The equations governing the physics of active magnetic regenerators are presented in simplified form. These are used as the basis for a numerical model which is compared to experimental results for validation.

Tom Burdyny, Andrew Rowe
Overview

• AMR Thermodynamics
  • Governing behaviour, losses, magnetic cycle
  • Goal: To find $T_{Span}$ and $Q_c$ as a function of $T_H$
• Model based on simplified theory
• Comparison to experimental results
• Current issues
• Future work
• Summary

Delft, October/Oktober 2011
Motivation

• What is the purpose of a model based on simplified theory?
  • Quick solutions
    • Immediate sensitivity analysis
  • Optimization
  • Parameters linked to equipment sizing and costs
  • Beneficial for all researchers in a group
Governing Equations\(^1\)

\[ m_f' c_p \frac{\partial T_f}{\partial t} + \dot{m} c_p \frac{\partial T_f}{\partial \bar{x}} = \frac{\partial}{\partial \bar{x}} \left( k_f A \frac{\partial T_f}{\partial \bar{x}} \right) + hA' (T_s - T_f) \]  
\( \text{Fluid:} \tag{1} \)

\[ M_s' c_B \frac{\partial T_s}{\partial t} + M_s T_s \left( \frac{\partial m}{\partial T} \right)_H \frac{\partial \mu_0 H}{\partial \bar{t}} = \frac{\partial}{\partial \bar{x}} \left( k_s A \frac{\partial T_s}{\partial \bar{x}} \right) + hA' (T_f - T_s) \]  
\( \text{Solid:} \tag{2} \)

Assuming \( T_f = T_s \) and \( k_{\text{eff}} = k_f + k_s \):

\[ \frac{\partial T}{\partial t} + \frac{\Phi}{R} \frac{\partial T}{\partial x} - \frac{1}{M_s' L c_B R} \frac{\tau_B}{\partial x} \left( \frac{k_{\text{eff}} A}{L} \frac{\partial T'}{\partial x} \right) = \frac{1}{R} \frac{\partial T_{\text{ad}}}{\partial t} \]  
\( \tag{3} \)

where \( R \) is the thermal mass ratio, \( \Phi \) is utilization and \( T_{\text{ad}} \) is the adiabatic temperature change of the material.

Against other terms conduction is then assumed to be small:

\[
\frac{\partial T}{\partial t} + \frac{\Phi}{R} \frac{\partial T}{\partial x} - \frac{\partial T}{\partial t} \left( \frac{L_c}{L_c + R} \frac{\partial T}{\partial t} \right) = \frac{1}{R} \frac{\partial T_a}{\partial t} \tag{3}
\]

Eq.(4) used to find the temperature at any point in the magnetic cycle relative to a single state point.

For convenience a secondary definition of utilization is:

\[
U_H = \frac{\Phi_H}{R_H} \tag{5}
\]

where the subscript H represents the utilization during a hot blow and C for a cold blow.

Energy Balance

\[ W' + Q'_p = \frac{dQ}{dx} \]

\[ W' = \frac{M_s' c_s}{\tau_c} \frac{\Delta T}{T} c_{RH} \left[ \left( \frac{R - 1}{R} \right) \Delta T + \frac{\Phi}{R} \frac{dT}{dx} \right] \] (6)

\[ Q'_p = Q'_{edd} + Q'_{amb} + Q'_{viscous} \] (7)

\[ Q = \frac{M_s' L c_s}{\tau_c} \left[ \frac{\Phi}{R} \Delta T - K \frac{dT}{dx} \right] \] (8)

Using the BC’s, \( T_{a0} \) and \( T_{a1} \), Eq.(10) can be solved for \( T_a(x) \)

\[ \frac{d^2 T}{dx^2} = \left( \frac{1}{K} \right) \left[ \frac{\Phi_{RH}}{R} \left( \frac{1}{c_{RH}} \frac{dT}{dx} - \frac{\Delta T}{T} \right) \frac{dT}{dx} - c_{RH} \left( \frac{R - 1}{R} \right) \frac{\Delta T^2}{T} - \frac{\tau_c}{M_s' L c_s} Q_p \right] \] (10)
Using Eq.(4) and $T_a(x)$, temperatures $T_b$, $T_c$, $T_d$ are found.

The fluid temperature, $T_f(x)$, is then: \[ T_f = \frac{1}{4} (T_a + T_b + T_c + T_d) \] (11)

\[ T_{\text{Span}} = T_H - T_C = T_f(x = L) - T_f(x = 0) \]
Cooling Power

• Cooling Power is post-calculated

\[ Q_c = Q_{amr} - Q_{losses} \] (12)

\[ Q_c = Q_{amr} - Q_{NTU} - Q_{amb} \] (13)

\[ Q_{amr} = \frac{M_s L c_s}{\tau_c} \left( \frac{\Phi}{R} \Delta T - K \frac{dT}{dx} \right) \] (14)

\[ Q_{NTU} = \frac{m_{disp}}{2} \omega c_p (T_H - T_C) \left( \frac{NTU/2}{(NTU/2)^2 + (m_f/m_{disp})^2} \right) \] (15)

\[ Q_{amb} = \left( \frac{0.16 W}{K} \right) (T_H - T_C) \] (16)

\(^3\)Tura, A. Cryogenic AMR Test Apparatus. CEC-ICMC, Keystone, Colorado, 2005
Model

Inputs

- $T_{a0}$ and $T_{a1}$ (Boundary Condition’s)
- Operating parameters, fluid/solid properties

Outputs

- $T_{Span}$
- Cooling power, work, efficiency
Experimental Data

- Data is from the SC-AMRTA at the University of Victoria
- Working fluid is Helium ($R \sim 1$)
- Materials are Gd, GdTb and GdEr flakes
- Experiments use either single, double or triple pucks of material
Model Results Summary

• Comparison to experimental results
  • $Q_c$ vs $T_{Span}$
  • $T_{Span}$ vs $T_H$
  • Regenerator temperatures profile
• Effect of utilization (U) on calculated results
Results: $Q_c$ vs $T_{Span}$ ($T_H=292K$)

AMR Cooling Power and Losses for Gd at $T_H = 292K$

- Black line: $Q_{amr}$
- Green line: $Q_{amr-QNTU}$
- Blue line: $Q_{amr-QNTU-Qamb}$
- Red dashed line: Experimental

Delft, October/Oktober 2011
Results: $Q_c$ vs $T_{\text{Span}}$ ($T_H=292\text{K}$)

AMR Cooling Power and Losses for Gd at $T_H = 292\text{K}$

$Q_c = 0\text{W}$
Results: $T_H$ vs $T_{Span}$ ($Q_c = 0\,\text{W}$)
Results: $T_H$ vs $T_{Span}$ ($Q_c = 0W$)
Results: Interface Temperature

Temperature Distribution along the Regenerator

Delft, October/Oktober 2011
Results: Effect of Utilization, U

• Near the Curie point curves destabilize for high U
• Prevents $T_{\text{Span}}$ from being calculated at $T_H = 310.7 K$
Results: Effect of Utilization

- $R = 1$ for He, $R = 1.5$ for water
- $\Phi = 0.27$ at 9.5 atm, $\Phi = 0.17$ at 6 atm
Comments

• Other losses
  • Demagnetization
  • Other heat leaks
• Model over predicts by a consistent amount
• Simulations take ~3s for each set of BC’s
• For a desired $T_H$ value multiple simulations (i.e. various BC’s) need to be performed and the data interpolated
  • One specific data point $(T_H, T_C)$ takes ~15s
Future Work

• Allow for the user to choose a specific field strength
• Account for further losses
• Layering
• Use model in conjunction with:
  • Costing
  • Optimization of parameters
Summary

• Simplified AMR thermodynamics were presented

• A model was created using this theory

• Model compared to experimental results
  • Shows similarities to experiments
  • Requires further data comparisons (GdTb)