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Bachelor End Project

Faculty of Applied Sciences
Applied Physics

Filling the emergency tank around the Small-scale, Large efficiency, Inherently safe, Modular Reactor

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Abstract

Nuclear reactors might be a solution for the climate problems, since they are low-carbon and relatively cheap. Unfortunately, the public opinion on nuclear energy is not good. This might change with the new generation of reactors, since there is even more attention to keep reactors safe. A main risk for any type of reactor is that the temperature becomes too high as that will damage safety barriers. Some new generation reactors do not need an external source to cool itself. An example of these reactors is the SLIMR. The SLIMR can cool passively due to the water pool that surrounds the SLIMR.

When an emergency shutdown occurs, the goal is to lose as much decay heat as fast as possible. However, these energy losses are unwanted in the nominal situation. For this purpose, a tank is invented around the reactor. This tank is a cylinder with an opening at the bottom and a pipe at the top connecting it to the atmosphere. A pump has to actively keep air inside the tank under the nominal situation under a high pressure, so that the reactor vessel is surrounded by air instead of water. The nominal heat losses decrease. When there is a shutdown, the pump no longer keeps air inside. Due to pressure differences, air will flow out of the tank to the atmosphere and water will flow in the tank at the bottom.

The time it takes to fill the tank under the assumption that the pressure in the tank became atmospheric instantly is already calculated. This assumption is only true in specific conditions. The goal of this research is to simulate this pressure as well and look at the influence of the exit geometry on those filling times.
The numerical model from this report can solve the height of the water, the velocities, pressure and densities in the tank for any time for different dimensions. The model however cannot solve the temperature. The assumption that the pressure drops to atmospheric pressure proves to be nearly right only under certain conditions. The influence of the exit geometry on the filling times is found as well. The conclusion is that a smaller exit pipe and more friction cause significant filling time extensions.

Further research might model temperatures along the tank and the reactor, to verify what conditions are needed to make sure this tank provides a safe solution.

Contents

Chapter 1

Introduction

1.1 Fukushima

The demand for energy is rising [3]. On the other hand, a 6.3% decarbonization rate is needed to fulfill the Paris Agreement [10]. Nuclear power could solve this problem since nuclear energy is low-carbon and relatively cheap. The main reason that nuclear energy isn't used more, is the public opinion on nuclear power. Civilians fear reactors since problems in a reactor can have disastrous consequences. Yet, the public opinion was improving until 2011 [7]. In 2011, the Fukushima disaster happened[2]. 130 kilometer of the coast of Japan, an earthquake with a strength of 9.0 occurred. The reactor went to a shutdown. The safety mechanism could withstand the seismic fluctuations, but the mechanism could not withstand the tsunami that was initiated by the earthquake. The tsunami destroyed the Residual Heat Removal system so that the residual heat could not be removed to the sea. A couple of reactors became so hot that the fuel melted through the core. There were also hydrogen explosions. These accidents caused radioactive materials to come in the atmosphere. The public opinion on nuclear energy has never been restored to the level it had in 2011[7].

1.2 Reactor Safety

Every reactor has a residual heat system containing an external energy drain. A reactor namely still produces decay energy after a shutdown. This decay heat in combination with the energy in the reactor on the moment of the shutdown might not get removed via the turbine. To still get rid of this energy, it has to be deposited in a storage or in water. In Fukushima, the system to deposit this residual heat was damaged causing leaks.

Already in 2004, a forum was found to look into a new generation of nuclear reactors[9]. Many reactors are aging and they need replacement or strong improvements to cope with the nuclear energy needs. This forum highlighted four research areas: sustainability, economics, safety and reliability, and proliferation resistance and physical protection[5]. One of the ideas to make reactors safe is to let them remove their energy passively to the environment and not via an external source. This is one of the ideas behind the smaller reactors, such as the Small Modular Reactors.[5]

1.2.1 SMR

The SMRs are partly scaled down versions of Large Reactors (LR) and partly entirely newly invented Generation IV reactors [12]. These SMRs lose more energy to the environment since their surface is relatively larger than their volume in comparison to LRs. The high energy loss can be used to remove the residual heat in case the reactor is shut down. The SMRs are surrounded by water to lose energy fast since this is a good coolant: water has a high heat transfer coefficient and a high specific energy capacity. Water surrounding the tank is therefore safe, but the heat losses during nominal operation are therefore relatively high in comparison to LRs. A smaller reactor has some economical advantages; since it is small, it can be produced off-location and transported to the site. Next to this, it can help with desalination of water and production of hydrogen. However, it also produces much less power than an LR. Besides, licensing of a reactor is hard, especially for new and smaller reactors like SMRs [13].

1.2.2 SLIMR

To optimize the reactor, whilst keeping it safe, Rohde proposed the Small-scale, Large efficiency, Inherently safe, Modular Reactor (SLIMR) [11]. It is an SMR, producing $350MW$, with the unique combination of the use of supercritical water, the use of Thorium and passive decay heat removal. For the scope of this project, the use of thorium and the supercritical water is not important, so the focus will be on the passive decay heat removal. The reactor has to be inherently safe; it does not need an active source to lose its decay heat. Veling already proved that under the design he proposed, it is possible to lose all decay heat if the reactor is submerged in a water pool [15]. To make the SLIMR supercritical, the pressure in the reactor has to be $25MPa$, but more importantly, the water has to surpass the temperature of $375°C$. Krijger proposed a different core design with a three-pass core. In his design, the core outlet temperature indeed exceeded this temperature, as it was $396°C$. The design of Veling is shown in figure 1.1. It still contains only one core pass.

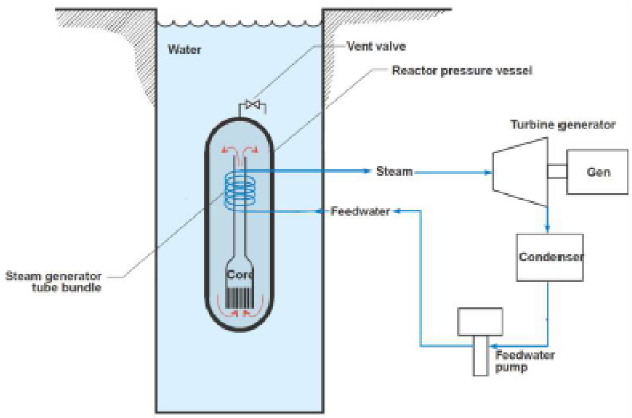

Figure 1.1: The design of the SLIMR as proposed by Veling [15]. It is not to scale.

1.3 Emergency Cooling

The reactor has to cool itself after a shutdown. It has to lose heat quickly for this purpose. However, the heat losses are unwanted when the reactor is operating nominally. Krijger [8] proposed to design a second cylinder around the reactor, which will be called the tank in the rest of the report. The goal of this tank is to keep the reactor surrounded by air in the nominal situation to minimize losses and surrounded by water at a shutdown to get the decay heat properly removed. The filling of the tank with water has to happen without any power source.
To keep the reactor surrounded by air, a pump has to push all water out of this tank and this pump in combination with valves keep the air under pressure so that the tank is fully filled by air under nominal conditions. However, if there is a shutdown, all electricity will be cut off. The tank will stop working and the air will flow out of the tank due to pressure differences. Since air flows out, the tank will be filled with water from the pool. Dimensions and a clarifying figure will be shown in section 2.1.

Veling already proved that his design will succeed in losing its decay heat to the pool without damaging the core [15]. The design is changed since the design includes an emergency cooling tank. This change causes multiple differences. Under nominal conditions, the tank loses heat in stationary conditions to air instead of to water. Secondly, there is a period in which the tank is filling with water from the pool. The reactor does not lose heat via the turbine any more. It loses heat partly via water and partly via air. Since this heat transfer will be lower than the production of energy in the reactor, the temperature of the reactor will rise. The time to fill the tank completely with water is called the filling time. At last, there is the period in which the tank is fully submerged in water. However, due to the tank, water flow will be influenced by the tank and the heat transfer will differ from Velings report.

1.4 Thesis Objective

The most important difference concerning heat losses compared to Veling is the filling part. This filling cannot become too long since the reactor becomes too hot then. This report focuses on the time to fill the tank: the filling time. It is follow-up research after Ettema [4]. Ettema already calculated the filling time of this tank under a couple of conditions. His conclusion of the filling time can be seen in figure 1.2.

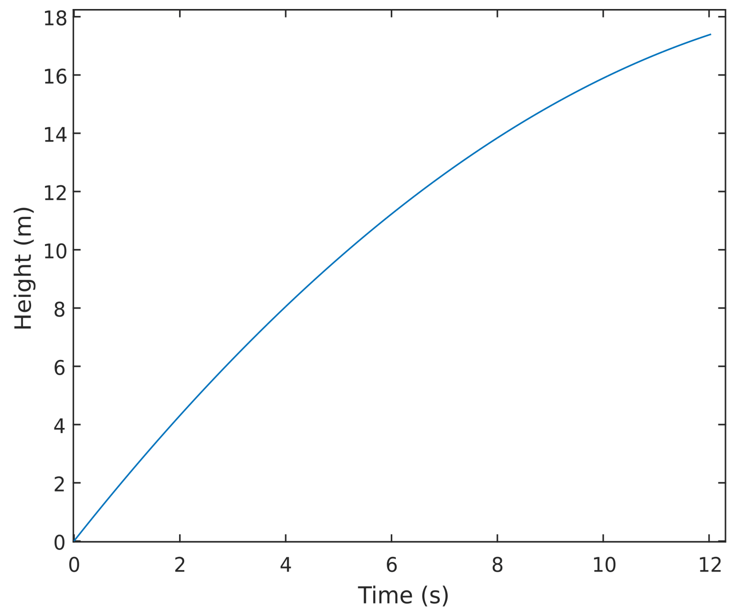

Figure 1.2: The height of the water in the tank plotted against the time, according to Ettema [4]. The filling time is the time when the whole reactor is submerged in water. It is the time when the height is 17.4 meter.

Ettema found a filling time of 12.04 seconds under his standard conditions. He assumed the air in the tank to be at atmospheric pressure instantaneously. This report does solve this pressure over time.
The objectives of this thesis are as follows:

Is it a reasonable assumption that the pressure immediately drops and if not, how big is its effect on the filling times of the tank?

What is the effect of the geometry of the outlet pipe on the filling times of the tank?

In chapter 2 the theory behind this thesis will be explained. The numerical methods behind the important equations will be elaborated in chapter 3. The results will be discussed in chapter 4. At last, the conclusions and recommendations will be given in chapter 5.

Chapter 2

Theory

This chapter covers the theory for the emergency filling of the tank with water. This starts with a recap of the SLIMR design as proposed in earlier research in section 2.1. This is followed by explaining the mechanical energy balance over the water as it is used in Ettema's report [4] in section 2.2. In comparison to his report, an extra variable appears: the pressure of the air in the tank. To solve this, the air in the tank is reviewed. First, it is important to look into properties of air in section 2.3. Second, the mechanical energy balance over the air in the tank will be reviewed in section 2.4. The temperature of the air is important for the mechanical energy balance. However, it is beyond the scope of this research to solve all heat transfer functions. It will be used that the temperature of the whole system is constant. Although it will not be used in the simulation, there is a small elaboration on the internal energy balance of the air 2.5. An overview of the gathered equations is shown in section 2.6.

2.1 SLIMR

2.1.1 Design of the SLIMR

The SLIMR with its tank can be seen in figure 2.1. Point 0 is the air just above the pool. The pressure there is atmospheric pressure. Point 1 is at the entrance (en) of the tank. Point 2 is the boundary between the water and the air in the tank in the annulus (an). The location of this point is therefore varying over time. Point 3 is just before the exit (ex) pipe, still in the annulus. Point 4 at the end is at the end of the exit pipe, the pressure there is again atmospheric pressure. The denotation of a variable is as follows: an example variable ξ_n means this variable ξ at point n. p_2 for instance is the pressure at point 2.
In the normal operating case, an active valve at point 4 keeps the pressure in the tank equal to the pressure of the water at $h = 0$. Therefore, the reactor will be surrounded by air and point 1 will be at the bottom of the tank.

Dimensions

Veling and Ettema already proposed a couple of dimensions for the reactor tank[15][4]. These are shown in table 2.1.

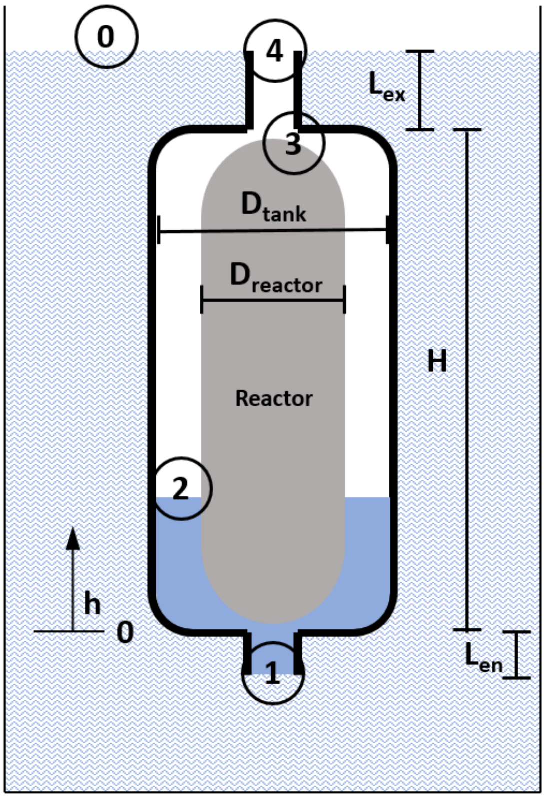

Figure 2.1: A schematic overview of the SLIMR with the emergency cooling tank. Not to scale.

Table 2.1: Table with the standard geometry.

Name	Abbreviation	Value
Length tank	H	$17.4m$
Length exit pipe	L_{ex}	$1m$
Diameter reactor	$D_{reactor}$	$3.2m$
Diameter tank	D_{tank}	$4.2m$
Diameter entrance	D_{en}	$\sqrt{4/\pi}m$
Diameter exit pipe	D_{ex}	Variable
Surface cross section tank	A_{tank}	$\frac{\pi D_{tank}^2}{4}$
Surface cross section annulus tank	A_{an}	$\frac{\pi(D_{tank}^2 - D_{reactor}^2)}{4}$
Surface cross section exit pipe	A_{ex}	$\frac{\pi D_{ex}^2}{4}$
Hydraulic diameter	D_h	$D_{tank} - D_{reactor}$

2.1.2 Simplifications of the SLIMR design

There are connections between the reactor and external components like the turbine. These connections go through the tank. The connections are omitted, because it is difficult to include them in the model and their effect on the filling times is negligible. The reactor has a spherical top and bottom. This is difficult to model as well. It is therefore assumed in the model that the top and bottom are cylindrical as well, so that the reactor is a cylinder extended along the height H of the tank.

2.2 Mechanical Energy Balance Water

This section is mainly an explanation of the theory in Ettema's report [4]. Some theory is new. This new theory is about the use of pressure and the use of the friction factor K.

2.2.1 Mechanical Energy Balance

The mechanical energy balance describes the mechanical energy in a predetermined volume, called the control volume (CV). It looks like an energy balance with at one side the changes in energy in the system and at the other side the sources that caused these changes.

$$\frac{dE_{mech}}{dt} = \frac{d\rho V\left(\frac{1}{2}v^2 + gz\right)}{dt} = \phi_{m,in}\left(e_m\right)_{in} - \phi_{m,out}\left(e_m\right)_{out} + \phi_w - \phi_m e_{fr} \tag{2.1}$$

In this equation, is $\frac{dE_{mech}}{dt}$ the change of mechanical energy over time. ρ is the density of the fluid in the CV. V is the volume of the control volume. The velocity in the CV is represented by v. g is the gravitational constant and z the height of the volume. $\phi_{m,in}$ is the mass flow going in the CV, $\phi_{m,out}$ is the mass flow going out of the CV. e_m is mechanical energy, consisting of kinetic energy and gravitational energy. ϕ_w is the work done on the system. $-\phi_m e_{fr}$ are the friction losses.
Especially the velocity v should be handled with care since normally there is not a uniform velocity v in the CV.
The reactor should be inherently safe. This means that no external power sources can provide mechanical energy. The ϕ_w term therefore cannot consist of external sources. It can only come from internal actions in the reactor tank.

Mass balance water

In the first CV which is the volume between point 1 and 2, the fluid is only water. Water is considered to be an incompressible fluid since its density varies very little over the pressures and temperatures considered in this report. The mass balance without chemical reactions is

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}. \tag{2.2}$$

$\frac{dm}{dt}$ is the derivative of the total mass in the CV over the time. $\dot{m}_{in}$ is the inlet flow of mass and $\dot{m}_{out}$ is the outlet flow of mass. Since the CV between point 1 and 2 does not have an outlet flow, $\dot{m}_{out} = 0$. Therefore, $\frac{dm}{dt} = \dot{m}_{in}$. Since the density is constant,

$$\frac{dm}{dt} = \rho_w \frac{dV}{dt} = \rho_w A_{an} v_2. \tag{2.3}$$

A_{an} is the surface of the annulus with as inner circle the reactor and as outer surface the tank. ρ_w is the density of water. This surface is the surface through which water passes. v_2 is the velocity of the water at point 2. The inlet flow is

$$\dot{m}_{in} = \rho_w v_1 A_{en}. \tag{2.4}$$

A_{en} is the surface of the entrance. v_1 is the average speed across this entrance surface. Therefore

$$\rho_w A_{en} v_1 = \rho_w A_{an} v_2 \tag{2.5}$$

The density of water ρ_w is constant. This is useful to get

$$v_1 A_{en} = v_2 A_{an}. \tag{2.6}$$

Inside the tank, there is no mass accumulation. All mass flowing in also has to flow out: $\rho A v = constant$. Since ρ_w and A_{an} are constant over the whole tank, v has to be constant over the whole tank as well. The velocity of the water in the whole tank is therefore v_2.

Derivative of the mechanical energy water

With the knowledge of water being an incompressible medium,the $\frac{d\rho V\left(\frac{1}{2}v^2+gz\right)}{dt}$ from equation 2.1 term can be rewritten.

$$\begin{aligned}
\frac{d\rho_w V\left(\frac{1}{2}v^2 + \int gdz\right)}{dt} &= \rho_w \frac{dV\left(\frac{1}{2}v^2 + \int gdz\right)}{dt} \\
&= \rho_w V \frac{d\left(\frac{1}{2}v^2 + \int gdz\right)}{dt} + \rho_w \left(\frac{1}{2}v^2 + \int gdz\right)\frac{dV}{dt} \\
&= \rho_w A_{an} h \frac{d\frac{1}{2}v_2^2 + \int_0^h gdz}{dt} + \rho_w \left(\frac{1}{2}v_2^2 + \int_0^h gdz\right) A_{an}\frac{dh}{dt} \\
&= \rho_w A_{an}\left(hv_2 a_2 + 0 + \frac{v_2^3}{2} + hgv_2\right) \\
&= \rho_w A_{an}\left(hv_2 a_2 + \frac{v_2^3}{2} + hgv_2\right)
\end{aligned} \tag{2.7}$$

ρ_w is the density of water, h is the height of the water in the tank, a is the derivative of v over the time. $\rho_w A_{an} h \frac{d\int_0^h gdz}{dt}$ is zero since the potential energy does not change under a constant volume. The $\rho_w A_{an} hgv_2$ term represents the changing potential energy over a changing volume.

Inlet and outlet flow water

The inlet flow and outlet flow of a volume change the derivative of the mechanical energy by $\phi_{m,in}\left(e_m\right)_{in} - \phi_{m,out}\left(e_m\right)_{out}$. For the CV of the water, there is no outlet flow, so $\phi_{m,out} = 0$. The inlet flow is equal to

$$\phi_{m,in} = \rho_w A_{en} v_1 = \rho_w A_{an} v_2. \tag{2.8}$$

The mechanical energy of a flow is

$$e_m = \frac{1}{2}v^2 + gz \tag{2.9}$$

The mechanical energy of the inlet flow is

$$(e_m)_{in} = (\frac{1}{2}v_1^2). \tag{2.10}$$

The gravitational part is zero, since the inlet height is $z = 0$.

Work water

The work done by the pressure in a system is given by [1]

$$\phi_w = \phi_m \int_2^1 \frac{1}{\rho} dp \tag{2.11}$$

with p being the pressure. The mass flow ϕ_m is equal to $\rho_w A_{an} v_2$. The density ρ is constant along the tank so the work is

$$\phi_m \int_2^1 \frac{1}{\rho} dp = \rho_w A_{an} v_2 \left(\frac{p_1}{\rho_w} - \frac{p_2}{\rho_w} \right) = A_{an} v_2 (p_1 - p_2) \tag{2.12}$$

Friction water

Two factors contribute to the friction. First, there is dissipation at the walls. Second, there is dissipation due to the enlargement of the water way.

$$\phi_m e_{fr} = \phi_m (f \frac{h}{D_h} \frac{1}{2} v^2 + K \frac{1}{2} v^2) \tag{2.13}$$

In this equation, f represents the friction factor, $D_{h,an}$ the hydraulic diameter of the annulus and K_{en} the resistance coefficient.

Friction factor

The factor factor depends on the Reynolds number.

$$Re = \frac{\rho v L}{\mu} \tag{2.14}$$

The Reynolds number is a dimensionless number indicating the ratio of inertial forces to viscous forces. A flow with a high Reynolds number is turbulent. A turbulent flow is barely bounded by its viscosity. The Reynolds number is highly relevant for the friction.

The characteristic length L is expressed for flow through tubes as the hydraulic diameter D_h. This hydraulic diameter is defined as

$$D_h = \frac{4A}{S}. \tag{2.15}$$

In equation (2.15), A is the surface of the cross section and S is the wetted perimeter. For a circular tube, $D_h = D$, for an annulus $D_h = D_{outercircle} - D_{innercircle}$. The hydraulic diameter in the tank is therefore

$$D_{h,an} = D_{tank} - D_{reactor} \tag{2.16}$$

The fluid is water at normal temperatures and the velocity will be in the order of $1\ \frac{m}{s}$[4]. The Reynolds number is in the order of 10^6. For this reason, the Haaland friction factor can be used, since it is accurate for $4 \cdot 10^3 < Re < 10^8$.

$$f = \left(-1.8log_{10} \left(\frac{6.9}{Re} + \left(\frac{\epsilon}{3.7D_h} \right)^{10/9} \right) \right)^{-2} \tag{2.17}$$

The wall roughness ϵ is assumed to be near zero, leaving the friction factor to be

$$f = \left(-1.8log_{10} \left(\frac{6.9}{Re} \right) \right)^{-2} \tag{2.18}$$

Resistance coefficient

The resistance coefficient K is a coefficient that quantifies the resistance when the geometry of the flow changes. For an enlargement of the water way, is K dependent on the ratio of the areas before and after the enlargement[6]. Ettema proposed $K = 2$. This is another adjustment to his report.

$$K_{en,1} = (\frac{A_{tank}}{A_{en}} - 1)^2. \tag{2.19}$$

The resistance coefficient has to be multiplied by a velocity. Depending on the velocity before of after the enlargement, this value is different. The $K_{en,1}$ is seen from the inlet. The resistance coefficient seen from the tank is

$$K_{en} = (\frac{A_{tank}}{A_{en}} - 1)^2 \frac{A_{an}^2}{A_{tank}^2}. \tag{2.20}$$

This is a different number since it is multiplied by a different velocity.

Total Mechanical Energy Balance Water

The theory is implemented in (2.1). The mechanical energy balance of the water then becomes

$$\begin{aligned} \rho_w A_{an} \left(h v_2 a_2 + \frac{v_2^3}{2} + h g v_2 \right) = \\ \rho_w A_{an} v_2 \left(\frac{p_1 - p_2}{\rho_w} + \frac{1}{2} v_1^2 - f \frac{h}{D_{h,an}} \frac{1}{2} v_2^2 - K_{en} \frac{1}{2} v_2^2 \right). \end{aligned} \tag{2.21}$$

The variables p_1 and v_1 can be excluded by the use of Bernoulli. The equation of Bernoulli is a special variant of the mechanical energy balance.
In case there is no friction, the fluid is incompressible, there is no external source and energy accumulation, equation (2.1) becomes

$$\frac{1}{2} v^2 + gz + \frac{p}{\rho} = cst. \tag{2.22}$$

This is used between points 0 en 1: $v_0 = 0$, $z_1 = 0$ and $z_0 = H + L_{ex}$. This leads to

$$\frac{1}{2}v_1^2 + \frac{p_1}{\rho_w} = g(H + L_{ex}) + \frac{p_0}{\rho_w}. \tag{2.23}$$

With this knowledge, the mechanical energy balance over the water between points 1 and 2 is complete.

$$\begin{aligned}&\rho_w A_{an}\left(hv_2 a_2 + \frac{v_2^3}{2} + hgv_2\right)\\ &= \rho_w A_{an} v_2\left(\frac{p_0 - p_2}{\rho_w} + g(H + L_{ex}) - f\frac{h}{D_{h,an}}\frac{1}{2}v_2^2 - K_{en}\frac{1}{2}v_2^2\right)\end{aligned} \tag{2.24}$$

If $g(H - h + L_{ex})\rho_w + p_0$ is bigger than or equal to p_2, there will a driving pressure, pushing the water up. However, the pressure p_2 and the velocity v_2 are unknown yet. For this reason, the air in the tank between points 2 and 4 has to reviewed.

2.3 Properties of air

The second control volume is between points 2 and 4. The fluid in the CV is air. This control volume consists of two parts: the air inside the tank between points 2 and 3 and the air in the outlet pipe. The pressure in the tank is assumed to be constant throughout the tank. It is not constant over time. The temperature of the air is assumed to be constant throughout the tank and the outlet pipe. The temperature is not constant, however in the research the dependence of temperature is not taken into account and the temperature is assumed to be constant over time. The air can be considered as an ideal gas [14]. This property can be used for calculating some state variables.
First of all, the ideal gas law is applicable for this situation.

$$pV = nRT \tag{2.25}$$

In equation (2.25), n is the amount of moles in the specified volume. R is the ideal gas constant. T is the temperature. Furthermore, the specific internal energy can be described by

$$u = \frac{f}{2}\frac{RT}{M} \tag{2.26}$$

with M being the molar mass of air. f being the total degrees of freedom. Air consists mainly of diatomic gases: N_2 and O_2, so the total degrees of freedom is approximately f of a diatomic gas: 5.

2.3.1 Pressure of air

The ideal gas law in equation (2.25) can be rewritten so that the pressure of the air is dependent of the density.

$$p = \frac{RT\rho_a}{M} \tag{2.27}$$

The pressure p_2 is a variable in 2.24 and it has to be solved. However, it depends on the density of the air ρ_a and on the temperature of air. ρ_a is also constant throughout the tank as p is constant throughout the tank. For this reason, the whole density in the tank is $\rho_{a,2} = \rho_{a,3}$. The derivative of the pressure is

$$\frac{dp_2}{dt} = \frac{R}{M}\frac{dT_2\rho_{a,2}}{dt} = \frac{R\rho_{a,2}}{M}\frac{dT}{dt} + \frac{RT}{M}\frac{d\rho_{a,2}}{dt} = \frac{p}{T}\frac{dT}{dt} + \frac{RT}{M}\frac{d\rho_a}{dt} \tag{2.28}$$

Note that the temperature is assumed to be constant. Therefore,

$$\frac{dp_2}{dt} = \frac{RT}{M}\frac{d\rho_{a,2}}{dt} \tag{2.29}$$

The variable $\rho_{a,2}$ has to be solved now.

2.3.2 Density of air

$\rho_{a,2}$ is the density of the air in the CV. It can be determined by $\rho = \frac{M}{V}$, with M the mass of air in the tank and V the volume consisting of air in the tank. This equality can be used to determine the changes in the density of the air in the CV.

$$\frac{d\rho_{a,2}}{dt} = \frac{dM}{Vdt} - \frac{MdV}{V^2dt} = \frac{-\phi_{m,out}}{V} + \rho_{a,2}\frac{\phi_V}{V} = \frac{-\rho_{a,4}v_4A_{ex} + \rho_{a,2}A_{an}v_2}{A_{an}(H-h)} \tag{2.30}$$

All terms in equation (2.30) are known, except for $\rho_{a,2}$, v_4 and v_2. Together with equation (2.24) there are now two equations and three unknown variables.
The density of the outflowing air is

$$\rho_{a,4} = \frac{Mp_4}{RT}. \tag{2.31}$$

Since p_4 is atmospheric pressure and T is assumed constant, this density is constant.

2.4 Mechanical Energy Balance Air

Another equation has to be found to solve v_4. For this purpose, the mechanical energy balance over the air between point 2 and 4 is taken.

$$\frac{d\rho V\left(\frac{1}{2}v^2 + gz\right)}{dt} = \phi_{m,in}\left(e_m\right)_{in} - \phi_{m,out}\left(e_m\right)_{out} + \phi_w - \phi_m e_{fr} \tag{2.32}$$

Air is not an incompressible fluid. This means that if the density changes in any predefined volume of air in the reactor, $\frac{dm}{dt} \neq 0$. Considering

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}, \tag{2.33}$$

the mass flow in this volume is not equal to the mass flow out of the volume. The velocity in the tank will therefore not be constant throughout the tank. For this reason, point 3 will have to be taken into account as well. The velocity of the air at point 2 is equal to the velocity of the water at point 2, v_2.

The density of the air in the exit tube is assumed to be ρ_4 and this density is constant. Since the density of the air in the exit pipe is constant, there is no mass accumulation $\frac{dm}{dt}$ in the exit tube. Therefore, according to equation (2.33),

$$\rho_{a,3}v_3A_{an} = \rho_{a,4}v_4A_{ex}. \tag{2.34}$$

For the sake of clarity, $\beta = \frac{\rho_{a,4}A_{ex}}{\rho_{a,3}A_{an}}$.

Derivative of the mechanical energy air between 2 and 3

The mechanical energy is located in the volume between point 2 and 3 and point 3 and 4. This subsection derives the mechanical energy of the air between points 2 and 3. It is assumed that the potential energy is negligible compared to the kinetic energy of the air. The velocity rises linearly from v_2 at h to v_3 at H if the density changes are constant over the volume. The velocity becomes $v(z) = (z-h)\frac{v_3-v_2}{H-h}+v_2$. Then, e_{mech} can be calculated.

$$\begin{aligned} e_{mech} &= \frac{< v^2 >}{2}\frac{\frac{1}{2}\int_h^H((z-h)\frac{v_3-v_2}{H-h}+v_2)^2dz}{H-h} \\ &= \frac{\frac{H-h}{2(v_3-v_2)}\frac{1}{3}((z-h)\frac{v_3-v_2}{H-h}+v_2)^3}{H-h}\Bigg|_h^H = \frac{1}{6}\frac{v_3^3-v_2^3}{v_3-v_2} = \frac{1}{6}(v_3^2+v_3v_2+v_2^2) \end{aligned} \tag{2.35}$$

The derivative of the mechanical energy then becomes

$$\begin{aligned} &\frac{d\rho_{a,2}Ve_{mech}}{dt} = \rho_{a,2}V\frac{de_{mech}}{dt} + \rho_{a,2}e_{mech}\frac{dV}{dt} + Ve_{mech}\frac{d\rho_{a,2}}{dt} \\ &= \rho_{a,2}A_{an}(H-h)\frac{1}{6}\left(2v_2a_2+v_3a_2+v_2a_3+2v_3a_3\right) \\ &- \rho_{a,2}e_{mech}A_{an}v_2 + e_{mech}(-\rho_{a,4}v_4A_{ex}+\rho_{a,2}A_{an}v_2) \\ &= \rho_{a,2}A_{an}(H-h)\frac{1}{6}\left(2v_2a_2+v_3a_2+v_2a_3+2v_3a_3\right) - e_{mech}\rho_{a,4}v_4A_{ex} \end{aligned} \tag{2.36}$$

For the $\frac{d\rho_{a,2}}{dt}$ term, equation (2.30) is used.

Derivative of the mechanical energy air between 3 and 4

For the volume between points 3 and 4, it is assumed that the density ρ_a and velocity v in the pipe are almost the same as the density $\rho_{a,4}$ and the velocity v_4 at the exit point 4.

$$\begin{aligned} &\frac{d\rho Ve_{mech}}{dt} = \rho V\frac{d\frac{1}{2}v_4^2}{dt} + V\frac{1}{2}v_4^2\frac{d\rho}{dt} \\ &= \rho_{a,4}LA_{ex}v_4a_4 - LA_{ex}\frac{1}{2}v_4^2\frac{\rho_{a,4}dT}{Tdt} \\ &= \rho_{a,4}LA_{ex}v_4a_4 \end{aligned} \tag{2.37}$$

The total change in the derivative of the mechanical energy is the sum of (2.36) and (2.37).

Inlet and outlet flow air

The inlet flow and outlet flow of a volume change the derivative of the mechanical energy by $\phi_{m,in}\left(e_m\right)_{in} - \phi_{m,out}\left(e_m\right)_{out}$. For the CV of the air, there is no inlet flow, so $\phi_{m,in} = 0$. The outlet mass flow is equal to

$$\phi_{m,out} = \rho_{a,4} A_{ex} v_4. \tag{2.38}$$

The mechanical energy of the outlet flow is

$$(e_m)_{in} = (\frac{1}{2} v_4^2). \tag{2.39}$$

Work air

Between point 2 and 3, there are no work terms causing mechanical energy, since the pressure does not change between points 2 and 3. The compression of the air by the water does influence the mechanical balance, but indirectly. The water pushing the air influences the density of the air in the tank and therefore the pressure of the air in the tank. There is a work term between points 3 and 4[1].

$$\phi_w = \phi_m \int_4^3 \frac{1}{\rho} dp \tag{2.40}$$

Since air is an ideal gas,

$$\phi_m \int_4^3 \frac{1}{\rho} dp = \rho_{a,4} A_{ex} v_4 \int_4^3 \frac{RT}{Mp} dp = \rho_{a,4} A_{ex} v_4 \frac{RT}{M} log\left(\frac{p_3}{p_4}\right) \tag{2.41}$$

applies.

Friction air

For the friction, the same logic applies as for the water.

$$\phi_m e_{fr} = \phi_m (f \frac{L_{ex}}{D_{ex}} \frac{1}{2} v_4^2 + K_{ex} \frac{1}{2} v_4^2) \tag{2.42}$$

The Reynolds number will again be in the Haaland-region, so the friction factor f is calculated via the same friction equation (2.18) as for the water. There is also some friction is the annulus. However, this is negligible since the velocity of the air in the exit pipe is much higher than the velocity of the air in the tank.
The resistance coefficient consists of two parts now. First, there is some kind of system making sure that the air stays in the tank under nominal conditions. When there is a shutdown, this will still have some effect on the geometry. The resistance caused by these components is K_{rest} and is unknown. Second, there is resistance since the surface through which the water passed narrows. This coefficient is dependent on the areas before and after the narrowing[6].

$$K_{ex} = 0.45(1 - \frac{A_{ex}}{A_{tank}}) + K_{rest} \tag{2.43}$$

Total mechanical energy balance air

The mechanical energy balance is known.

$$\rho_{a,2}A_{an}(H-h)\frac{1}{6}\left(2v_2a_2+v_3a_2+v_2a_3+2v_3a_3\right)-\left(\frac{1}{6}v_2^2+\frac{1}{6}v_2v_3+\frac{1}{6}v_3^2\right)\rho_{a,4}v_4A_{ex}$$
$$+\rho_{a,4}LA_{ex}v_4a_4- = \rho_{a,4}v_4A_{ex}(-\frac{1}{2}v_4^2-f\frac{L_{ex}}{D_{ex}}\frac{1}{2}v_4^2-K_{ex}\frac{1}{2}v_4^2+\frac{RT}{M}log\left(\frac{p_3}{p_4}\right)) \tag{2.44}$$

It is important to note that a_3 and a_4, as well as v_3 and v_4 are coupled via $\beta=\frac{\rho_{a,4}A_{ex}}{\rho_{a,3}A_{an}}$, as explained in equation 2.34. v_4 can be solved now as well.

2.5 Internal Energy Balance

The temperature of the control volumes influences the equations gathered in the previous sections. In this report, T will be assumed to be constant. For possible follow-up research, the founding will be discussed briefly. Only the internal energy balance for the air will be discussed.

2.5.1 Internal Energy Balance Air

The internal energy balance is shown in equation (2.45).

$$\frac{d\rho Vu}{dt}=\phi_{m,in}u_{in}-\phi_{m,out}u_{out}+\phi_q+\phi_m e_{fr}-\phi_m\int pd\left(\frac{1}{\rho}\right) \tag{2.45}$$

u is the specific internal energy and ϕ_q is the heat transfer in equation 2.45. At first, the derivative of the internal energy is reviewed. The specific internal energy for the air is equal to $\frac{5}{2}\frac{RT}{M}$ as can be seen in equation (2.26).

$$\begin{aligned}\frac{d\rho_a Vu}{dt}&=\rho_{a,2}\frac{5}{2}\frac{RT}{M}\frac{dV}{dt}+\frac{5}{2}\frac{RT}{M}V\frac{d\rho_a}{dt}+\rho_a V\frac{d\frac{5}{2}\frac{RT}{M}}{dt}\\&=\rho_{a,2}\frac{5}{2}\frac{RT}{M}(-v_2A_{an})+\frac{5}{2}\frac{RT}{M}(-A_{ex}v_4\rho_{a,4}+A_{an}v_2\rho_{a,2})+A_{an}(H-h)\rho_{a,2}\frac{5R}{2M}\frac{dT}{dt}\\&=-A_{ex}v_4\rho_{a,4}\frac{5}{2}\frac{RT}{M}+A_{an}(H-h)\rho_{a,2}\frac{5R}{2M}\frac{dT}{dt}\end{aligned} \tag{2.46}$$

For the $\frac{d\rho_{a,2}}{dt}$ term, equation (2.30) is used.
Once again, there is no inlet flow of air. Therefore, $\phi_{m,in}u_{in}-\phi_{m,out}u_{out}=-\phi_{m,out}u_{out}$. ϕ_q includes the heat transfer from the reactor and heat losses to the tank and its surroundings. These are however not included in this report. The friction subtracts

mechanical energy in the mechanical energy balance (2.1) and adds internal energy to the internal energy balance. The last term is

$$- \phi_m \int_2^4 pd\left(\frac{1}{\rho}\right) = A_{ex}\rho_{a,4}v_4 \int_2^4 \frac{RTp}{Mp^2}dp = A_{ex}\rho_{a,4}v_4 \frac{RT}{M}\ln\frac{p_4}{p_2}. \tag{2.47}$$

$$\begin{aligned}\frac{d\rho V u}{dt} &= \phi_{m,in}u_{in} - \phi_{m,out}u_{out} + \phi_q + \phi_m e_{fr} - \int pd\left(\frac{1}{\rho}\right)\\ &= -A_{ex}v_4\rho_{a,4}\frac{5}{2}\frac{RT}{M} + A_{ex}v_4\rho_{a,4}\left(f_{ex}\frac{L}{2D_{h,ex}}v_2^2 + K_{tot,ex}\frac{1}{2}v_2^2 + \frac{RT}{M}\ln\frac{p_4}{p_2}\right) + \phi_q\end{aligned} \tag{2.48}$$

Equation (2.46) and (2.48) combined leads to equation (2.49)

$$A_{an}(H-h)\rho_{a,2}\frac{5R}{2M}\frac{dT}{dt} = A_{ex}v_4\rho_{a,4}\left(f_{ex}\frac{L}{2D_{h,ex}}v_2^2 + K_{tot,ex}\frac{1}{2}v_2^2 + \frac{RT}{M}\ln\frac{p_4}{p_2}\right) + \phi_q \tag{2.49}$$

2.6 Overview

This sections gives an overview of the important equations. These equations will be used in chapter 3.

The mechanical energy over the water between point 1 and 2 leads to equation (2.24):

$$\begin{aligned}&\rho_w A_{an}\left(hv_2a_2 + \frac{v_2^3}{2} + hgv_2\right)\\ &= \rho_w A_{an}v_2\left(\frac{p_0-p_2}{\rho_w} + g(H+L_{ex}) - f\frac{h}{D_{h,an}}\frac{1}{2}v_2^2 - K_{en}\frac{1}{2}v_2^2\right)\end{aligned} \tag{2.50}$$

In equation (2.50), there are two variables: the velocity of the water in the tank v_2 and the pressure of the air in the tank p_2. This pressure p_2 is dependent on the density of the air in the tank via equation (2.29).

$$\frac{dp_2}{dt} = \frac{RT}{M}\frac{d\rho_{a,2}}{dt}. \tag{2.51}$$

There are now three variables, v_2, p_2 and $\rho_{a,2}$ and two equations. The density $\rho_{a,2}$ can be expressed as in equation (2.30):

$$\frac{d\rho_{a,2}}{dt} = \frac{-\rho_{a,4}v_4A_{ex} + \rho_{a,2}A_{an}v_2}{A_{an}(H-h)} \tag{2.52}$$

In this equation, another new variable appeared: v_4. This variable is solved via the mechanical energy balance over the air in the tank as described in equation (2.44).

$$\begin{aligned}
&\rho_{a,2}A_{an}(H-h)\frac{1}{6}\left(2v_2a_2+\beta v_4a_2+\beta v_2a_4+2\beta^2v_4a_4\right)\\
&-\left(\frac{1}{6}v_2^2+\frac{1}{6}\beta v_2v_4+\frac{1}{6}\beta^2v_4^2\right)\rho_{a,4}v_4A_{ex}+\rho_{a,4}LA_{ex}v_4a_4\\
&=\rho_{a,4}v_4A_{ex}(-\frac{1}{2}v_4^2-f\frac{L_{ex}}{D_{ex}}\frac{1}{2}v_4^2-K_{ex}\frac{1}{2}v_4^2+\frac{RT}{M}log\left(\frac{p_2}{p_4}\right))
\end{aligned} \tag{2.53}$$

Finally, there are four equations and four variables: v_2, p_2, $\rho_{a,2}$ and v_4.

Chapter 3

Numerical Methods

This chapter will cover the bridge between the theory and the experiment as it is been programmed. The system is not solvable analytically and it has to be solved numerically. Therefore, the system has to be discretized. First, some background knowledge on numerical methods is discussed in section 3.1. Then, the method to make the equations solvable is discussed in section 3.2. Finally, some boundary conditions are discussed in section 3.3.

3.1 Numerical Approach

A continuous line is discretized by taking points every Δt seconds. Function $y(t)$ is numerically y_n where a number n corresponds to a time t. The time t and n are coupled via $t = n\Delta t$.
A continuous derivative is

$$y'(x) = \lim_{h \to 0} \frac{y(x+h) - y(x)}{h}. \tag{3.1}$$

In numerical form, this becomes

$$y'_n = \frac{y_{n+1} - y_n}{\Delta t}. \tag{3.2}$$

for $\Delta t \to 0$.

3.2 Numerical Solving

The equations from section 2.6 have to be discretized. This can be done in multiple ways. If a derivative of a variable depends on the variable, it can be solved explicitly or implicitly. If the derivative is solved explicitly, the equation looks like

$$y'_{n+1} = \frac{y_{n+1} - y_n}{\Delta t} = f(x_{n+1}, y_n) \quad \text{or} \quad y'_{n+1} = \frac{y_{n+1} - y_n}{\Delta t} = f(x_n, y_n). \tag{3.3}$$

When the derivative and thus y_{n+1} is solved, it depends on y_n.
If the derivative is solved implicitly, the equation looks like

$$y'_{n+1} = f(x_{n+1}, y_{n+1}) \quad \text{or} \quad y'_{n+1} = f(x_n, y_{n+1}) \tag{3.4}$$

When this derivative and thus y_{n+1} is solved, it depends on y_{n+1}. As long as the time step Δt is small enough, both the explicit and the implicit method suffice. Ettema found that both methods are good for $\Delta t \leq 10^{-4}$. If Δt are made smaller, the calculating times raise. For this reason, is $\Delta t = 10^{-4}$ in the report. In the rest of the report, the explicit model is used.
The derivatives depend on other variables as well. These can also be solved in multiple ways. The height of the water in the tank, for instance, is dependent on the velocity of the water in the tank by

$$\frac{dh}{dt} = v_2 \tag{3.5}$$

This can be solved by

$$h_{n+1} = h_n + v_{2,n}\Delta t \quad \text{or} \quad h_{n+1} = h_n + v_{2,n+1}\Delta t \tag{3.6}$$

One by one, each variable will be solved for $n + 1$. When all variables are known for an n, a variable, for instance v_{n+1}, is solved with all variables on time step n. The next variable that is solved, for instance h_{n+1}, depends on all variables on time step n except for v. This next variable h_{n+1} depends on v_{n+1}. This is done until all variables are known for an $n + 1$. The flowchart can be seen in figure 3.1. At first, some constants are given. The geometry has to be set beforehand, as well as the initial conditions. These affect some state variables. Then, the time loop can start. The order in which all variables is solved does not effect the solution for a small Δt. The order is selected for as little numerical problems around $t = 0$.
The variables μ for water and air and ρ for water vary over time because of their dependence on the temperature and the pressure. However, there are assumed to be constant since the differences are negligible for the conditions in the study.

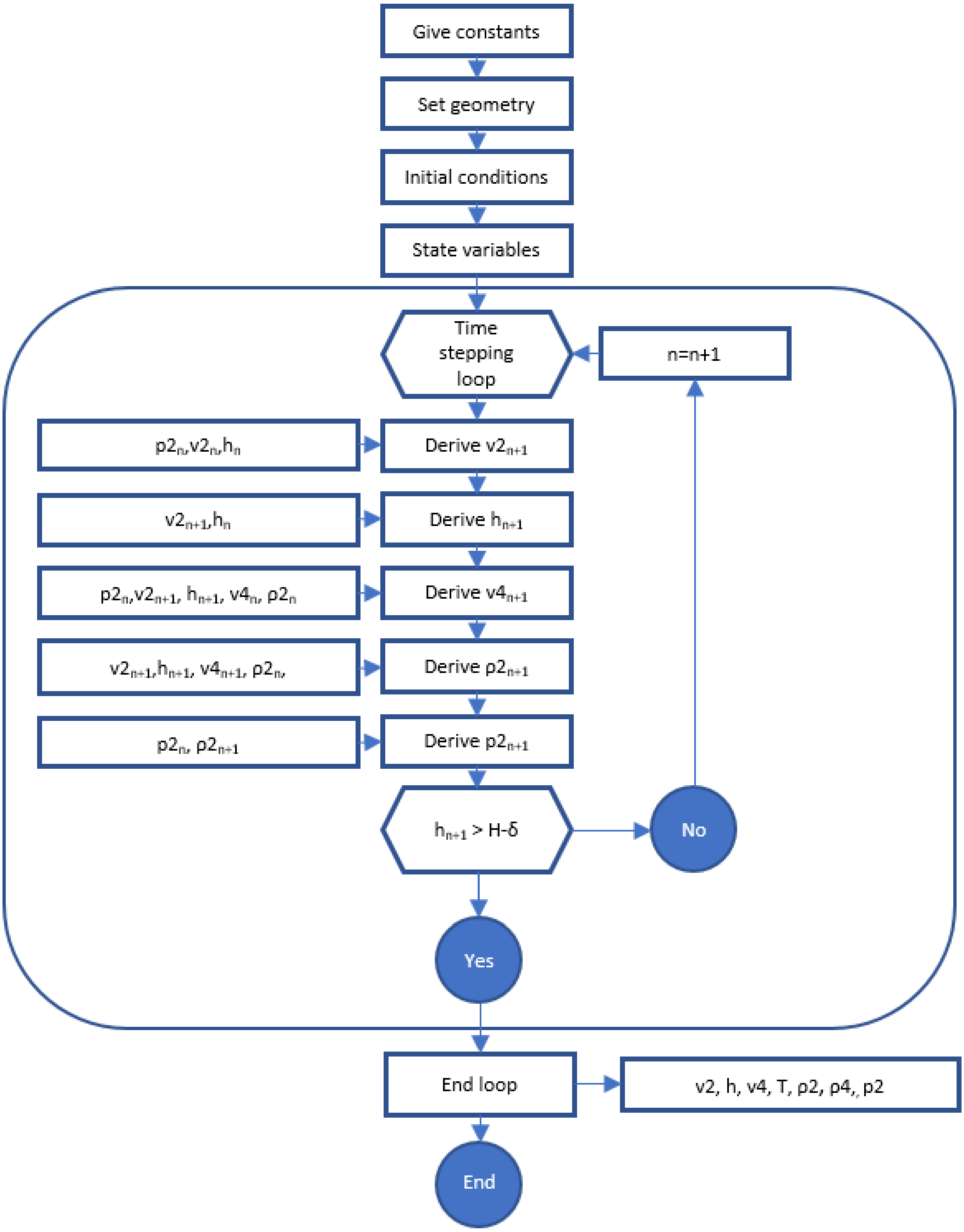

Figure 3.1: The flowchart containing the order in which all variables are solved.

3.2.1 Equations discretized

The equations of section 2.6 first have to be rewritten so that the derivative is on one side of the equation and the others terms on the other side. Then, the equations have to be discretized. Equation (2.50) becomes

$$a_2 = \frac{dv_2}{dt} = \frac{p_0 - p_2}{h\rho_w} + g\frac{H + L_{ex} - h}{h} - \frac{v_2^2}{2}\left(\frac{(1 + K_{en}\frac{A_{an}^2}{A_{en}^2})}{h} + \frac{f}{D_{h,an}}\right). \tag{3.7}$$

Its discretized version then becomes

$$v_{2,n+1} = v_{2,n} + \Delta t\left(\frac{p_0 - p_{2,n}}{h_n\rho_w} + g\frac{H + L_{ex} - h_n}{h_n} - \frac{v_{2,n}^2}{2}\left(\frac{(1 + K_{en}\frac{A_{an}^2}{A_{en}^2})}{h_n} + \frac{f_{n,an}}{D_{h,an}}\right)\right). \tag{3.8}$$

The height is then calculated by

$$h_{n+1} = h_n + v_{2,n+1}\Delta t \tag{3.9}$$

and the acceleration becomes

$$a_{n+1} = \frac{v_{2,n+1} - v_{2,n}}{\Delta t} \tag{3.10}$$

Equation (2.53) becomes equation (3.11)

$$\begin{aligned} a_4 =& \frac{-\rho_{a,2}A_{an}(H-h)(2a_2v_2 + \beta a_2 v_4)}{\rho_{a,2}A_{an}(H-h)(2\beta^2 v_4 + \beta v_2) + 6A_{ex}L\rho_{a,4}v_4} \\ &+ \frac{6A_{ex}v_4\rho_{a,4}\left(-\frac{1}{2}v_4^2(1 + f\frac{L_{ex}}{D_{ex}} + K_{ex}) + \frac{v_2^2 + \beta v_4 v_2 + \beta^2 v_4^2}{6} + \frac{RT}{M}log\left(\frac{p_2}{p_4}\right)\right)}{\rho_{a,2}A_{an}(H-h)(2\beta^2 v_4 + \beta v_2) + 6A_{ex}L\rho_{a,4}v_4} \end{aligned} \tag{3.11}$$

Numerically, $\beta_n = \frac{A_{ex}\rho_{4,n}}{A_{an}\rho_{2,n}}$. Then,

$$\begin{aligned} &v_{4,n+1} = v_{4,n} + \\ &\Delta t\frac{-\rho_{2,n}A_{an}(H-h_n)(2a_{2,n+1}v_{2,n+1} + \beta_n a_{2,n+1}v_{4,n})}{\rho_{2,n}A_{an}(H-h_n)(2\beta_n^2 v_{4,n} + \beta_n v_{2,n+1}) + 6A_{ex}L\rho_{4,n}v_{4,n}} + \\ &\Delta t\frac{6A_{ex}v_{4,n}\rho_{4,n}\left(-\frac{1}{2}v_{4,n}^2(1 + f_n\frac{L_{ex}}{D_{ex}} + K_{ex}) + \frac{v_{2,n}^2 + \beta_n v_{4,n}v_{2,n+1} + \beta_n^2 v_{4,n}^2}{6} + \frac{RT_n}{M}log\left(\frac{p_{2,n}}{p_4}\right)\right)}{\rho_{2,n}A_{an}(H-h_n)(2\beta_n^2 v_{4,n} + \beta_n v_{2,n+1}) + 6A_{ex}L(\rho_{4,n}v_{4,n})}. \end{aligned} \tag{3.12}$$

The density of the air in the tank as in equation (2.52) is

$$\frac{d\rho_2}{dt} = \frac{-\rho_{a,4}v_4A_{en} + \rho_{a,2}A_{an}v_2}{A_{an}(H-h)} \tag{3.13}$$

The discretized version of this density is

$$\rho_{2,n+1} = \rho_{2,n} + \Delta t\frac{-\rho_{4,n}v_{4,n+1}A_{en} + \rho_{2,n}A_{an}v_{2,n+1}}{A_{an}(H-h_n)}. \tag{3.14}$$

Finally, the pressure is

$$\frac{dp_2}{dt} = \frac{RTd\rho_{a,2}}{Mdt}. \tag{3.15}$$

The discretized version of the pressure is

$$p_{n+1} = p_n + \frac{RT_{n+1}}{M}(\rho_{2,n+1} - \rho_{2,n}). \tag{3.16}$$

All variables v_2, v_4, $\rho_{a,2}$ and p_2 are now discretized and solvable.

3.3 Initial conditions

When the reactor shuts down, the valve opens. This is at is $t = 0$. The height of the water in the tank is $h(t = 0) = 0$, the water does not have an initial velocity and the air does not have an initial velocity, $v_2(t = 0) = 0$ and $v_3(t = 0) = 0$. The pressure in the tank is equal to $p_2(t = 0) = p_0 + (H + L_{ex})g\rho_w$. The density is therefore $\rho_{a,2} = \frac{Mp_2(t=0)}{RT}$. For $t = 0$, equations (3.8) and (3.12) will divide zero by zero. Therefore, h and v_3 get a small value assigned. This can not be infinitely small, due to the time steps not being infinitely small.
When the height h approaches H, equation (3.14) will be divided by a number approaching zero. Therefore, the loop cannot be continued until $h > H$. A small number δ under H, the loop will stop ($h > H - \delta$). This is also shown in figure 3.1. The filling time ft will then be calculated by

$$ft = t_{end} + \frac{\delta}{v_{2,end}}. \tag{3.17}$$

t_{end} is the time when the condition $h > H - \delta$ becomes true. $v_{2,end}$ is the velocity of the water on that moment.

3.4 Different geometries

To be able to compare different geometries, the loop as shown in figure 3.1 will be executed for multiple values of the concerning geometry. This will be done by changing one variable each time with respect to a default geometry. Two geometrical factors will be differed: D_{ex} and K_{rest}. D_{ex} is the diameter of the exit pipe. When this diameter is small, the mass flow of air out of the tank is small. This mass flow is namely equal to $\rho_{a,4}A_{ex}v_4$. The density of the air in the tank drops less then with a bigger diameter and so does the the pressure. A geometry with a small diameter is for this reason expected to have higher filling times. The resistance coefficient K_{rest} is a coefficient explaining how many energy is dissipated due to valves and other components at the exit. Ideally, this is zero. A higher resistance coefficient is expected to resist the air to flow out of the tank. Then again, the density of the air in the tank stays higher and thus the pressure. A geometry with a higher resistance coefficient is expected to have higher filling times. The exit diameter D_{ex} will be varied from logarithmically from $D_{ex} = 0.05m$ to $D_{ex} = 1m$ in 10 steps. The resistance coefficient K_{rest} will be varied from $K_{rest} = 0$ to $K_{rest} = 5$ linearly in 10 steps.

Chapter 4

Results

In this chapter, the results will be discussed. At first, the default settings are shown in section 4.1. The process of filling the tank will in reviewed in section 4.2. In section 4.3, the effects of the outlet on the pressure and the filling times will be discussed.

4.1 Default Geometry

At first, the change of all the variables over time will be discussed for a default geometry. The dimensions are extracted from the report of Ettema and Veling [4][15]. There are two new variables: D_{ex} and K_{rest}. The resistance coefficient K_{en} in equation (3.8) is also different because another method is used to define it. In Ettema's report, $K_{en} = 67$. For the default geometry, however, is $K_{en} = 29$. The default settings are seen in table 4.1.

Table 4.1: Table with the default settings

Name	Abbreviation	Value
Length tank	H	$17.4m$
Length exit pipe	L_{ex}	$1m$
Diameter reactor	$D_{reactor}$	$3.2m$
Diameter tank	D_{tank}	$4.2m$
Diameter entrance	D_{en}	$\sqrt{4/\pi}m$
Diameter exit pipe	D_{ex}	$0.1m$
Surface cross section tank	A_{tank}	$\frac{\pi D_{tank}^2}{4}$
Surface cross section annulus tank	A_{an}	$\frac{\pi(D_{tank}^2 - D_{reactor}^2)}{4}$
Surface cross section exit pipe	A_{ex}	$\frac{\pi D_{ex}^2}{4}$
Hydraulic diameter	D_h	$D_{tank} - D_{reactor}$
Water temperature	T_{water}	$25°C$
Air temperature	T_{air}	$40°C$
Resistance coefficient entrance	K_{en}	$(\frac{A_{tank}}{A_{en}} - 1)^2 \frac{A_{an}^2}{A_{tank}^2}$
Resistance coefficient exit pipe	K_{nar}	$0.45(1 - \frac{A_{ex}}{A_{tank}})$
Friction valve and pump	K_{rest}	0
Total resistance coefficient exit	K_{ex}	$K_{nar} + K_{rest}$
Velocity water in the tank at $t = 0$	$v_2(t = 0)$	$0 \ \frac{m}{s}$
Velocity air in the exit pipe at $t = 0$	$v_4(t = 0)$	$0.001 \ \frac{m}{s}$
Height water in the tank at $t = 0$	$h(t = 0)$	$0.001 \ m$
Step size time in program	Δt	$0.0001 \ s$
Step size time in program	δ	$0.05 \ m$

4.2 Filling the tank

The filling time is 134 seconds, as can be seen in figure 4.1. Even if K_{en} is set to 67, the K_{en} value Ettema proposed, the filling time is 134 seconds. The assumption that the pressure becomes atmospheric pressure instantaneously after the shutdown in Ettema [4] was wrong since the filling time was only 12.4 seconds under the default settings under this assumption and under $K_{en} = 67$. His model calculated a filling time of 7.8 seconds when $K_{en} = 29$. In the default geometry, $D_{ex} = 0.1m$. For larger exit pipe diameters, the assumption of Ettema might be correct. This will be discussed in the section 4.3.

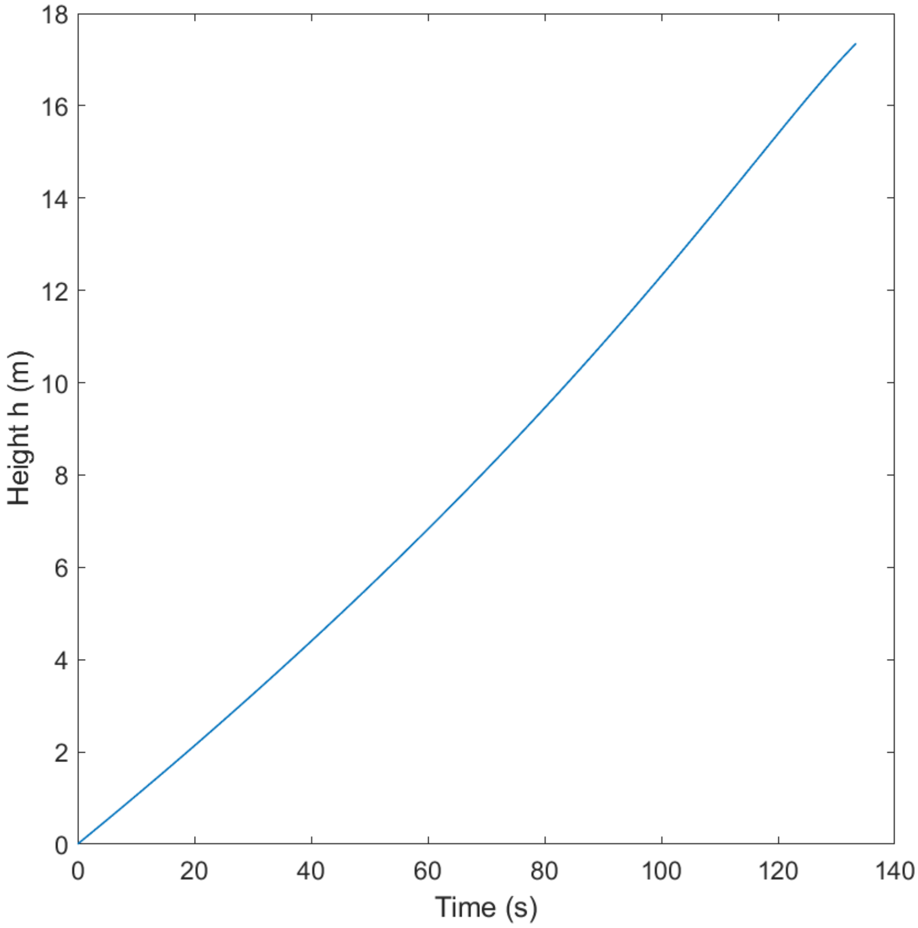

Figure 4.1: The height h of the water in the tank against the time. The filling time can be calculated by extrapolating the height to $h = 17.4$.

4.2.1 Pressure in the tank

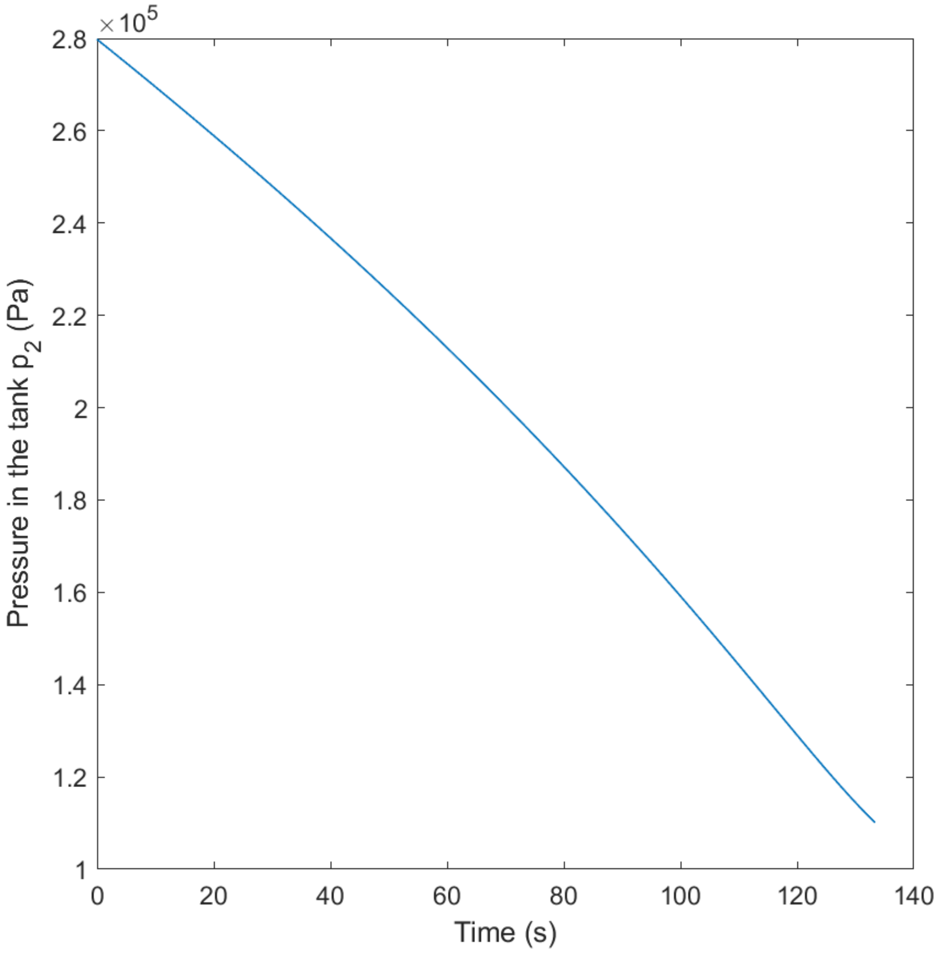

Figure 4.2: The pressure of the air in the tank against the time.

The pressure does not drop to atmospheric pressure instantaneously, it drops slowly and becomes only atmospheric pressure when the tank is totally filled with water. It is interesting to see how large the force is on the water due to pressure differences. This force depends on the difference in pressure between $p_0 + \rho_w g(H + L_{ex} - h)$ and p_2. It can be seen in equation (3.8) that this difference is the driving force for the water to fill the tank. This difference can be seen in figure 4.3. The difference between $p_0 + \rho_w g(H + L_{ex} - h)$ and p_2 never reaches 400 Pa. If the pressure in the tank p_2 would drop to atmospheric pressure, this difference would start at $2.84bar - 1bar = 1.84bar$. Since the pressure difference is much smaller, the acceleration of the water is much smaller, the velocity of the water is much smaller and thus is the filling time larger.

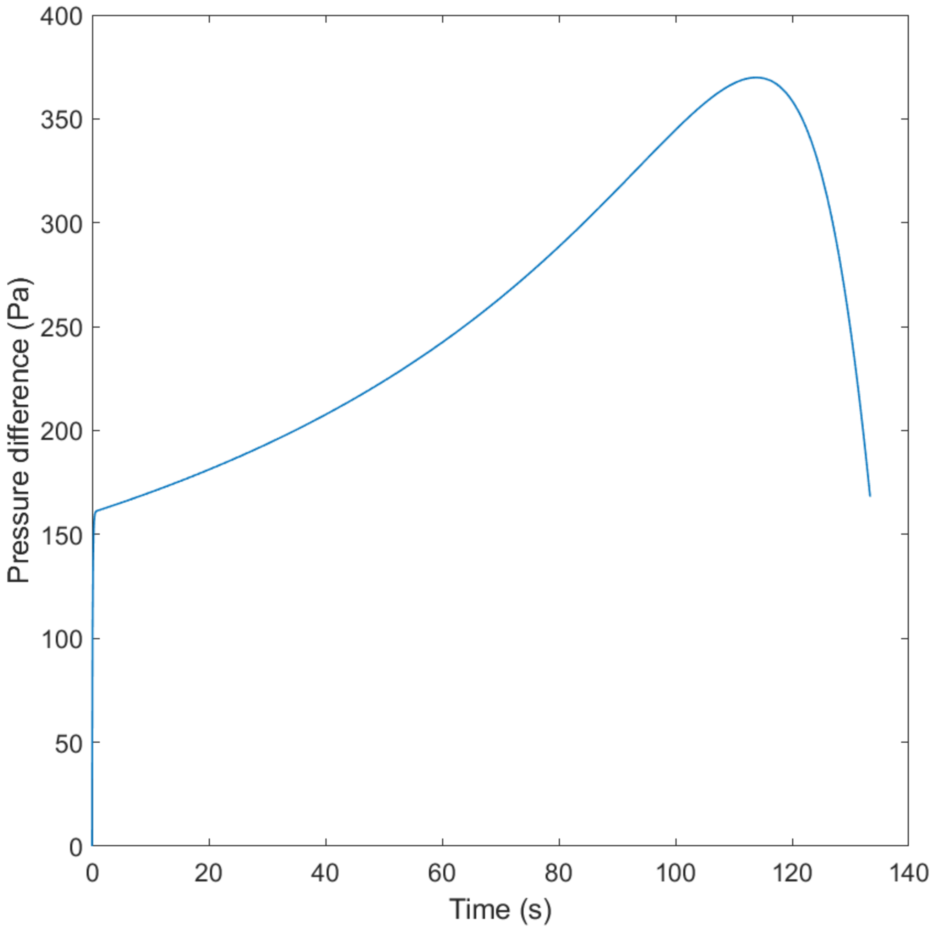

Figure 4.3: The driving pressure, $(p_0 + \rho_w g(H + L_{ex} - h)\text{-}p_2)$ against the time.

The density is linearly dependent on the pressure since there are no temperature changes as can be seen in equation (2.51). The density is shown in figure 4.4.

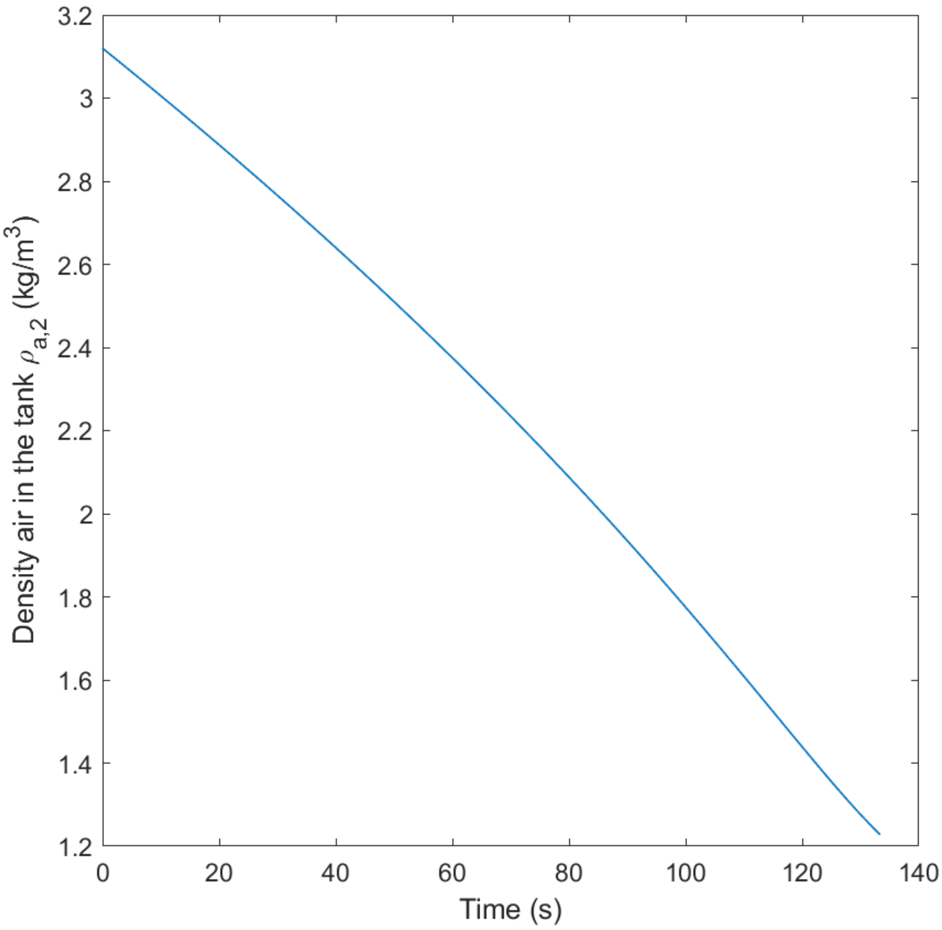

Figure 4.4: The density of the air in the tank against the time.

4.2.2 Velocity

Velocity water

The velocity seems to be almost constant over time, considering figure 4.1. This is not quite true, as can be seen in figure 4.5 and 4.6.

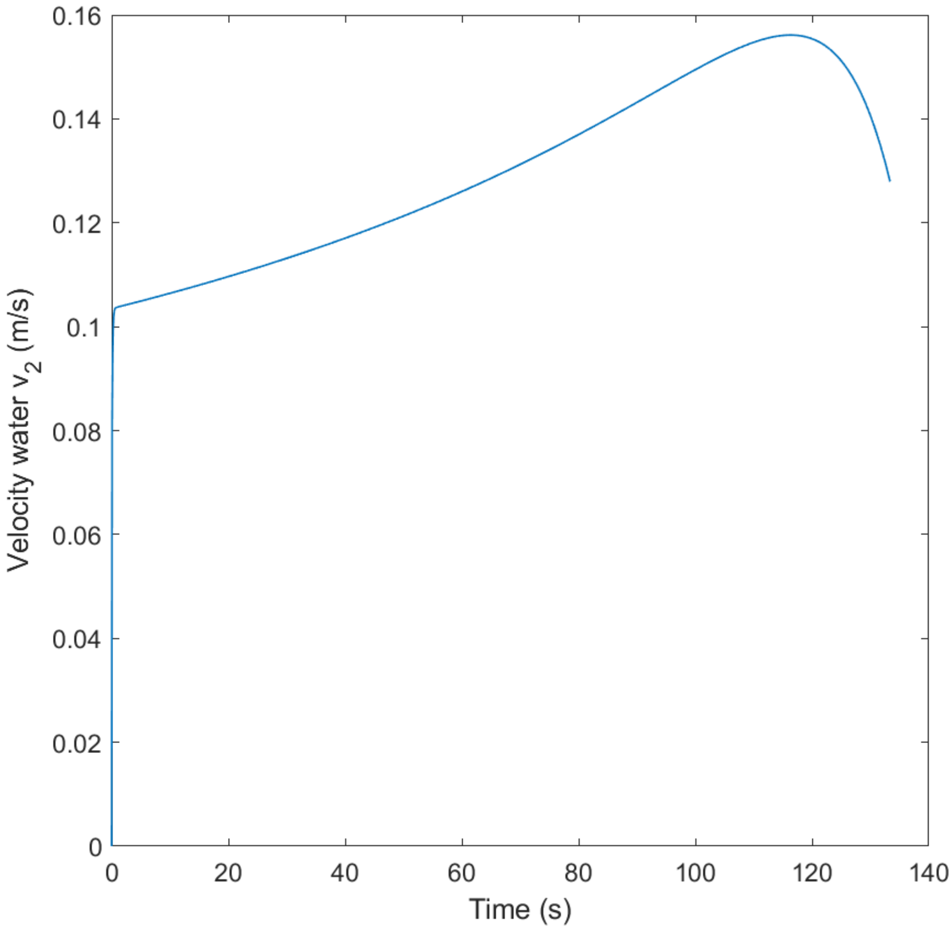

Figure 4.5: The velocity of the water against the time.

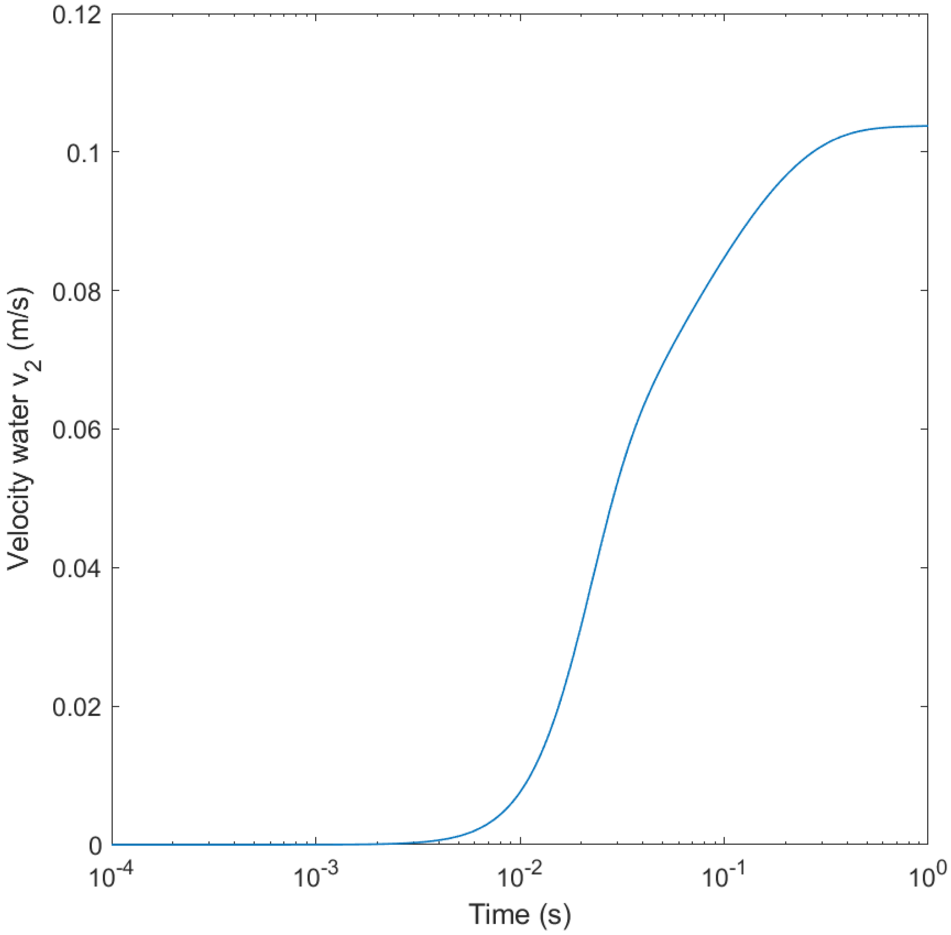

Figure 4.6: The first second of the velocity of the water against the time, on a logarithmic scale.

In the first second, the velocity of the water rises very quickly. The quick increase of the velocity stops after a second. The velocity increases still, but on a much smaller scale. The velocity of course peaks; when the height of the water increases, the driving pressure $(p_0 + \rho_w g(H + L_{ex} - h) - p_2)$ decreases as can be seen in figure 4.3. The friction brakes the velocity than more than the driving pressure pushes it up. The quick increase of the velocity and then the stagnation can be explained by figure 4.7. The acceleration is equal to

$$a_2 = \frac{p_0 - p_2}{h\rho_w} + g\frac{H + L_{ex} - h}{h} - \frac{v_2^2}{2}\left(\frac{(1 + K_{en}\frac{A_{an}^2}{A_{en}^2})}{h} + \frac{f}{D_{h,an}}\right). \tag{4.1}$$

This contains a driving force

$$\frac{p_0 - p_2}{h\rho_w} + g\frac{H + L_{ex} - h}{h} \tag{4.2}$$

and a friction part

$$-\frac{v_2^2}{2}\left(\frac{(1+K_{en}\frac{A_{an}^2}{A_{en}^2})}{h}+\frac{f}{D_{h,an}}\right). \tag{4.3}$$

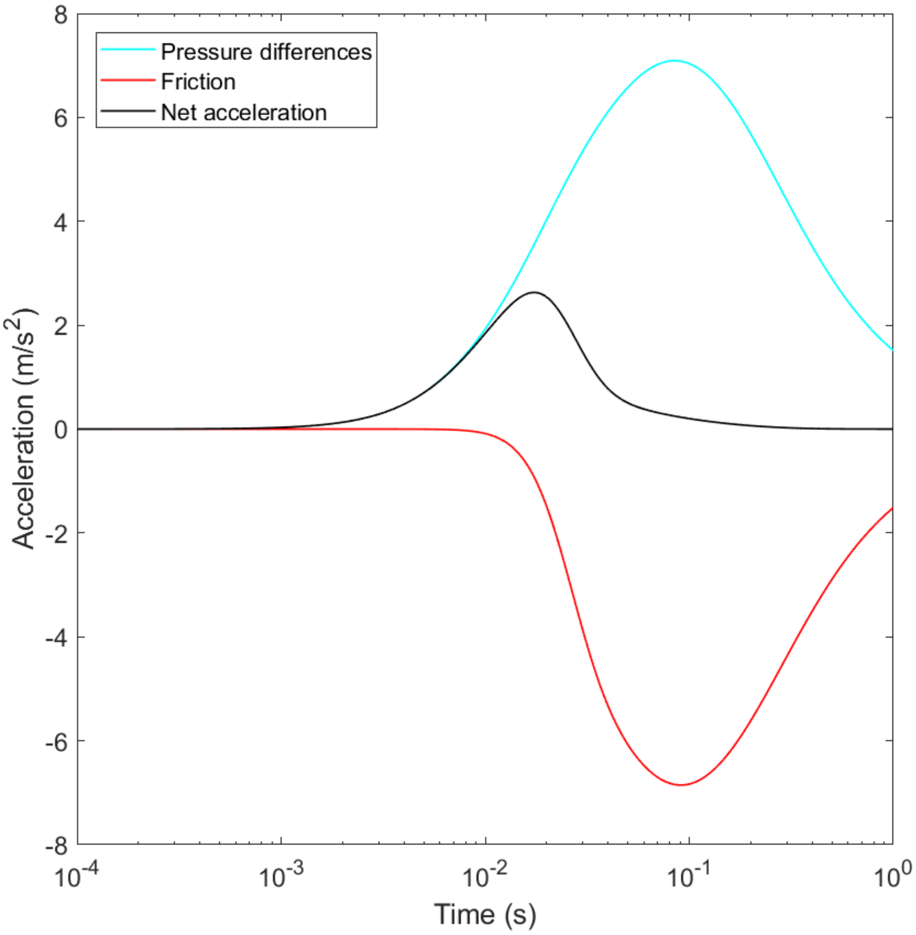

Figure 4.7: The first second of the acceleration of the water against the time, on a logarithmic scale. The blue line expresses the acceleration due to the driven force. The red line expresses the friction. The black line is the sum of the blue and the red line and expresses the net acceleration.

The friction decreases later than the driven forces increases. The velocity rises in this time increasing the friction. At the end, both accelerations become smaller since h increases. If the friction due to the broadening of the water channel is put to zero, so $K_{en} = 0$ in the water control volume, the velocity would behave even more unexpected, as seen in figure 4.8.

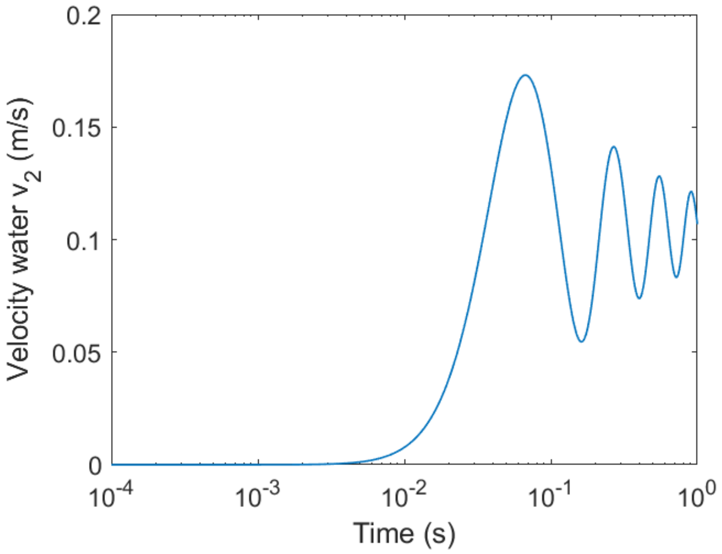

Figure 4.8: The first second of the velocity of the water in the tank against the time, on a logarithmic scale. The resistance coefficient K_{en} is put to zero in the mechanical energy balance of the water.

As can be seen in figure 4.8, is the velocity oscillating. The acceleration figure 4.9 also shows this oscillation.

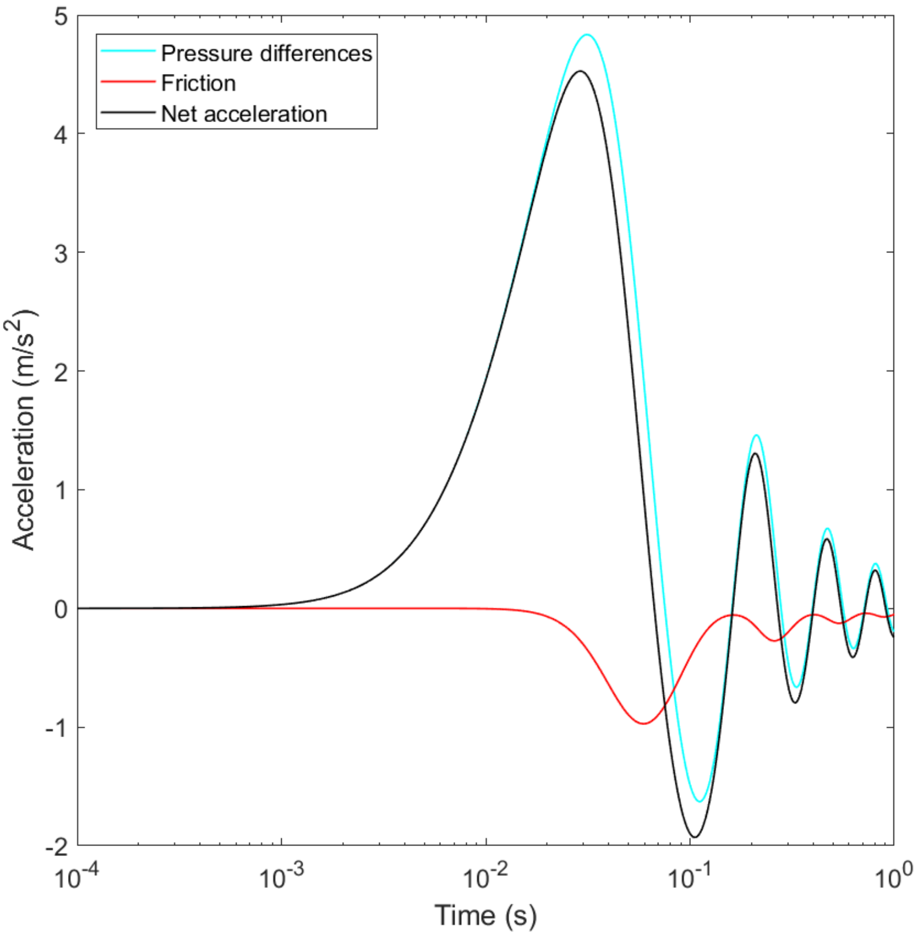

Figure 4.9: The first second of the acceleration of the water in the tank against the time, on a logarithmic scale. The resistance coefficient K_{en} is put to zero in the mechanical energy balance of the water.The blue line expresses the acceleration due to the driven force. The red line expresses the friction. The black line is the sum of the blue and the red line and expresses the net acceleration.

The driving pressure even becomes smaller than zero. An explanation for this behavior is that due to the pressure difference, the velocity increases according to equation (3.8). This effects the pressure in the tank via equations (3.16) and (3.14). The pressure in the tank becomes higher than the pushing pressure of the water. Because of the higher pressure in the tank, does the velocity decreases. The density decreases via equation (3.16) and thus decreases the pressure via equation (3.14). This creates an effect like a spring.

Velocity air

The velocity of the outflowing air in the exit pipe is shown in figure 4.10.

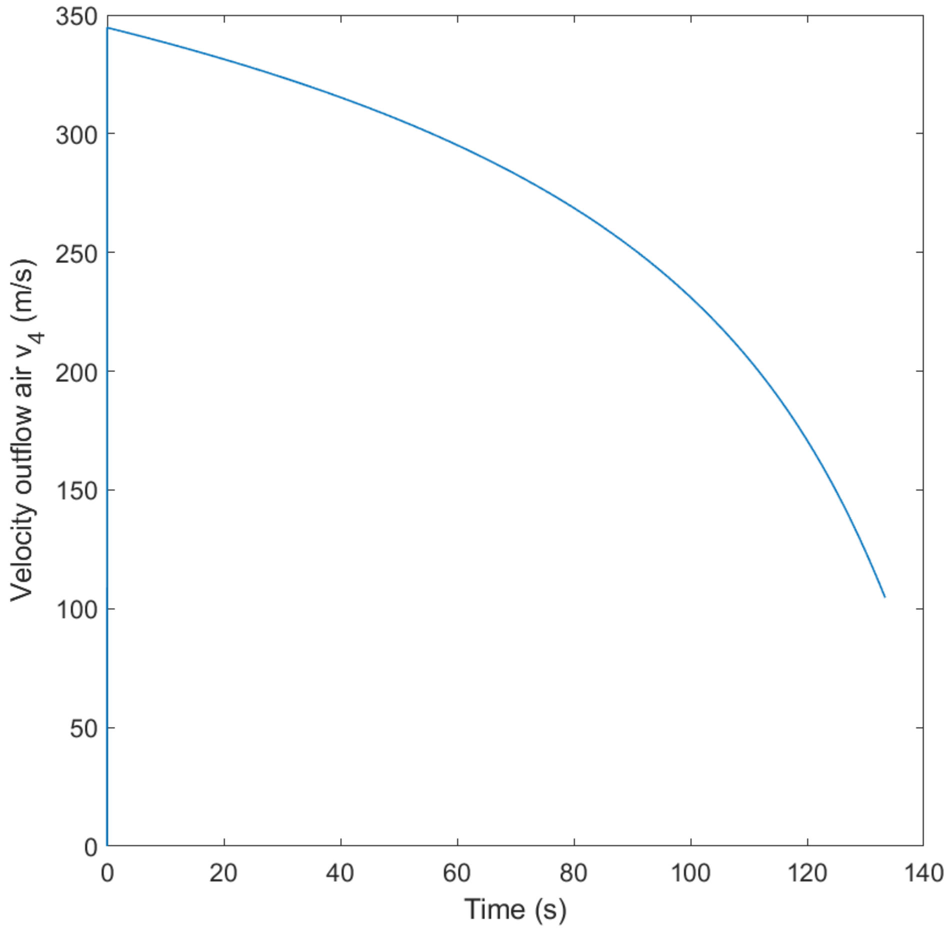

Figure 4.10: The velocity of the outflow of the air against the time.

Note that the air flow reaches velocities higher than the sound speed.
The velocity of the air in the exit pipe can be converted to the velocity of the air in the tank just before the exit pipe v_3. Figure 2.1 explained point 3. The difference between v_3 and v_2 expresses to what extended the density of the air in the tank drops. In figure 4.11, v_2 and v_3 are reviewed.

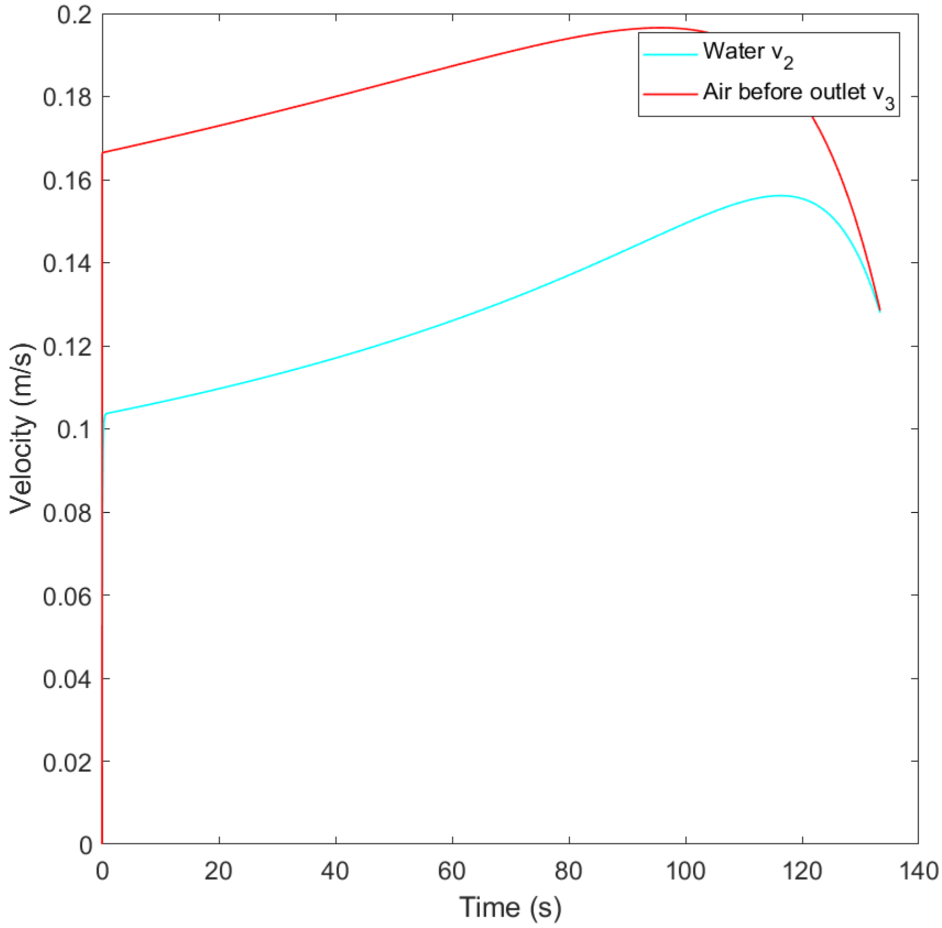

Figure 4.11: The velocity of the outflow of the air in the tank and the velocity of the water against the time, on a logarithmic scale.

It can be seen that the velocity at the top of the tank is higher than the velocity of the water in the tank. This velocity v_2 is also equal to the velocity of the air just above the water level in the tank. Therefore, the density drops and thus the pressure as can be seen in figures 4.2 and 4.4. Near the end, they become equal, since the density in the tank approaches the atmospheric pressure.

4.3 Changing the outlet

There are two factors in the outlet influencing the filling times. The most important one is the diameter of the exit pipe D_{ex}. If this diameter gets smaller, fewer mass flows out of the system, therefore stays the density of the air in the tank higher via equation (3.16) and thus the pressure via equation (3.14). This effects the acceleration of the water via (3.8), therefore increasing the filling time. Next to this, there is a valve and a pump somewhere attached to the pipe. These cannot be implemented in the system, but they can be resembled by a K_{rest}-factor. The default $D_{ex} = 0.1m$ and $K_{rest} = 0$.

4.3.1 Exit diameter

The exit diameter against the filling times is plotted in figure 4.12.

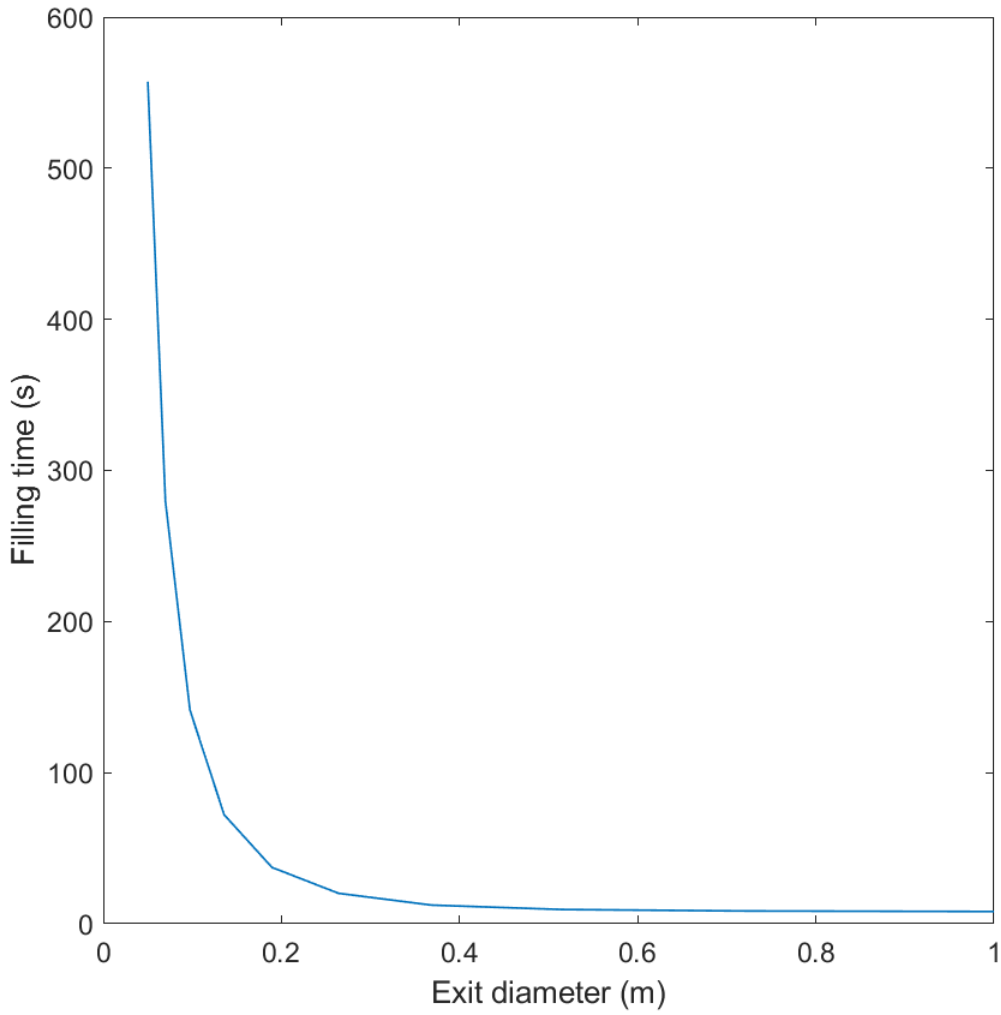

Figure 4.12: The exit diameter and its belonging filling time.

The filling time appears to inversely related to the exit diameter with an asymptote at a filling time of $7.8s$, the filling time found by the model of Ettema [4]. If the exit diameter is taken large enough, the filling time approaches the filling times calculated in Ettema's report. The filling times ft are fit to the exit diameter D_{ex}, according to

$$ft = a(D_{ex})^b + c. \tag{4.4}$$

This fit has an R-square of 1.0000. The fit is therefore very good.

Variable	Fit	95 % confidence interval
a	0.9929	(0.9102, 1.076)
b	-2.11	(-2.138, -2.082)
c	5.259	(3.798, 6.72)

$$ft = 0.9929(D_{ex})^{-2.11} + 5.259 \tag{4.5}$$

The code blows up if the exit diameter is set too small or too large due to numerical problems. However, figure 4.12 shows that the filling time gets extremely large when the exit diameter gets smaller. It is also reviewed how fast the pressure drops. If the pressure drops fast, Ettema's theory [4] might be valid. The 1.2 bar time shows the moment the pressure in the tank crossed the 1.2 bar. The pressure never reached atmospheric pressure, but it might not even reach 1.1 bar since the highest point of the tank is still 1 meter under the water level. Therefore, the 1.2 bar time is reviewed. This time is useful to verify if the pressure drops much fast than the height rises.

Table 4.2: Table with the 1.2 bar time and the filling time for different exit pipe diameters

D_{ex} (m)	1.2 bar time (s)	Filling time (s)
0.05	524.9	557.2
0.07	263.9	279.6
0.10	133.3	141.4
0.14	67.7	72.1
0.19	34.4	37.3
0.26	17.3	20.2
0.37	8.4	12.3
0.51	3.8	9.4
0.72	1.7	8.4
1.00	0.8	8.1

The 1.2 bar time is close to the filling time until $D_{ex} = 0.37$. For $D_{ex} \leq 0.37$ does the pressure drop very fast and does the filling time enter its asymptote. For this moment, the exit pipe is no longer the main resistor, but other components such as the entrance pipe are.

4.3.2 Friction factor pumps and valves

It is also insightful to get an idea of the effect of K_{rest} for the exact geometry including pumps and valves at the top. This is shown in figure 4.13.

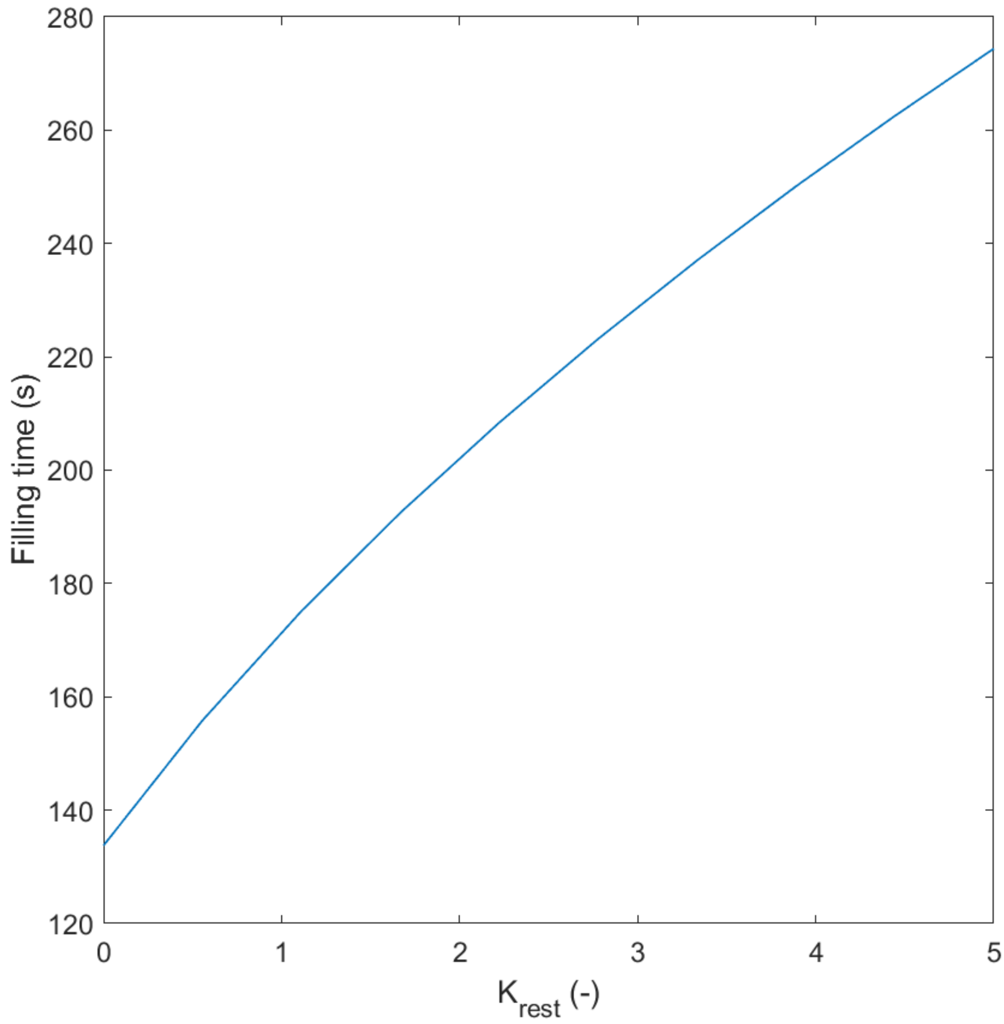

Figure 4.13: The K_{rest} against its filling time.

Figure 4.13 shows that K_{rest} has a strong effect on the filling times. If K_{rest} would be 1 for instance, the filling time would already be 20 seconds longer. The codes blows up for K_{rest} too high, but it can already be seen in figure 4.13 that the filling times are increasing when K_{rest} is higher. This relation should continue until K_{rest} is infinitely high, since the filling time should be infinitely then as well since the air will never get out. It is therefore important that the geometry of valves and pumps has a small resistance coefficient.

Chapter 5

Conclusions and Recommendations

The SLIMR is a new type of reactor. Its opportunities look promising. It is kept under water so that the water will cool the reactor passively in case of a shutdown. However, this heath transfer is unwanted in a normal situation. Therefore, a tank has been proposed to keep the reactor surrounded by air. In case of an emergency shutdown, the pump stops working and water will enter the reactor. Ettema calculated the filling time assuming that the pressure in the air dropped immediately to atmospheric pressure. In this follow-up research, the pressure is taken into account, as well as the addition of an exit pipe.

5.1 Conclusions

The model that has been made to solve the height of the water in the emergency tank works as it should. It gives a clear overview on the processes in the tank during a shutdown.
The assumption that the pressure drops to atmospheric pressure instantaneously is obviously not a realistic one. This assumption suffices if the exit diameter is large. The assumption is wrong for exit pipe diameters smaller than half a meter. The filling times can become more than 50 times higher with a model that takes the pressure into account compared to the model that assumed that the pressure is atmospheric instantaneously. The filling time is inversely dependent on the exit pipe diameter via $ft = 0.9929(D_{ex})^{-2.11} + 5.259$.
The effect of the resistance coefficient K_{rest} on the filling time has been taken into reviewed. This coefficient has a strong negative influence on the filling times, as expected.
The assumption that the gravitational energy would be negligible is true, since it is much smaller than $\frac{1}{2}v_4^2$. The Reynolds numbers as they were estimated in section 2 turn out to be right.

5.2 Recommendations

The reactor should be able to lose its decay heat properly. If the reactors internals gets too hot, safety barriers will be damaged. It is important to know if the reactor can still lose its decay heat in case of an emergency tank. Therefore, further research has to be done to investigate all heat transfers in the reactor vessel and the emergency tank. The water and the air will heat up due to the decay heat of the reactor. This temperature increase has effect on the mechanical energy balances as well. The pressure and the densities are directly related to the temperature. These temperatures can be implemented in the model of this report to calculate the state variables each time step.
The implementation of the temperature has to be made for the steady state situation, for the filling situation and for the filled situation. This last situation has a strong similarity with Velings report [15]. However, his research did not take a tank into account. This tank changes the flow of the water and thus the heat removal.
The velocities and the pressure in the tank are oscillating when the friction due to K_{en} is assumed to be zero. The reason for the oscillation might be investigation. The effect might cause resonance or another effect. This can be reviewed as well.

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