly reason as is mention in the point kinetics theory where equation (2.13) is used for normalisation. The reactivity is the net production in the average core of a reactor, and says something about whether the core is breeding or not. So if the reactivity is 0 there is the same amount of production as loss and if it is less than 0 there is more loss then production in the core.

### 4.2.3 Delayed Critical Mass Fraction

Then the delayed production matrix part gives

$$\begin{aligned} \left\{ \rightarrow \omega_{cm} \rightarrow P_d \psi_T(t) \right\} & \rightarrow P - L \rightarrow \psi_T(t) \\ \left\{ \rightarrow \omega_{cm} \rightarrow P \psi_T(t) \right\} & \rightarrow P \rightarrow \psi_T(t) \end{aligned}$$

$$R_T(t) = \beta_T(t) R_T(t)$$

(4.12)

$$\beta_T(t) = \begin{cases} \rightarrow \omega_{cm} \rightarrow P_d \psi_T(t) \\ \rightarrow \omega_{cm} \rightarrow P \psi_T(t) \end{cases}$$

(4.13)

where $\beta_T$ is the delayed critical mass fraction which described the amount of critical mass delayed per generation time. For further detail see section 2.2.
4.2.4 Delayed Critical Mass Source

The second term on the RHS is

\[
\begin{align*}
S_{Td}(t) &= \begin{cases} 
- \epsilon_m \rightarrow w_T \cdot \hat{S}_{Td}(t) \\
- \epsilon_m \rightarrow w_T \cdot \hat{P}_T \psi_T(t) 
\end{cases} 
\end{align*}
\]

(4.14)

which is the reduced delayed critical mass source. This describes the critical mass that is delayed in its appearance per generation time by precursors. An elaborate explanation can be found in section 2.2. For the isotope of importance the delayed source can also be given by

\[
\hat{S}_{Td}(t) = \lambda_r \hat{C}_{Ti}(t)
\]

(4.15)

where \( \hat{C}_{Ti}(t) \) is the accumulating amount of precursor isotope \( i \). The change in precursor population can be described by

\[
\frac{d}{dt} \hat{C}_{Ti}(t) = -\lambda_r \hat{C}_{Ti}(t) + \left[ P_T \hat{\Lambda}_T(t) \right]_i
\]

(4.16)

The first term on the RHS gives the decay of the precursor and the second term gives the production of it. With use of the the factorization from equation (3.15) applying critical mass weighting and normalised with equation (3.16), equation (4.16) becomes

\[
\frac{d}{dt} \frac{w_{T_i}^{cm} \hat{C}_{Ti}(t)}{w_{T_i}^{cm} \psi_T(t)} = -\lambda_r \frac{w_{T_i}^{cm} \hat{C}_{Ti}(t)}{w_{T_i}^{cm} \psi_T(t)} + \frac{w_{T_i}^{cm} P_T \psi_T(t)}{w_{T_i}^{cm} \psi_T(t)} R_T(t)
\]

(4.17a)

\[
\frac{dc_{Ti}}{dt} = -\lambda_r c_{Ti} + \beta_{Ti} \frac{w_{T_i}^{cm} P_T \psi_T(t)}{w_{T_i}^{cm} \psi_T(t)} R_T(t)
\]

(4.17b)

\[
\frac{dc_{Ti}}{dt} = -\lambda_r c_{Ti} + \frac{\beta_{Ti}}{\Lambda_T(t)} R_T(t)
\]

(4.17c)

with

\[
c_{Ti} = \frac{w_{T_i}^{cm} \hat{C}_{Ti}(t)}{w_{T_i}^{cm} \psi_T(t)}
\]

(4.18)

which is the reduced precursor population. So now the reduced delayed source, \( s_{Td}(t) \), can be expressed in term of the reduced precursor population, \( c_{Ti}(t) \), after the following steps

\[
s_{Td}(t) = \sum_i s_{Td_i}(t) = \sum_i \frac{w_{T_i}^{cm} \hat{S}_{Td_i}(t)}{w_{T_i}^{cm} \psi_T(t)} = \sum_i \frac{w_{T_i}^{cm} \lambda_r \hat{C}_{Ti}(t) \Lambda_T(t)}{w_{T_i}^{cm} \psi_T(t)} = \Lambda_T(t) \lambda_r c_{Ti}(t)
\]

(4.19)

So the delayed source in terms of precursors becomes

\[
s_{Td}(t) = \Lambda_T(t) \sum_i \lambda_r c_{Ti}(t)
\]

(4.20)

The importance of these steps will follow further on in the chapter, see section 4.3.4.
4.2.5 Discharge

The third term on the RHS of equation (4.7) represents the discharge. The discharge is the amount of critical mass of the average reactor core that is being discharged per generation time. The accumulating discharge in figure 3.1 can be described as

\[ \arrowvec{\kappa_T}(t) = \arrowvec{D_T}(t) \arrowvec{N_T}(t) \]  
(4.21)

where \( \arrowvec{D} \) is the discharge matrix, with the fraction of a reactor that is being discharged into the external fuel cycle on the diagonal and zeros elsewhere. This fraction is the same for every isotope, due to the fact that a fraction of the core is being discharged, which means the same fraction for every isotope. The amount of isotopes being discharged is not the same, because every isotope is present in a different amount. Substituting equation (4.21) into the term from equation (4.7), with \( \arrowvec{N_T}(t) = R_T(t) \arrowvec{\psi_T}(t) \) gives

\[ \left\{ \arrowvec{w_{cm_T}}, \arrowvec{D_T}(t) \arrowvec{\psi_T}(t) \right\} R_T(t) = \delta_T(t) R_T(t) \]  
(4.22)

This gives the discharge parameter,

\[ \delta_T(t) = \left\{ \arrowvec{w_{cm_T}}, \arrowvec{D_T}(t) \arrowvec{\psi_T}(t) \right\} \]  
(4.23)

4.2.6 Charge

The accumulating charge in figure 3.1 and equation (3.11) can be split into

\[ \arrowvec{\xi_T}(t) = \arrowvec{\xi_{TC}}(t) + \arrowvec{\xi_{TB}}(t) + \arrowvec{S_T}(t) \]  
(4.24)

which represent the accumulating core charges, blanket charges and independent sources. Inserting equation (4.24) into the last term on the RHS of equation (4.7) gives

\[ \left\{ \arrowvec{w_{cm_T}}, \arrowvec{\xi_T}(t) \right\} = \left\{ \arrowvec{w_{cm_T}}, \arrowvec{\xi_{TC}}(t) \right\} + \left\{ \arrowvec{w_{cm_T}}, \arrowvec{\xi_{TB}}(t) \right\} + \left\{ \arrowvec{w_{cm_T}}, \arrowvec{S_T}(t) \right\} \]  
(4.25)

The core and blanket charges can be expressed by coupling parameters. The coupling parameters determine what happens to the fuel, so how much of the fuel in the reactor is going to the next reactor, to which type of reactor and does it go from core to core or from blanket to core.
4.2.7 Core Coupling

The accumulating core charge can be seen as the sum of accumulating charges of all different cores,
\[ \xi_{TC}(t) = \sum_{T'} \xi_{T'C\rightarrow T}(t) \] (4.26)

Each charge is the amount of critical mass that comes from the core of a reactor type \( T' \) and is being charged into the core of a reactor type \( T \) after being reprocessed in the external fuel cycle. The time the reprocessing takes is described by using \( \tau \), which is called the reprocessing time. This accumulating charge of a core into the next core can be described by
\[ \xi_{T'C\rightarrow T}(t) = C_{T'C\rightarrow T}(t)N_{T'}(t - \tau) \] (4.27)

with \( C \) the coupling matrix, which contains the fraction of the fuel of a reactor core that is being charged into the next reactor core on the diagonal and zeros elsewhere. So the charge, \( \xi \), is the amount of fuel that is being charged into the reactor core of type \( T \) from the fuel from a reactor core of type \( T' \) from time \( \tau \) before. Taking the first term of the RHS of equation (4.25) and inserting equation (4.27) in combination with the factorisation from equation (3.15) gives
\[ \left\{ \begin{array}{l} w_{T}, \xi_{TC}(t) \\ w_{T}, \xi_{T'C\rightarrow T}(t) \end{array} \right\} = \sum_{T'} \left\{ \begin{array}{l} \rightarrow cm_{T'}, \xi_{T'C\rightarrow T}(t - \tau) \\ \rightarrow cm_{T'}, P_{T'}(t) \end{array} \right\} R_{T'}(t - \tau) = \sum_{T'} \xi_{T'C\rightarrow T}R_{T'}(t - \tau) \] (4.28)

So the core coupling parameter is defined as
\[ \xi_{T'C\rightarrow T} = \left\{ \begin{array}{l} \rightarrow cm_{T'}, C_{T'C\rightarrow T}\psi_{T'}(t - \tau) \\ \rightarrow cm_{T'}, P_{T'}(t) \end{array} \right\} \] (4.29)

Schematically this can be seen in figure 4.1a

\[ \text{Figure 4.1: Scheme of the coupling coefficients.} \]
4.2.8 Blanket Coupling

The blanket coupling is the amount of critical mass that is being charged from the blanket of a reactor type $T'$ to a reactor core type $T$ during the generation time. This is shown schematically in figure 4.1b.

The accumulating blanket charge can be seen as the sum of the accumulating blanket charges of all different blankets,

$$\dot{\xi}_{TB}(t) = \sum_{T'} \dot{\xi}_{T'B\rightarrow T}(t)$$  \hspace{1cm} (4.30)

Each charge is the amount of critical mass that comes from the blanket of a reactor type $T'$ and is being charged into the core of a reactor type $T$ after being reprocessed for a time $\tau$ in the external fuel cycle. This accumulating charge of a blanket into the core of the next reactor can be described by

$$\dot{\xi}_{T'B\rightarrow T}(t) = C_{T'B\rightarrow T}(t)\dot{N}^{\infty}_{TB}$$  \hspace{1cm} (4.31)

Similarly to the factorisation of equation (3.15), and how the discharge and core coupling are determined, the accumulation of blanket fuel can be described by splitting it into a blanket amplitude function and the equilibrium blanket composition, which looks as follows

$$\dot{N}_{TB}(t) = B_T(t)\dot{N}^{\infty}_{TB}$$  \hspace{1cm} (4.32)

where $B_T(t)$ is the number of blankets and $\dot{N}^{\infty}_{TB}$ is the equilibrium blanket composition. The equilibrium composition can be used because the composition of a blanket reaches equilibrium within one cycle length, see figure 3.2b. Taking the second term on the RHS of equation (4.25) and inserting equation (4.31) combined with equation (4.32) gives

$$\begin{Bmatrix} w_{T}, \dot{\xi}_{TB}(t) \end{Bmatrix} = \sum_{T'} \begin{Bmatrix} w_{T'}, C_{T'B\rightarrow T}(t)\dot{N}^{\infty}_{TB} \end{Bmatrix}$$

This results in the blanket coupling parameter,

$$\varepsilon_{T'B\rightarrow T} = \frac{\begin{Bmatrix} w_{T'}, \dot{N}^{\infty}_{TB} \end{Bmatrix}}{\begin{Bmatrix} w_{T'}, \dot{\psi}_{T}(t) \end{Bmatrix}}$$  \hspace{1cm} (4.34)

4.2.9 Independent Source

The last term in the charge equation (4.25) is the critical mass charge from an independent source per generation time into a reactor.

$$s_T(t) = \frac{\begin{Bmatrix} w_{T'}, \dot{S}_{T}(t) \end{Bmatrix}}{\begin{Bmatrix} w_{T'}, \dot{\psi}_{T}(t) \end{Bmatrix}}$$  \hspace{1cm} (4.35)
An independent source can be used to maintain reactor operation if needed. Taking all the parameters together equation (4.7) becomes

\[
\Lambda_T(t) \frac{dR_T(t)}{dt} = [\rho_T(t) - \beta_T(t) - \delta_T(t)]R_T(t) + \sum_{T'} \varepsilon_{T' C \rightarrow T}(t) R_{T'}(t - \tau) + \sum_{T'} \varepsilon_{T' B \rightarrow T}(t) B_{T'}(t - \tau) + s_{Td}(t) + s_T(t)
\]

(4.36)

This is called the final point kinetics balance equation and is almost the same as the fuel cycle kinetics equation. This equation needs to be solved in the same way as the point kinetics equations (2.28a) and (2.28b) are solved by adding an extra precursor equation. In the final fuel cycle kinetics equation this is not the case, which will be explained in the upcoming section where the final steps in the derivation of the fuel cycle kinetics equation will be taken.

4.3 Fuel Cycle Kinetics Equation

To get from equation (4.36) to the fuel cycle kinetics equation a few extra steps are needed. In this section these last steps will be derived.

4.3.1 Time Dependence Parameters

The parameters in equation (4.36) are all time dependent. The equation can be solved by updating the parameters during time, but to make finding the solution computationally cheaper it can also be done by taking the parameters as constant. To determine the asymptotic growth rate, as in figure 3.4b, with equation (4.36) the equilibrium core composition, \( \psi(\infty) \) has to be used. This means that the parameters become \( \Lambda(\infty), \rho_T(\infty), \delta_T(\infty), \varepsilon_{T' C \rightarrow T}(\infty) \) and \( \varepsilon_{T' B \rightarrow T}(\infty) \). These equilibrium parameters are used as constant in the fuel cycle kinetics equation.

4.3.2 Initial Condition

As explained in the previous section the equilibrium composition is used to determine the asymptotic growth rate. Due to neglecting the time dependency in the beginning a constant error occurs in the accumulation of reactors. This error can be corrected by introducing a pseudo-initial condition. This is sketched in figure 4.2 where \( R_{T0} \) is the initial condition and \( R_{T1} \) the pseudo-initial condition. The pseudo-initial condition can be determined with the use of a breeding bonus. The breeding bonus is constructed with the use of the difference between the absolute breeding worths and the absolute critical mass worths weighted with the initial reactor core composition. The absolute breeding worths are determined from the relative breeding worths, \( w^* \), which follow from the adjoint eigenvalue problem described in the thesis of Hanan [12]. The adjoint eigenvalue problem looks like

\[
C^* w^* = \gamma_{\infty} H w^*
\]

(4.37)

where \( C \) is the production matrix minus the loss matrix. The matrix \( H \) describes the difference between the discharge composition of the core and the residing composition. Due to the fact that this difference is very small it can be treated as the unit matrix, so it does not participate in the eigenvalue problem [11].
With the relative breeding worths from the adjoint eigenvalue problem the absolute breeding worths are as follows

\[ w^* T = \frac{w^* T}{\psi_T(\infty)} \]  \hspace{1cm} (4.38)

The absolute breeding worths say how the fuel at initial state contributes to the critical mass in asymptotic state. The absolute critical mass worths are determined in the same way with the use of the critical mass weights from equation (3.19)

\[ w_{cm}^* T = \frac{w_{cm}^* T}{\psi_T(\infty)} \]  \hspace{1cm} (4.39)

The absolute critical mass worths say how the fuel contributes to the critical mass at that specific time, here the initial state. The difference between the two absolute worths creates the difference that occurred due to the use of the equilibrium composition. So weighting the difference between the two absolute worths with the initial composition gives the breeding bonus

\[ b_T = \frac{w_{cm}^* T - w_{cm}^* T}{\psi_T(0)} \]  \hspace{1cm} (4.40)

With the use of the breeding bonus the pseudo-initial condition can be determined

\[ R_{T1} = R_{T0}(1 + b_T) \]  \hspace{1cm} (4.41)
4.3.3 Blanket Delay

Equation (4.36) consists of a number of reactors, $R_T(t)$, and a number of blankets, $B_T(t)$. The number of blankets, $B_T(t)$, is not the same as the number of reactors, $R_T(t)$. This is the case because a newly started breeder FBR has a delay in the blanket fuel discharge, because there is a build up of the fuel in a blanket that is prescribed by the fuel management program. To solve equation (4.36) it is necessary to express the number of blankets, $B_T(t)$, in terms of the number of reactors, $R_T(t)$. The rate of increase of blanket inventory can be described by

$$\frac{d\rightarrow N_B(t)}{dt} = \pi_B^\infty [\rightarrow N_B - \rightarrow N_B(t)]$$  \hspace{1cm} (4.42)$$

where $\pi_B^\infty$ is the equilibrium production rate of the blanket and $\rightarrow N_B$ is the equilibrium composition of the blanket. So the rate of increase in equation (4.42) is the difference between the production in the blanket in equilibrium and the production in the blanket at the time in question. The production rate follows from a similar eigenvalue problem from which the asymptotic growth rate can be determined [12]. The eigenvalue problem is the generalised eigenvalue problem of equation (4.37)

$$C_T \rightarrow \psi(\infty) = \gamma_H \rightarrow \psi(\infty)$$  \hspace{1cm} (4.43)$$

from here the asymptotic growth rate can be determined by

$$\gamma_H = \left\{ \frac{\rightarrow w_{cm}}{\rightarrow w_{cm} \rightarrow \psi(\infty)} \right\}$$  \hspace{1cm} (4.44)$$

In a similar way, it follows from Hanan [12] that the blanket production rate is

$$\pi_B^\infty = \left\{ \frac{\rightarrow w_{cm}}{\rightarrow w_{cm} \rightarrow \psi(\infty)} \right\}$$  \hspace{1cm} (4.45)$$

where $\rightarrow \kappa_B^\infty$ is the equilibrium blanket discharge. Integration of equation (4.42) gives the blanket composition

$$\rightarrow N_B(t) = \rightarrow N_B^\infty \left( 1 - \exp[-\pi_B^\infty t] \right)$$  \hspace{1cm} (4.46)$$

Equation (4.46) can also be written as follows

$$\frac{\rightarrow N_B(t)}{\rightarrow N_B^\infty} = 1 - \exp[-\pi_B^\infty t] = \theta_T(t)$$  \hspace{1cm} (4.47)$$

where $\theta$ is called the blanket fraction. The growth of blankets can be described by

$$B_T(t) = R_{T0} \theta_T(t) + \int_0^t \frac{dR_T(t')}{dt'} \theta_T(t - t') dt'$$  \hspace{1cm} (4.48)$$

where the first term on the RHS is the blanket inventory build up for the initial condition reactors. The second term describes the blanket inventory build up in new reactors. Applying
integration by parts gives,

\[
B_T(t) = R_T(0) \theta_T(t) + \left[ R_T(t) \theta_T(t - t') \right]_0^t + \pi_B^\infty \int_0^t R_T(t') \exp\left[ -\pi_B^\infty (t - t') \right] dt' \tag{4.49a}
\]

\[
B_T(t) = \pi_B^\infty \int_0^t R_T(t') \exp\left[ -\pi_B^\infty (t - t') \right] dt' \tag{4.49b}
\]

Which then leads to

\[
\frac{dB_T(t)}{dt} = \pi_B^\infty [R_T(t) - B_T(t)] \tag{4.50}
\]

At large times the number of blankets can be described similarly to the number of reactors in equation (3.24) which when solved gives

\[
B_T(t) = B_T(\infty) \exp[\gamma t] \tag{4.51}
\]

Combining equations (4.50) and (4.51) shows how the number of blankets can be written as function of the number of reactors.

\[
B_T(t) = \frac{\pi_B^\infty}{\pi_B^\infty + \gamma} R_T(t) \tag{4.52}
\]

### 4.3.4 Delayed Isotopes

To solve equation (4.36) an extra precursor equation is needed. This section explains how this extra precursor equation can be made redundant. In the change of precursors in time, equation (4.17c), depends on the relationship between the precursors, \(c_T_i(t)\), and the number of reactors, \(R_T(t)\). For large times both depend on a single exponential, the same as in equation (3.25), so

\[
c_T_i(t) = c_T_i(\infty) \exp[\gamma t] \tag{4.53}
\]

\[
R_T(t) = R_T(\infty) \exp[\gamma t] \tag{4.54}
\]

Inserting these equations into equation (4.17c) gives

\[
\frac{dc_T_i}{dt} = -\lambda_{ri} c_T_i(\infty) \exp[\gamma t] + \frac{\beta_T_i R_T(\infty) \exp[\gamma t]}{\Lambda_T t} \tag{4.55}
\]

solving this for \(c_T_i(t)\) gives

\[
c_T_i = \frac{\beta_T_i R_T(t)}{\Lambda_T t} \frac{1}{[\lambda_{ri} + \gamma]}. \tag{4.56}
\]

Inserting this into equation (4.20) gives

\[
s_{Ti}(t) = \sum_i \beta_T_i R_T(t) \frac{\lambda_{ri}}{[\lambda_{ri} + \gamma]} \tag{4.57}
\]

For the delayed source only radioactive decay that creates plutonium isotopes is of importance. The radioactive decay of the plutonium itself is already in the loss matrix, see equation (3.6b). The radioactive decay of highest importance is then the one of \(^{239}\text{Pu}\) which has the half-life \(t_{1/2} = 2.356\) days which leads to the value \(98.26 \frac{1}{\text{cycle}} [13] \) for the radioactive decay. This considered with an asymptotic growth of \(\gamma = 8%\) gives the following approximation

\[
\frac{\lambda_{ri}}{[\lambda_{ri} + \gamma]} = \frac{98.26}{[98.26 + 0.08]} \sim 1 \tag{4.58}
\]

A reasonable approximation for the delayed source is therefore

\[
s_{Ti}(t) \sim \sum_i \beta_T_i R_T(t) = \beta_T R_T(t) \tag{4.59}
\]
4.3.5 Fuel Cycle Kinetics

With the use of the equilibrium composition and the delayed source equation (4.59), equation (4.36) becomes the fuel cycle kinetics equation,

\[
\Lambda_T(\infty) \frac{dR_T(t)}{dt} = \left[\rho_T(\infty) - \delta_T(\infty)\right] R_T(t) + \sum_T \varepsilon_{T' \rightarrow T} (\infty) R_{T'}(t - \tau) \\
+ \sum_T \varepsilon_{T' \rightarrow T} (\infty) B_{T'}(t - \tau) + s_T(t)
\] (4.60)

which can be solved with the use of the blanket delay and with the pseudo-initial condition.

4.4 Symbiotic Deployment

In section 4.2 the coupling between the cores and blankets is explained for a single type of reactor. For a symbiotic deployment of FBRs and LWRs a sample coupling is shown in figure 4.3.

\[\text{Figure 4.3: Scheme of the coupling coefficients in a symbiotic deployment of FBRs and LWRs.}\]

It can be seen that there is coupling from the core of an LWR' to the core of an LWR, there is coupling from the core of an FBR' to the core of an FBR and that there is coupling from the blanket of an FBR' to both the core of an FBR and LWR. There is no coupling from the core of an FBR' to an core of an LWR. The reason for this is that the higher plutonium isotopes coming from an FBR have a negative impact on the operation of an LWR. By burning the cleaner plutonium in the LWR improves the performance. The fuel cycle kinetics equation for the FBRs is here

\[
\Lambda_F(\infty) \frac{dR_F(t)}{dt} = \left[\rho_F(\infty) - \delta_F(\infty)\right] R_F(t) + \sum_F \varepsilon_{F' \rightarrow F} (\infty) R_{F'}(t - \tau) \\
+ \sum_F \varepsilon_{F' \rightarrow F} (\infty) B_{F'}(t - \tau) + \sum_{L'} \varepsilon_{L' \rightarrow F} (\infty) R_{L'}(t - \tau)
\] (4.61)

In equation (4.61) the independent source is no longer there. The reason for this is that in a symbiotic system like this there is no need for an independent source to maintain reaction operation. The fuel cycle kinetics equation for the LWRs becomes

\[
\Lambda_L(\infty) \frac{dR_L(t)}{dt} = \left[\rho_L(\infty) - \delta_L(\infty)\right] R_L(t) + \sum_{L'} \varepsilon_{L' \rightarrow L} (\infty) R_{L'}(t - \tau) + \sum_F \varepsilon_{F' \rightarrow L} (\infty) B_{F'}(t - \tau)
\] (4.62)

For the same reason as for the FBRs there is no independent source.
CHAPTER 5

Results

In this chapter we will look into the results. First the results for a symbiotic system of FBRs and LWRs are treated that were at hand from Maudlin’s thesis [3]. Further the results from a single deployment of FBRs and for a symbiotic deployment of FBRs and PWRs are shown. For the single deployment of FBRs and the symbiotic deployment of FBRs and PWRs the results from the fuel cycle kinetics equations will be checked with results obtained from TRITON calculations. TRITON is a module of the SCALE code system that provides a realistic approach to transport and depletion analysis of reactor fuel assemblies [4]. The last sections will be about how a demanded asymptotic growth can be fulfilled by a symbiotic system.

5.1 Maudlin’s Thesis

Maudlin’s thesis [3] is used as a test case for solving the fuel cycle kinetics equations. Maudlin researched a symbiotic deployment of FBRs and LWRs. The results that are obtained by solving the fuel cycle kinetics equations with the data from Maudlin’s thesis are compared with the results obtained by Maudlin.

5.1.1 Fuel Cycle Kinetics

For solving the fuel cycle kinetics equations the parameters, the pseudo-initial conditions and the blanket delay are needed. The parameters are already determined by Maudlin and given in his thesis. The blanket production rate, $\pi_{\infty B}$ and the breeding bonuses, $b_F$ and $b_L$ are also given. With these the blanket delay and pseudo-initial conditions can be determined. The parameters given in Maudlin’s thesis are shown in tables 5.1 and 5.2. They were determined in the same way as explained in section 4.2.
5.1. MAUDLIN’S THESIS

### Results

#### Table 5.1: Parameters for an FBR in symbiotic deployment from [3].

<table>
<thead>
<tr>
<th>Time</th>
<th>$\rho$</th>
<th>$\delta$</th>
<th>$\varepsilon_{F'\rightarrow F}$</th>
<th>$\varepsilon_{L'\rightarrow F}$</th>
<th>$\varepsilon_{F'\rightarrow B}$</th>
<th>$\Lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.4804</td>
<td>1.255</td>
<td>1.219</td>
<td>0.3679</td>
<td>0.4044</td>
<td>4.485</td>
</tr>
<tr>
<td>$\infty$</td>
<td>-0.3888</td>
<td>1.275</td>
<td>1.244</td>
<td>0.3616</td>
<td>0.3975</td>
<td>4.408</td>
</tr>
<tr>
<td>$\Delta%$</td>
<td>19.1</td>
<td>1.6</td>
<td>2.0</td>
<td>-1.7</td>
<td>-1.7</td>
<td>-1.7</td>
</tr>
</tbody>
</table>

Maudlin used the equilibrium composition to determine the reactivity. For the other parameters Maudlin used the initial composition. He decided to do this because there where only small differences compared to the use of the equilibrium composition, see tables 5.1 and 5.2.

The breeding bonuses from Maudlin which were determined with equation (4.40) are $b_F = -0.0491$ for the FBR and $b_L = 0.0229$ for the LWR. With the initial conditions $R_{F0} = 4$ and $R_{L0} = 2$ used by Maudlin this gives the pseudo-initial conditions

\[
R_{F1} = R_{F0}(1 + b_F) = 3.8036 \quad (5.1a)
\]
\[
R_{L1} = R_{L0}(1 + b_L) = 2.0458 \quad (5.1b)
\]

The blanket delay follows from equation (4.52). The blanket production rate, $\pi_B^\infty = 0.63$ and the asymptotic growth $\gamma_\infty = 2.01\%$, are given in Maudlin’s thesis. Filling this in the equation gives

\[
B_F(t) = \frac{\pi_B^\infty}{\pi_B^\infty + \gamma_\infty} R_F(t) = 0.969 R_F(t) \quad (5.2)
\]

The values from the last sections with a fuel lag time, $\tau = 1$ year, and no reprocessing losses, $\epsilon = 0$, are used to solve the fuel cycle kinetics equations (4.61) and (4.62). In figure 5.1a we see the result of the symbiotic deployment, which has an asymptotic growth of $\gamma_\infty = 2.01\%$ and the ratio after 100 years between LWRs and FBRs is 0.427.
5.2 FBR DEPLOYMENT

In the upcoming sections all the data will be self created. In this section the results treated are of the growth of a system of FBRs. The design of the FBR that is used is similar to the one Ott and Borg used [11]. The results are determined in two ways, one using fuel cycle kinetics and one simulated numerically using TRITON. The TRITON results are used to check the results of the fuel cycle kinetics equations. This is done by creating an error function to check the difference between the results of the fuel cycle kinetics equations and the results from TRITON.
5.2.1 Fuel Cycle Kinetics

For solving the fuel cycle kinetics first the parameters have to be constructed. As explained in section 4.3.1 the equilibrium composition is used to determine the parameters. This equilibrium composition is obtained from the TRITON calculations.

Of further importance for creating the parameters is that after every cycle the whole reactor, blanket and core, is being discharged into the external fuel cycle. All the fuel is being reprocessed without any losses, $\epsilon = 0$, and is charged back instantly, so the reprocessing time $\tau = 0$ cycles. All the fuel is being charged into the core of an FBR and a new clean blanket of pure $^{238}$U is constructed. This means that the discharge matrix $D_F$, the core coupling matrix $C_F^{C\rightarrow F}$ and the blanket coupling matrix $C_F^{B\rightarrow F}$ have 1 on the diagonal. All the discharge and coupling matrices have the following form

$$A = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{pmatrix}$$

The values for the FBR deployment in matrix (5.3) are then the values given in table 5.3. With these matrices and the use of the equilibrium composition the parameters become as in table 5.4.

Before the fuel cycle kinetics equations can be solved the pseudo-initial condition and the blanket delay have to be determined.

The breeding bonus for this FBR deployment is

$$b_F = \begin{cases} \left[ \overrightarrow{w F} - \overrightarrow{w cm} \right], \overrightarrow{\psi F}(0) \\ \overrightarrow{w cm}, \overrightarrow{\psi F}(0) \end{cases} = -0.0879$$

which leads to the pseudo-initial condition

$$R_{F1} = R_{F0}(1 + b_F) = 0.9121$$

Due to the discharge of the whole blanket at the end of a cycle and charge of a new clean blanket at the beginning of a cycle there is no blanket inventory build up. For this reason there is also no blanket delay. This means that the number of blankets equals the number of reactors. $B_F(t) = R_F(t)$

Combining all this in the fuel cycle kinetics equation (4.60) and solving the equation gives an asymptotic growth of $\gamma_{\infty} = 6.84\%$ for both the results with and without pseudo-initial condition. Both results are given in figure 5.2a. In the figure can be seen the number of FBRs as a function of time in cycles.
5.2. **FBR Deployment**

(a) Growth of reactors in an FBR deployment determined by fuel cycle kinetics.

(b) The accumulation of FBRs in an FBR deployment determined using TRITON.

Figure 5.2: The growth of reactors in an FBR deployment.

### 5.2.2 TRITON Calculations

To check the outcome of the solution of the fuel cycle kinetics equation the same FBR is also numerically calculated using TRITON. TRITON [4] is a module of the SCALE [14] system that is developed by Oak Ridge National Laboratory. In this thesis the capability of TRITON is used to couple the module KENO [15] with the module ORIGEN-S [16]. TRITON can perform 3-D depletion calculations with ORIGEN-S based on the KENO Monte Carlo transport module. In KENO almost all geometric shapes can be constructed and processed using quadratic equations. The KENO criticality transport code determines the fluxes which are used by ORIGEN-S. ORIGEN-S performs point depletion and decay analyses to obtain the isotopic concentrations.

In the TRITON calculations the reactor is constructed in KENO and consists out of multiple blocks called materials, see figure 5.3. All the different elements present in a reactor, like fuel, cladding etc. are homogenised per material. In the fuel cycle kinetics equation the core and the blanket are taken as a whole.

(a) Axial slice of a reactor.

(b) Radial slice of a reactor.

Figure 5.3: Axial and radial slice of a block model of a fast breeder reactor.

From the TRITON calculations the accumulation of reactors can be determined in the way that is explained in section 3.4.2. This results in figure 5.2b, which also gives the number of FBRs as a function of time in cycles. Then to determine the asymptotic growth rate equation
(3.25) is used on the point where the result in figure 5.2b reaches asymptotic growth. This gives an asymptotic growth rate $\gamma_{\infty} = 6.89\%$. This is very close to the asymptotic growth that was determined from the fuel cycle kinetics $\gamma_{\infty} = 6.84\%$. So the fuel cycle kinetics theory is accurate in determining the asymptotic growth for an FBR deployment.

### 5.2.3 Error

To see if the fuel cycle kinetics theory also is accurate for the number of accumulating reactors an error function is created. The error between the number of reactor from the results of the fuel cycle kinetics equation and the TRITON calculation is determined by

$$\text{Error} = \frac{R_{\text{Kinetics}} - R_{\text{TRITON}}}{R_{\text{TRITON}}} \ast 100 \quad (5.6)$$

This is done for the fuel cycle kinetics solution with and without pseudo-initial condition. The result for the error without pseudo-initial condition is shown in figure 5.4a and the result with pseudo-initial condition is shown in figure 5.4b. Both errors are given in percent as a function of time in cycles. What can be seen is that the error in the case of use of a pseudo-initial condition is far lower than without. On the other hand the behaviour of the error is the same. Both of these facts were to be expected from looking at figure 5.2a. What can be seen is that the pseudo-initial condition only decreases or increases the number of reactors and does not change the asymptotic behaviour of the system. This is also explained in figure 4.2. Further it can be seen that the error for both with and without pseudo-initial condition does not stabilise. The reason for this is that there remains a difference in asymptotic growth between the fuel cycle kinetics solution and the TRITON solution. An explanation for the remaining difference between the fuel cycle kinetics solution and the TRITON solution can be that the TRITON solution only gives a solution at the end of every cycle where the fuel cycle kinetics is a continuous function.

From this it can be concluded that also for determining the number of reactors in an FBR deployment the fuel cycle kinetics theory is accurate with the use of a pseudo-initial condition.

---

**Figure 5.4:** Error between the number of FBRs between the TRITON calculation and the Fuel Cycle Kinetics with and without use of the pseudo-initial condition.
5.3 Symbiotic Deployment

In this section a symbiotic deployment of FBRs and PWRs is researched. The FBR reactor design from the previous section is used and a PWR is self designed. The coupling between the reactors is done in the same way as is done in the symbiotic deployment from Maudlin’s thesis only with use of different coupling matrices.

The deployment of a self designed symbiotic system of reactors is the first step towards modelling the fuel behaviour of a reactor park. If the fuel cycle kinetics theory can handle this, it probably can also handle a reactor park with more then two types of reactors at different times in their cycles.

5.3.1 Fuel Cycle Kinetics

As in the FBR deployment all the fuel of the cores, FBRs and PWRs, and blankets is discharged at the end of a cycle into the external fuel cycle. There are no reprocessing losses, $\epsilon = 0$, and all the fuel is reprocessed and charged without a delay into the reactors, $\tau = 0$ cycles. These are not very realistic assumptions, but these needed to be made because of implementation problems in the TRITON management code.

The coupling is the same is given in figure 4.3 only now with the use of a PWR as a specific type of LWR. The coupling values are chosen in such a way that a working symbiotic system is created but without any goal beyond that. For the coupling of the fuel in a blanket of an FBR half of the fuel goes to a core of an FBR. The other half is going to a core of a PWR. So $C_{F^p B \rightarrow F}$ and $C_{F^p B \rightarrow P}$ have 0.5 on the diagonal. The core of an FBR is fully coupled to the next FBR. The core of a PWR is for 90% charged into an FBR and for 10% charged into a PWR. So $C_{P^p C \rightarrow F}$ has 0.9 on the diagonal and $C_{P^p C \rightarrow P}$ has 0.1 on the diagonal. See for the values for matrix (5.3) tables 5.5 and 5.6.

\[
\begin{array}{|c|c|c|c|}
\hline
 & A & F & C \\
\hline
 a & 1 & 0.5 & 0.5 \\
\hline
\end{array}
\]

\textbf{Table 5.5:} Values for an FBR in a symbiotic deployment of FBRs and PWRs.

\[
\begin{array}{|c|c|c|c|}
\hline
 & A & P & C \\
\hline
 a & 1 & 0.1 & 0.9 \\
\hline
\end{array}
\]

\textbf{Table 5.6:} Values for a PWR in a symbiotic deployment of FBRs and PWRs.

With the use of these matrices and the equilibrium compositions the parameters for the FBR and the PWR can be determined. These are given in tables 5.7 and 5.8.

\[
\begin{array}{|c|c|c|c|c|}
\hline
 & \rho & \delta & \epsilon_{F^p C \rightarrow F} & \epsilon_{F^p B \rightarrow P} \\
\hline
 -0.250 & 4.637 & 4.637 & 0.3459 & 0.7207 & 4.637 \\
\hline
\end{array}
\]

\textbf{Table 5.7:} Asymptotic parameters of an FBR in a symbiotic deployment.

\[
\begin{array}{|c|c|c|c|c|}
\hline
 & \rho & \delta & \epsilon_{P^p C \rightarrow P} & \epsilon_{P^p C \rightarrow F} \\
\hline
 -3.037 & 12.388 & 1.239 & 6.816 & 12.388 \\
\hline
\end{array}
\]

\textbf{Table 5.8:} Asymptotic parameters of a PWR in a symbiotic deployment.
5.3. SYMBIOTIC DEPLOYMENT

Again before solving the fuel cycle kinetics equations the pseudo-initial conditions and also the blanket delay have to be determined. The breeding bonuses for the symbiotic system of FBRs and PWRs are

\[ b_F = \left\{ \frac{w^* F - w_{cm} F}{w_F, \psi_F(0)} \right\} = -0.1593 \] (5.7)

\[ b_P = \left\{ \frac{w^* P - w_{cm} P}{w_P, \psi_P(0)} \right\} = -0.1996 \] (5.8)

these values and the initial conditions \( R_{F0} = 4 \) and \( R_{P0} = 2 \) give the pseudo-initial conditions,

\[ R_{F1} = R_{F0}(1 + b_F) = 3.3626 \] (5.9)

\[ R_{P1} = R_{P0}(1 + b_P) = 1.6008 \] (5.10)

Also in this symbiotic deployment the whole blanket is being discharged at the end of a cycle, the same as in the FBR deployment discussed previously. Due to this there is no fuel build up in the blanket and so no blanket delay. Thus the number of blankets equals the number of fast reactors \( R_F(t) = B_F(t) \).

Combining all this the fuel cycle kinetics equations (4.61) and (4.62) give an asymptotic growth rate \( \gamma_\infty = 8.95\% \) for both with and without the use of the pseudo-initial conditions. See figure 5.5a. Both results contain the number of FBRs and PWRs as a function of time in cycles.

(a) The accumulation of FBRs and PWRs in a symbiotic deployment determined by fuel cycle kinetics.

(b) The accumulation of FBRs and PWRs in a symbiotic deployment determined using TRITON.

Figure 5.5: The growth of reactors in a symbiotic deployment of FBRs and PWRs.
5.3.2 TRITON Calculations

Using the theory again from section 3.4.2 the accumulation of reactors can be determined from the TRITON calculation. This results in figure 5.5b which has an asymptotic growth rate $\gamma_\infty = 8.69\%$. Also this result contains the number of FBRs and PWRs as a function of time in cycles. The asymptotic growth of the TRITON calculation is quite close to the asymptotic growth obtained with the fuel cycle kinetics equations $\gamma_\infty = 8.95\%$. The difference is bigger than in the FBR deployment. This bigger difference is caused by the discontinuous behaviour of the TRITON solution. In the TRITON calculation the number of reactors is determined at the end of every cycle where in the fuel cycle kinetics solutions the number of reactors is a continuous function. In figure 5.5b the big influence of the time steps, at the end of every cycle, on the solution can be seen. It creates a big discontinuity in the first cycles.

5.3.3 Error

Again the errors, see equation (5.6), between the numbers of reactors are determined for the solution with and without the use of the pseudo-initial conditions. The errors for the result without the use of the pseudo-initial condition are given in figure 5.6a and with the pseudo-initial condition in figure 5.6b. The error is given separately for the FBRs and the PWRs in percentage as a function of time in cycles. As mentioned in the previous section there is a big discontinuity in the first cycles of the PWRs and a smaller one for the FBRs. This is caused by the fact that the TRITON solution is not continuous, where the fuel cycle kinetics solution is. Another confirmation that the error is due to the time dependence can be seen in the behaviour of the FBRs. The FBRs do not have a big discontinuous jump as in the PWRs in their behaviour, which translates to a much smaller error between the two solutions. Due to the fact that the reactors are coupled to each other the large discontinuity in the PWRs has also an effect on the behaviour of the FBRs.

![Figure 5.6](image1.png)  ![Figure 5.6](image2.png)

(a) Error between TRITON and Fuel Cycle Kinetics without pseudo-initial condition.  
(b) Error between TRITON and Fuel Cycle Kinetics with pseudo-initial condition.

**Figure 5.6:** Errors in the numbers of FBRs and PWRs between the TRITON calculation and the Fuel Cycle Kinetics with and without use of the Pseudo-Initial Condition.
5.4 Growth of Nuclear Energy

In the previous two sections the fuel cycle kinetics solutions of the FBR deployment and the symbiotic deployment were compared with the TRITON calculations, which were done first. This was done to verify the fuel cycle kinetics theory. In this section it is done the other way around, so now first the fuel cycle kinetics solution will be determined, which will than be checked by the TRITON calculations. This is where the true power of the fuel cycle kinetics theory comes to hand. With the use of the fuel cycle kinetics equations multiple set-ups with different coupling matrices and ratio between initial number of reactors can be run and checked in a short amount of time. The search for suitable coupling matrices and initial ratios between the reactors is an iterative process. This means that after every change in the set-up of the fuel cycle kinetics equations the solution is checked against the demanded result. If the fit is not close enough the set-up is changed again until the solution of the fuel cycle kinetics equations fits the demand. When a set-up is found that gives the desired behaviour, then these values are implemented in TRITON and the results are compared with the fuel cycle kinetics solution.

In this section the same symbiotic deployment of FBRs and PWRs is used as in section 5.3. The question of interest is: Can the fuel cycle kinetics equations find a set-up that has a predetermined asymptotic growth? For this question two asymptotic growths are studied. They can be derived from figure 1.1, which contains the expected growth of nuclear energy demand in the next 100 years. This prediction has an upper and a lower boundary. From the data in table A.4 asymptotic growths of $\gamma_\infty = 4.06\%$ for the upper boundary and $\gamma_\infty = 1.99\%$ for the lower
boundary can be derived for the growth until 2060. These asymptotic growths are determined per cycle, so some of the values of symbiotic system from the previous section can be used.

### 5.4.1 System set-up

Before starting the iterative process of finding suitable coupling matrices the behaviour of the system is adjusted by changing the matrices. The reason for this is to prevent the drop in the beginning of figure 5.5b and so finding a suitable starting point for the iteration. The drop needs to be prevented because it was responsible for the large error in the beginning of figures 5.6a and 5.6b. Only the $\mathcal{C}_{F^\prime B \rightarrow P}$, $\mathcal{C}_{F^\prime F \rightarrow F}$, $\mathcal{C}_{P^\prime C \rightarrow P}$ and $\mathcal{C}_{P^\prime C \rightarrow F}$ matrices and the ratio between the initial number of reactors are changed in finding a starting set-up and in the iteration process. The reason for this is that these values determine the amount of critical mass going to the PWRs. It is necessary to send more critical mass to the PWRs to prevent the drop. There is no coupling from the FBR cores to the PWR cores, $\mathcal{C}_{F^\prime C \rightarrow P}$, the reason for this is explained in section 4.4.

**Figure 5.8:** Behaviour of the system changing the set-up.

(a) Change of the behaviour of the system by changing the coupling of the blankets of the FBRs to the cores of the PWRs.

(b) Change of the behaviour of the system by changing the coupling of the cores of the PWRs to the next cores of the PWRs.

(c) Change of the behaviour of the system by changing the ratio of the initial number of FBRs and PWRs.

(d) Starting set-up for finding the pre-determined asymptotic growth.
The first step performed is increasing the amount of critical mass that comes from the blankets of the FBRs and goes to the cores of the PWRs, see figure 5.8a. What can be seen is that the drop decreases and that there is also a small change in asymptotic behaviour. The decrease in drop and asymptotic behaviour results in a lower number of FBRs and a higher number of PWRs as a function of time per cycle. The reason for the decrease in drop comes from the fact that there is now more critical mass going to the PWRs. For the same reason the asymptotic growth becomes less, because the breeding of fuel happens in the FBRs and with less critical mass going to the FBRs there is also less growth. With the blanket coupling to the PWRs, \[ C_{F'B\rightarrow P} = 1, \]

another coupling matrix must be changed to get rid of the drop. Now the coupling of the critical mass going from PWR cores to the next PWR cores is changed, see figure 5.8b. The same as with the blanket coupling happens. More critical mass is going to the PWRs so the drop becomes even less and the asymptotic growth decreases further. The number of FBRs still decreases. The number of PWRs on the other hand increases for the first few cycles and decreases for the last cycles. The increase of PWRs in the beginning is caused by the drop becoming smaller and the decrease at the end is caused by the lower asymptotic growth. The last set-up value to change is the ratio between the initial number of FBRs and PWRs, see figure 5.8c. By increasing the initial number of FBRs there is more critical mass going to the PWRs and the drop vanishes. What can be seen from figure 5.8c is that by changing the initial values the asymptotic behaviour does not change, only the number of FBRs and PWRs as a function of time in cycles changes.

In figure 5.8d the results for a suitable starting set-up, no drop in the beginning and a smaller asymptotic growth, for the iteration process are shown. The starting set-up contains the values for the matrices founded from the figure 5.8. Figure 5.8a gives \[ C_{F'B\rightarrow F} = 1, \] figure 5.8b gives \[ C_{F'B\rightarrow P} = 1 \] and figure 5.8c gives the ratio of \[ \frac{2}{9} PWRs : FBRs \] as initial numbers of reactors.

### 5.4.2 Fuel Cycle Kinetics Iteration I

With the suitable starting set-up found from the previous section the iterative process can be performed. For the iteration the same symbiotic system is used as in section 5.3 so also the same equilibrium composition is used. This was also used in finding the values for the coupling matrices of the starting set-up. The values for the coupling matrices of the starting set-up are given in tables 5.9 and 5.10. The coupling matrix \( C_{F'B\rightarrow F} \) is set to zero because all the critical mass from the blanket is going to the PWRs and the total amount coming from the blankets is equal to 1. Together with a ratio of 2 PWRs and 9 FBRs as the initial condition the starting set-up of the iterative process is created. The fuel cycle kinetics solution for this starting set-up is shown in figure 5.8d. This system has an asymptotic growth \( \gamma_{\infty} = 3.56\% \). The parameters are not shown here because these will change during the iteration, due to changes in the coupling matrices. The changes in the coupling matrices give a clearer view of what the changes are.

<table>
<thead>
<tr>
<th>FBR</th>
<th>A</th>
<th>D_F</th>
<th>C_{F'C→F}</th>
<th>C_{F'B→F}</th>
<th>C_{F'B→P}</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.9: Values for the starting set-up of an FBR in a symbiotic deployment of FBRs and PWRs for figure 5.8d.

<table>
<thead>
<tr>
<th>PWR</th>
<th>A</th>
<th>D_P</th>
<th>C_{P'C→P}</th>
<th>C_{P'C→F}</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>0.75</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.10: Values for the starting set-up of a PWR in a symbiotic deployment of FBRs and PWRs for figure 5.8d.

Because the same symbiotic system is used as in section 5.3, also here the whole blanket is being discharged at the end of a cycle. Thus the number of blankets equals the number of fast reactors \( R_F(t) = B_F(t) \).
Because the use of the same system also the breeding bonuses are the same.

\[ b_F = \left\{ \begin{array}{l} \rightarrow w_F - \rightarrow \psi_F(0) \\ \rightarrow \psi_F(0) \end{array} \right\} = -0.1593 \quad (5.11) \]

\[ b_P = \left\{ \begin{array}{l} \rightarrow w_P - \rightarrow \psi_P(0) \\ \rightarrow \psi_P(0) \end{array} \right\} = -0.1996 \quad (5.12) \]

these values and the initial conditions from the ratio of 9 FBRs and 2 PWRs, \( R_{F0} = 9 \) and \( R_{P0} = 2 \), give the pseudo-initial conditions,

\[ R_{F1} = R_{F0}(1 + b_F) = 7.5663 \quad (5.13) \]

\[ R_{P1} = R_{P0}(1 + b_P) = 1.6008 \quad (5.14) \]

5.4.2.1 Upper Boundary

The goal of the iteration is to find the upper and lower boundary determined from the data in table A.4. These data represent the upper and lower predictions of the expected growth of the number of nuclear reactors. The expected growth for the upper boundary can be represented by an asymptotic growth of \( \gamma_\infty = 4.06\% \) per cycle. To determine the parameters that fulfill the fuel cycle kinetics solution for the upper and lower asymptotic growth iteration has to be applied on the coupling matrices. The iteration only takes into account changes in \( C_{F' \rightarrow F} \), \( C_{P' \rightarrow P} \) and \( C_{P' \rightarrow C} \) for reasons explained before.

Starting with the set-up given in figure 5.8d iteration is done. The iterative process is done by trial and error. The iteration towards the upper boundary gives the values given in tables 5.11 and 5.12 for the coupling matrices. What can be seen in table 5.11 is that total coupling from the blankets is not equal to 1 any more. The reason for this is that in the iteration a small decrease in asymptotic growth was still needed to fit the demand. The best way to change the asymptotic growth by a small amount is by changing the coupling from the blankets. With the coupling from the blankets going to the PWRs already set to 1 it was chosen to take 1% of the critical mass coming from the blankets out during reprocessing. With these matrices the parameters become as shown in tables 5.13 and 5.14. Solving the fuel cycle kinetics with these parameters gives an asymptotic growth \( \gamma_\infty = 4.08\% \) shown in figure 5.9a. In the figure the numbers of FBRs and PWRs are given as functions of time in cycles. This asymptotic growth is almost identical to the asymptotic growth of \( \gamma_\infty = 4.06\% \) which was asked for the upper boundary.

<table>
<thead>
<tr>
<th>FBR</th>
<th>A</th>
<th>D_F</th>
<th>( C_{F' \rightarrow F} )</th>
<th>( C_{F' \rightarrow P} )</th>
<th>( C_{F' \rightarrow C} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.99</td>
<td></td>
</tr>
</tbody>
</table>

**Table 5.11:** Values for the upper boundary of the energy demand for an FBR in a symbiotic deployment of FBRs and PWRs.

<table>
<thead>
<tr>
<th>PWR</th>
<th>A</th>
<th>D_P</th>
<th>( C_{P' \rightarrow P} )</th>
<th>( C_{P' \rightarrow C} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.79</td>
<td>0.21</td>
<td></td>
</tr>
</tbody>
</table>

**Table 5.12:** Values for the upper boundary of the energy demand for a PWR in a symbiotic deployment of FBRs and PWRs.
5.4. GROWTH OF NUCLEAR ENERGY

5.4.1. Upper Boundary

The accumulation of FBRs and PWRs in a symbiotic deployment with an asymptotic growth \( \gamma_{\infty} = 4.08\% \).

Figure 5.9: The upper and lower asymptotic growth of the energy demand by a symbiotic system of FBRs and PWRs determined with the fuel cycle kinetics equations.

\[
\begin{array}{cccccc}
\rho & \delta & \varepsilon_{F'\rightarrow F} & \varepsilon_{F'\rightarrow B} & \varepsilon_{F'\rightarrow P} & \Lambda \\
-0.250 & 4.637 & 4.637 & 0 & 1.427 & 4.637 \\
\end{array}
\]

Table 5.13: Asymptotic parameters for the upper boundary of the energy demand for an FBR in a symbiotic deployment.

\[
\begin{array}{cccccc}
\rho & \delta & \varepsilon_{P'\rightarrow P} & \varepsilon_{P'\rightarrow F} & \Lambda \\
\end{array}
\]

Table 5.14: Asymptotic parameters for the upper boundary of the energy demand for a PWR in a symbiotic deployment.

5.4.2.2 Lower Boundary

The same iteration process is done only now towards the lower boundary. Also the lower boundary follows from the data in table A.4 and can represented by an asymptotic growth of \( \gamma_{\infty} = 1.99\% \) per cycle. The iterative process towards the lower boundary then gives the values in tables 5.15 and 5.16 for the coupling matrices. Now even 20% of the critical mass coming from the blankets to the PWRs is taken out during reprocessing. These coupling matrices give then the parameters given in tables 5.17 and 5.18. The solution of the fuel cycle kinetics equations gives an asymptotic growth \( \gamma_{\infty} = 1.99\% \) see figure 5.9b which shows again the numbers of FBRs and PWRs as functions of time in cycles. This is exactly the demanded asymptotic growth of \( \gamma_{\infty} = 1.99\% \) for the lower boundary.

\[
\begin{array}{cccc}
\Lambda & D_F & C_{F'\rightarrow F} & C_{F'\rightarrow B} \\
a & 1 & 1 & 0.8 \\
\end{array}
\]

Table 5.15: Values for the lower boundary of the energy demand for an FBR in a symbiotic deployment of FBRs and PWRs.

\[
\begin{array}{cccc}
\Lambda & D_P & C_{P'\rightarrow P} & C_{P'\rightarrow C} \\
a & 1 & 0.75 & 0.25 \\
\end{array}
\]

Table 5.16: Values for the lower boundary of the energy demand for a PWR in a symbiotic deployment of FBRs and PWRs.
5.4. GROWTH OF NUCLEAR ENERGY

<table>
<thead>
<tr>
<th>FBR</th>
<th>PWR</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ</td>
<td>δ</td>
</tr>
<tr>
<td>-0.250</td>
<td>4.637</td>
</tr>
</tbody>
</table>

Table 5.17: Asymptotic parameters for the lower boundary of the energy demand for an FBR in a symbiotic deployment.

Table 5.18: Asymptotic parameters for the lower boundary of the energy demand for a PWR in a symbiotic deployment.

5.4.3 TRITON Calculations Iteration I

To check if the coupling matrices determined from the iterative process for the upper and lower boundary are correct they are implemented in TRITON. This means that the values for the upper boundary, tables 5.11 and 5.12, and the lower boundary, tables 5.15 and 5.16, are inserted in the TRITON calculation.

This results in an asymptotic growth of $γ_∞ = 5.64\%$ for upper boundary and an asymptotic growth of $γ_∞ = 3.9\%$ for the lower boundary both with the coupling matrices from the fuel cycle kinetics iteration. This means that the asymptotic growths from the TRITON calculations are much higher than the asymptotic growths found from the iteration done with fuel kinetics equations. This was not entirely expected. The reason for this difference is that due to the drastic changes in the coupling matrices and in the initial ratio of the FBRs and PWRs the equilibrium compositions and critical mass weights of the reactors have changed. So an extra iteration step is needed to find the demanded asymptotic growths for the upper and lower boundary. The changes for the coupling matrices in the iterative process will now be smaller so a large change in equilibrium compositions and critical mass weights is not expected. So the values found in this TRITON calculation for the equilibrium compositions and critical mass weights will be used in the next iteration step.

(a) The upper boundary with an asymptotic growth $γ_∞ = 5.64\%$ determined with TRITON.

(b) The lower boundary with an asymptotic growth $γ_∞ = 3.9\%$ determined with TRITON.

Figure 5.10: The upper and lower asymptotic growth of the energy demand by a symbiotic system of FBRs and PWRs determined with TRITON.
5.4. GROWTH OF NUCLEAR ENERGY

5.4.4 Fuel Cycle Kinetics Iteration II

As a starting point for the next iteration step the values of the coupling matrices for the lower boundary found in the last TRITON calculation are chosen. This is done because the result for this TRITON calculation has an asymptotic growth of \( \gamma_\infty = 3.9\% \) which is very close to the asymptotic growth of \( \gamma_\infty = 4.06\% \) demanded for the upper boundary. Solving the fuel cycle kinetics equations with the values from tables 5.15 and 5.16 and the new equilibrium composition and critical mass weights from the last TRITON calculations gives an asymptotic growth of \( \gamma_\infty = 4.23\% \). This solution is shown in figure 5.11.

![Figure 5.11](image)

**Figure 5.11:** A symbiotic system of FBRs and PWRs with an asymptotic growth \( \gamma_\infty = 4.23\% \) determined by fuel cycle kinetics with the equilibrium compositions and critical mass weights from the TRITON calculation of section 5.4.

The asymptotic growth of \( \gamma_\infty = 3.9\% \) of the TRITON calculation is slightly lower than the asymptotic growth of \( \gamma_\infty = 4.23\% \) of the fuel cycle kinetics solution with the equilibrium compositions and critical mass weights from the TRITON calculation. For this reason it is chosen in this iteration step to let the asymptotic growth of the fuel cycle kinetics solution to be slightly higher then the demanded asymptotic growths of \( \gamma_\infty = 4.06\% \) for the upper boundary and \( \gamma_\infty = 1.99\% \) for the lower boundary. With this done the asymptotic growths obtained from the TRITON calculations will become as close as possible to the demanded ones.

Before the iterative process can be done the pseudo-initial conditions have to be recalculated. This has to be done because the reactor compositions and critical mass weights have changed, so also the breeding bonuses have changed. They are now as follows

\[
b_F = \left\{ \begin{array}{l}
\left\{ \frac{\psi_F}{w_F}, \frac{\psi_F}{w_F}(0) \right\} \\
\{ \rightarrow, \psi_F(0) \}
\end{array} \right. = -0.242
\]

(5.15)
\begin{equation}
bp = \left\{ \left[ \overrightarrow{w_P} - \overrightarrow{w_P} \right], \overrightarrow{\psi_P}(0) \right\} = -0.2489 \tag{5.16}
\end{equation}

The ratio of 9 FBRs and 2 PWRs, \( R_{F0} = 9 \) and \( R_{P0} = 2 \), is kept the same so this gives the pseudo-initial conditions,

\[ R_{F1} = R_{F0}(1 + b_F) = 6.822 \tag{5.17} \]

\[ R_{P1} = R_{P0}(1 + b_P) = 1.5022 \tag{5.18} \]

5.4.4.1 Upper Boundary

With this starting set-up first iteration is performed towards the upper boundary. This gives the values in tables 5.19 and 5.20. These coupling matrices lead to the parameters in tables 5.21 and 5.22 for the fuel cycle kinetics equations. The fuel cycle kinetics equations give then an asymptotic growth of \( \gamma_\infty = 4.32\% \). This result is shown in figure 5.12a. Shown are the numbers of FBRs and PWRs as functions of time in cycles. This is close to the demanded asymptotic growth of \( \gamma_\infty = 4.06\% \). Keep in mind that the asymptotic growth of the solution of the fuel cycle kinetics equations is deliberately higher than the demanded asymptotic growth.

<table>
<thead>
<tr>
<th>FBR</th>
<th>( \Lambda )</th>
<th>( D_F )</th>
<th>( C_{F'G\rightarrow F} )</th>
<th>( C_{F'B\rightarrow F} )</th>
<th>( C_{F'B\rightarrow P} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>0</td>
<td>0.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\textit{Table 5.19:} Values for the upper boundary of the energy demand for an FBR in a symbiotic deployment of FBRs and PWRs.

<table>
<thead>
<tr>
<th>PWR</th>
<th>( \Lambda )</th>
<th>( D_P )</th>
<th>( C_{P'G\rightarrow P} )</th>
<th>( C_{P'C\rightarrow F} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>0.79</td>
<td>0.21</td>
<td></td>
</tr>
</tbody>
</table>

\textit{Table 5.20:} Values for the upper boundary of the energy demand for a PWR in a symbiotic deployment of FBRs and PWRs.

<table>
<thead>
<tr>
<th>FBR</th>
<th>( \rho )</th>
<th>( \delta )</th>
<th>( \varepsilon_{F'G\rightarrow F} )</th>
<th>( \varepsilon_{F'B\rightarrow F} )</th>
<th>( \varepsilon_{F'B\rightarrow P} )</th>
<th>( \Lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.215</td>
<td>4.568</td>
<td>4.568</td>
<td>0.768</td>
<td>4.568</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\textit{Table 5.21:} Asymptotic parameters for the upper boundary of the energy demand for an FBR in a symbiotic deployment.

<table>
<thead>
<tr>
<th>PWR</th>
<th>( \rho )</th>
<th>( \delta )</th>
<th>( \varepsilon_{P'C\rightarrow P} )</th>
<th>( \varepsilon_{P'C\rightarrow F} )</th>
<th>( \Lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.368</td>
<td>8.322</td>
<td>6.574</td>
<td>1.867</td>
<td>8.232</td>
<td></td>
</tr>
</tbody>
</table>

\textit{Table 5.22:} Asymptotic parameters for the upper boundary of the energy demand for a PWR in a symbiotic deployment.
5.4. GROWTH OF NUCLEAR ENERGY

(a) The accumulation of FBRs and PWRs in a symbiotic deployment with an asymptotic growth $\gamma_\infty = 4.32\%$.

(b) The accumulation of FBRs and PWRs in a symbiotic deployment with an asymptotic growth $\gamma_\infty = 2.23\%$.

Figure 5.12: The upper and lower asymptotic growth of the energy demand by a symbiotic system of FBRs and PWRs determined with the fuel cycle kinetics equations.

5.4.4.2 Lower Boundary

For the iteration towards the lower boundary the values in tables 5.23 and 5.24 are obtained for the coupling matrices. These coupling matrices lead to the parameters in tables 5.25 and 5.26 for the fuel cycle kinetics equations. The fuel cycle kinetics equations give then an asymptotic growth of $\gamma_\infty = 2.23\%$. The result is shown in figure 5.12b. Shown are the numbers of FBRs and PWRs as functions of time in cycles. Also for the lower boundary the obtained asymptotic growth of $\gamma_\infty = 2.23\%$ is close to the demanded asymptotic growth of $\gamma_\infty = 1.99\%$. Again keep in mind that the result is deliberately higher.

Table 5.23: Values for the lower boundary of the energy demand for an FBR in a symbiotic deployment of FBRs and PWRs.

<table>
<thead>
<tr>
<th></th>
<th>$\Lambda$</th>
<th>$D_F$</th>
<th>$C_{F' \rightarrow F}$</th>
<th>$C_{F' \rightarrow B}$</th>
<th>$C_{F' \rightarrow P}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FBR</td>
<td>a</td>
<td>1</td>
<td>0</td>
<td>0.75</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.24: Values for the lower boundary of the energy demand for a PWR in a symbiotic deployment of FBRs and PWRs.

<table>
<thead>
<tr>
<th></th>
<th>$\Lambda$</th>
<th>$D_F$</th>
<th>$C_{P' \rightarrow F}$</th>
<th>$C_{P' \rightarrow B}$</th>
<th>$C_{P' \rightarrow P}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PWR</td>
<td>a</td>
<td>1</td>
<td>0.85</td>
<td>0.15</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.25: Asymptotic parameters for the lower boundary of the energy demand for an FBR in a symbiotic deployment.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\delta$</th>
<th>$\epsilon_{F' \rightarrow F}$</th>
<th>$\epsilon_{F' \rightarrow B}$</th>
<th>$\epsilon_{F' \rightarrow P}$</th>
<th>$\Lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.207</td>
<td>4.564</td>
<td>4.564</td>
<td>0</td>
<td>0.625</td>
<td>4.564</td>
</tr>
</tbody>
</table>

Table 5.26: Asymptotic parameters for the lower boundary of the energy demand for a PWR in a symbiotic deployment.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\delta$</th>
<th>$\epsilon_{P' \rightarrow F}$</th>
<th>$\epsilon_{P' \rightarrow B}$</th>
<th>$\epsilon_{P' \rightarrow P}$</th>
<th>$\Lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.243</td>
<td>8.292</td>
<td>7.048</td>
<td>1.322</td>
<td>8.292</td>
<td></td>
</tr>
</tbody>
</table>
5.4.5 TRITON Calculations Iteration II

To check the asymptotic growths obtained from iteration they are checked again by TRITON calculations performed with the same coupling matrices. The values used in TRITON are shown in tables 5.19 and 5.20 for the upper boundary and with tables 5.23 and 5.24 for the lower boundary.

The results for the upper boundary of the TRITON calculation have an asymptotic growth of $\gamma_\infty = 4.11\%$, see figure 5.13a, which is almost identical to the demanded asymptotic growth of $\gamma_\infty = 4.06\%$ for the upper boundary. So it can be concluded that the small changes in the coupling matrices have not affected the equilibrium compositions and critical mass weights much.

The result from the TRITON calculation for the asymptotic growth of the lower boundary is $\gamma_\infty = 2.28\%$, see figure 5.13b. This result is higher as expected and higher than the demanded asymptotic growth $\gamma_\infty = 1.99\%$. The reason for this is that for the lower boundary the changes in the coupling matrices have been larger than for the upper boundary. So the equilibrium compositions and critical mass weights have changed more than in the upper boundary which has effect on the asymptotic growth.

![Graphs showing upper and lower asymptotic growth](image)

(a) The upper boundary with an asymptotic growth $\gamma_\infty = 4.11\%$ determined with TRITON.

(b) The lower boundary with an asymptotic growth $\gamma_\infty = 2.28\%$ determined with TRITON.

**Figure 5.13:** The upper and lower asymptotic growth of the energy demand by a symbiotic system of FBRs and PWRs determined with TRITON.

5.4.6 Error

The errors between the fuel cycle kinetics solutions from the last iteration and the TRITON calculations with the coupling matrices from this iteration are shown in figures 5.14a and 5.14b. The large errors in the beginning are caused by the use of the pseudo-initial condition which creates an immediate error for the first few cycles. Also the still existing small dip in the beginning of the TRITON calculations contributes to the error. What can be seen is that the error in the beginning is far smaller than in figure 5.6. The reason for this is the minimisation of the dip in the beginning. Most of the error left in the beginning is caused by the lowering of the initial number of reactors by the pseudo-initial condition. The use of the pseudo-initial condition creates a larger error in the beginning than without the pseudo-initial condition. This error though is only for the first cycles after that the error is smaller, see the explanation of figure 4.2. Further it can be seen that the errors still do not entirely stabilise, this is still caused...
by the small differences in asymptotic growths. These smaller differences cause the growth in
the errors to be much smaller compared to the errors in figure 5.6. The larger errors in the lower
boundary compared to the errors for the upper boundary are caused by the larger difference
in asymptotic growths between the fuel cycle kinetics solution and the TRITON solution. The
reason for the error in the FBRs being larger than in the PWRs for both the upper and lower
boundary is caused by the number of reactors. The number of FBRs is much larger than the
number of PWRs which creates a larger error.

Figure 5.14: Errors in the numbers of FBRs and PWRs between the TRITON calculation and the Fuel Cycle Kinetics.

For the upper boundary the results from the TRITON calculations and the result from the
fuel cycle kinetics equations are very close. Although there is a larger difference between the
result from the TRITON calculations and the result from the fuel cycle kinetics equations for the
lower boundary it is still quite close. This proves the value of using the fuel cycle kinetics theory
on this sort of problem. With the use of the fuel cycle kinetics theory a suitable set-up could be
found for determining the upper and lower boundary for a demanded asymptotic growth within
a short amount of time. If every set-up in the search had to be calculated by TRITON it would
have taken tremendous amounts of time.

5.4.7 Fuel Cycle Kinetics vs. Fuel Cycle Kinetics

In this section the equilibrium compositions, critical mass weights and coupling matrices from
the last TRITON calculations, section 5.4.5, are used to solve the fuel cycle kinetics equations.
These solutions will be compared to the solutions of the fuel cycle kinetics equations from of
section 5.4.4 which were solved with the equilibrium compositions and critical mass weights
from the TRITON calculations from section 5.4.3. So the difference lies now in the equilibrium
compositions and critical mass weights. The coupling matrices are the same for both fuel cycle
kinetics solutions. This is done to show that the largest contribution to the error is from the
difference in time behaviour of the fuel cycle kinetics results, which is continuous, and that of
the TRITON results which is discontinuous.
The accumulation of FBRs and PWRs in a symbiotic deployment with an asymptotic growth $\gamma_\infty = 4.45\%$.

The accumulation of FBRs and PWRs in a symbiotic deployment with an asymptotic growth $\gamma_\infty = 2.55\%$.

Figure 5.15: The upper and lower asymptotic growth of the energy demand by a symbiotic system of FBRs and PWRs determined with the fuel cycle kinetics equations.

The fuel cycle kinetics solutions with equilibrium composition and critical mass weights from the last TRITON calculations give an asymptotic growth of $\gamma_\infty = 4.45\%$ for the upper boundary, see figure 5.15a. This is slightly higher than the asymptotic growth of $\gamma_\infty = 4.32\%$ determined in section 5.4.4. The lower boundary determined with the values from the last TRITON calculation gives an asymptotic growth of $\gamma_\infty = 2.55\%$, see figure 5.15b which is a little bit higher than the asymptotic growth of $\gamma_\infty = 2.23\%$ also determined in section 5.4.4. The difference is smaller for the upper boundary because there are only small changes in the coupling matrices and for that only small changes in the equilibrium compositions and critical mass weights. The difference between the asymptotic growths for the lower boundary is more than the difference between asymptotic growths of the higher boundary. This is expected because the changes in the coupling matrices are larger and so the difference in the equilibrium compositions and critical mass weights will be larger.

5.4.8 Error

The errors between the fuel cycle kinetics solutions with different equilibrium compositions and critical mass weights, from section 5.4.3 and 5.4.5, are very small for both the upper and lower boundaries, see figures 5.16a and 5.16b. The errors are small because the fuel cycle kinetics solutions are almost identical. They only have a slightly different pseudo-initial conditions and asymptotic growths. The difference in pseudo-initial condition can be seen from the small error at cycle 0 in figures 5.16a and 5.16b. Also the difference in asymptotic growths follows from the figures, where can be seen that the error does not stabilise. The reason why the errors for the upper boundary increase faster is due to the larger number of reactors present for the upper boundary.
5.4. GROWTH OF NUCLEAR ENERGY

(a) Error between Fuel Cycle Kinetics solutions for the upper boundary.

(b) Error between Fuel Cycle Kinetics solutions for the lower boundary.

Figure 5.16: Errors in the numbers of FBRs and PWRs between the Fuel Cycle Kinetics solutions.

The minimal errors between the fuel cycle kinetics solutions shows that an extra iteration step will not have a big influence on the numbers of reactors in the fuel cycle kinetics solutions and therefore is not performed. The errors between the fuel cycle kinetics solutions and the TRITON solutions are still present. The reason is that the TRITON solutions have a different time behaviour than the fuel cycle kinetics solutions. The TRITON solutions have a discontinuous behaviour, where the fuel cycle kinetics solutions are continuous.
Discussion and Conclusions

6.1 Discussion

In chapter 5 the fuel cycle kinetics theory was tested in practice. The first case treated was with the parameters from Maudlin’s thesis [3] and is shown in section 5.1. In the original theory from Maudlin’s thesis there is not a clear explanation of how the core and blanket coupling between different reactor types is performed. Before the fuel cycle kinetics equations could be solved, with the parameters at hand, the equations first had to be constructed in the intended form, see section 4.4. With a clear understanding of the coupling and implementing it in the right way in the fuel cycle kinetics equations, solving the problem was straightforward and identical results to the results from Maudlin were obtained. So the fuel cycle kinetics theory works on a known problem.

With the information on the coupling between reactors and solving of the fuel cycle kinetics equations, the second test case, section 5.2, a deployment of only FBRs was performed. In this section practical complications in creating the parameters came to the surface. These complications consisted of how to construct the equilibrium compositions and critical mass weights, due to small fluctuations in the equilibrium compositions between cycles see figure 3.2a. The solution chosen was to average both the equilibrium composition and the critical mass weights over 5 cycles from the point that equilibrium is reached. Another complication was how the discharge and reprocessing needed to be performed. For simplification it was decided to discharge all the fuel from the reactor, blanket and core, and then charge it back into the next reactor. This total discharge was chosen because otherwise the reactors would get partially different compositions and this would complicate the problem significantly. With the implementation of the use of multiple batches in the reactor the discharge could be split into smaller discharges and charges, but still the problem would exist that the reactor consists out of multiple batches with different compositions. First solving of the fuel cycle kinetics equations for a deployment of FBRs was performed without the pseudo-initial condition. This gave a similar asymptotic growth as in the TRITON calculation, only the number of reactors did not correspond. By solving the fuel cycle kinetics equation with the pseudo-initial condition both the expected asymptotic growth was reached and the number of reactors corresponded between the fuel cycle kinetics solution and the TRITON solution within an acceptable margin of error. The use of the pseudo-initial
6.1. DISCUSSION

In section 5.3 the third test case was performed. This test case was a symbiotic deployment of FBRs and PWRs. With the knowledge from the test cases of sections 5.1 and 5.2 the construction of the fuel cycle kinetics equations and solving did not create large problems. The errors in the number of reactors in the symbiotic deployment with pseudo-initial conditions were a bit larger than in the FBR deployment. The reason for this larger difference is the continuous behaviour of fuel cycle kinetics solutions and the discontinuous behaviour of the TRITON solutions. This creates the large error in the early cycles. Also the use of two fuel cycle kinetics equations, one for FBRs and one for PWRs, gives an accumulation error through the coupling. Further it was seen that the error does not stabilise because of the difference in asymptotic growths between the fuel cycle kinetics solutions and TRITON solutions.

In the fourth test case again a symbiotic deployment of FBRs and PWRs was treated, section 5.4. In this section the coupling in the fuel cycle kinetics equations between the FBRs and PWRs was altered in such a way that two demanded asymptotic growths were reached. These asymptotic growths were checked by implementing the coupling matrices found from the fuel cycle kinetics equation into TRITON. The demanded asymptotic growths were determined at the hand of the upper and lower boundary of the expected nuclear energy growth in the upcoming years. The first step that was taken was the minimisation of the influence of the discontinuous behaviour of the TRITON solution in order to reduce the error. This was done by removing the dip in the first fuel cycles by changing the coupling matrices and the initial ratio of FBRs and PWRs, see figure 5.8. Then an iteration was done on the coupling in the fuel cycle kinetics equations between the reactors to determine the two set boundaries. When the found coupling matrices were implemented in TRITON the results gave an unexpected difference on the first attempt. This was caused by to the fact that the iteration process was done with the equilibrium compositions and critical mass weights of the first symbiotic deployment. Due to the large changes in the coupling and the ratio between the reactors the equilibrium compositions and critical mass weights were changed considerably.

A second iteration was needed to get the two asymptotic growths for the set boundaries. As a starting point the equilibrium compositions and critical mass weights from the lower asymptotic growth of the first iteration were taken. This lower asymptotic growth was already close to the demanded upper asymptotic growth, so the small changes in the coupling matrices needed to get to the demanded asymptotic growths should not affect the equilibrium compositions and critical mass weights much. First the iteration towards the demanded upper boundary was done. The coupling matrices from this iteration were again used for the TRITON calculation. The result now was only slightly different then the demanded asymptotic growth. So the small changes in the coupling matrices did not affect the equilibrium compositions and critical mass weights much. For the lower demanded asymptotic growth rate the coupling matrices had to change more and for this reason also the equilibrium compositions and critical mass weights changed. Due to this larger change the result from the TRITON calculation was higher than the demanded asymptotic growth. The difference was not very large but larger than for the upper demanded asymptotic growth.

As a comparison the fuel cycle kinetics equations were solved again with equilibrium compositions and critical mass weights of the last performed TRITON calculation. These fuel cycle kinetics solutions were compared to the fuel cycle kinetics solutions obtained in the last iteration process. The errors between the two solutions were both for the upper and lower boundary very small. The reason for this is that the behaviours of both solutions are almost identical. There was only a small difference between the pseudo-initial conditions which created a small error at the beginning of the first cycle. The small difference between the asymptotic growths can be
seen from the fact that the error does not stabilise.

After performing the test cases one downside of the fuel cycle kinetics theory was noticed and this is that the solutions are highly sensitive to small errors in the parameters that construct the equations. The smallest error in the creation of the parameters can cause a large difference in the number of reactors.

## 6.2 Conclusions

- The fuel cycle kinetics theory is a computationally cheap method. As a comparison, in the last symbiotic system the TRITON calculations for the FBR took about 3 hours per cycle and the PWR about 1 hour per cycle. These calculations can be run simultaneously, so the longest, the FBR, determines the total time a calculation takes. For 30 cycles the total runtime was 90 hours which is far more than the 1 minute the fuel cycle kinetics calculations take.

- The accuracy of the fuel cycle kinetics solution is sufficient for most points. The place where a larger error occurs can be assigned to the continuous behaviour of the fuel cycle kinetics solution compared to the discontinuous output of the TRITON calculations. In general the TRITON calculations are a more closer approximation to reality so if the TRITON calculations would give a continuous output then this would give a smaller error between the TRITON and the fuel cycle kinetics solution.

- The points mentioned above make the fuel cycle kinetics model a very powerful tool to research complex reactor scenarios accurately and in a short amount of time.

- The solution of the fuel cycle kinetics is highly sensitive to small errors in the parameters. Just a small error in the calculation of a parameter can give a huge deviation in the number of reactors. If for instance in the FBR deployment the reactivity is 1% lower, the asymptotic growth is 0.06% smaller. This means that after 100 cycles the number of FBRs is 877 instead of 934 FBRs. It is therefore necessary to determine the parameters as carefully and accurately as possible.

## 6.3 Future Work

There are two major points of interest to look into in the future. One is to implement the still missing steps from the fuel cycle kinetics theory in the model and the second is to extend the theory and model. Both steps enhance the accuracy and make the model more applicable to realistic problems.

1. In the model used to generate the results not all the theory of the fuel cycle kinetics is used. By implementing these missing steps the fuel cycle kinetics solutions should give even more accurate and realistic results.

   - One of the missing steps is the reprocessing time, $\tau$. In the model used throughout the thesis it is assumed that the reprocessing and charging is done instantaneously. This is not a realistic assumption, so for a more realistic prediction the reprocessing time has to be accounted for.

   - In the model also the reprocessing losses, $\epsilon$, are set to zero. Again this is not what happens in reality. So also the reprocessing losses have to be implemented to make the model more realistic.
6.3. FUTURE WORK DISCUSSION AND CONCLUSIONS

- A single batch core is assumed in the model in this thesis. In reality a reactor consists of multiple batches. In the fuel cycle kinetics theory this can easily be implemented, see equation (3.6c). The problem will be in implementing a multiple batch core in the TRITON calculations to check the results, because the use of multiple batches will give the reactor cores different compositions.

- In the model used in the thesis the blanket was discharged after every cycle, which meant that there was no blanket inventory build up and thus that the number of blankets equalled the number of fast reactors, $B_F(t) = R_F(t)$. The radial blankets can be taken out at any point, so for instance after every cycle. On the other hand the axial blanket follows the same proceedings as the core. So if there is a multi batch core as explained in the previous point, parts of the blanket remain multiple cycles in the reactor. In this case there is an inventory build up and the blanket delay has to be accounted for.

- In this thesis the equilibrium composition used was obtained from TRITON calculations that were already performed, so no extra research was performed on this item. In section 5.4 this caused a problem due to the fact that the equilibrium composition of the reactors changes quite a lot when the coupling between the reactors changes drastically. So the equilibrium composition needs to be determined beforehand for changing conditions. A method by which this can be achieved is the eigenvalue problem from equation 4.43.

$$\overrightarrow{C}_\infty \psi(\infty) = \gamma_\infty \overrightarrow{H} \psi(\infty) \quad (6.1)$$

- One point that is not a missing step in the fuel cycle kinetics theory itself but has direct relation to the results of it is the larger error that was produced by the discontinuous behaviour of the TRITON results. This was due to the fact that it produced only output at the end of the cycle, see figure 5.6. A way to resolve this could be by creating a continuous output of the TRITON calculations. This could lower or even dissolve the large error in the beginning of figure 5.6 and help to minimise the error in other systems.

2. A few points are of interest to extend the theory. They enhance the accuracy of the theory but also more important to allow the possibility of using the theory on more realistic scenarios.

- A method for enhancing the accuracy would be to expand the method of how the critical mass weighting is done. At the moment the weighting in the fuel cycle kinetics is done for the core as a whole and also for the blanket as a whole. In a realistic scenario the core and the blanket consist out of multiple materials, see figures 5.3a and 5.3b. To make the fuel cycle kinetics solution even more accurate it could be considered to do the weighting for the different materials separately. This would enhance the accuracy because the beginning of cycle composition is different in every material, which means the composition changes in a different way. This can give slightly different results compared to taking the core and blanket as a whole. A way to do the different weighting is to treat every material as a separate sub-core or sub-blanket. This means that for every sub-core and sub-blanket coupling matrices have to be created and also all the parameters have to be determined separately for every material. Taken all the parameters from the sub-cores and sub-blankets together gives than the fuel cycle kinetics equations for the reactors in total. Doing this will enhance the accuracy. It will increase the calculation time, but due to the fact that the fuel cycle kinetics calculations take very short amount of time this will not be problematic.
Another way to enhance the accuracy can be by eliminating the approximation done in equation 4.58. Eliminating this approximation will ask for the addition of an extra equation in the form of a precursor equation similar to the one used in the point kinetics theory, see equation 2.28b. This will increase the calculation time because now there will be two equations for every reactor type, but due to the short calculation time of the fuel cycle kinetics equations this should not cause a problem.

In the model only critical mass weights are used, for the reason explained in section 3.4.1. It can be of interest to look at the effect of using different types of weighting in the fuel cycle kinetics equations. Other types of weighting can enhance accuracy if the fuel cycle kinetics theory is used for other purposes then finding the asymptotic growth.

To make the model more realistic an interesting thing to do is to make use of multiple reactor designs and types. In the model used in this thesis only two types of reactors are used, see figure 3.1, at different points in a cycle with use of the same fuel. In reality every reactor type is designed in a different way and makes use of a different fuel and thus has a different fuel composition. This will increase the number of fuel cycle kinetics equations, one for every reactor, and with this the calculation time. Also this increase should not create a problem.

![Diagram of a reactor park with different designed FBRs and LWRs.](image)

**Figure 6.1:** Scheme of a reactor park with different designed FBRs and LWRs.

The difficulty will be in the weighting of the different fuel compositions of the different types of reactors and the coupling between them.

With the use of a more extended reactor park the effects of different coupling and different ratios between reactors can be researched to improve for instance, the efficient use of fuel or the minimisation of the production of minor actinides.
Bibliography


WNA Nuclear Century Outlook Data

The Nuclear Century Outlook is an estimation of the worldwide growth of nuclear power in the 21st Century and is performed by the World Nuclear Association, WNA. The outlook sets a high and a low boundary, as can be seen in figure A.1. The graph is based on the data from tables A.1, A.2 and A.3.

Figure A.1: Prediction of the maximum and minimum energy production by nuclear reactors
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**Table A.1:** Current Nuclear Programs
Table A.2: Nations Planning Nuclear Potential Entrants

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J.A.B. van Rhijn 67 Delft University of Technology
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*Table A.4: World Total*