Risk Analysis of Coastal Protections at Tidal Coasts

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Abstract

A probabilistic design concept for coastal protections supplementing the traditional deterministic design approach was worked out and applied near Wilhelmshaven and Bremerhaven at the German North Sea coast. Using this modern design concept the probability of failure of the coastal protection system was calculated taking into account wave overtopping at the main dike being the most important failure mechanism. Today’s recurrence interval is approximately 1000 years. In the event of climate change – raising water-levels and increasing wind speeds – the recurrence interval is significantly reduced. An increase of the water-level of 0.50 m will reduced the recurrence interval to 40 %, while an increase in wind-speed of 10 % leads to a 20 % reduction of the recurrence interval. All results were analysed with respect to uncertainties caused by the extrapolation of wind and water-level statistics to extreme events revealing the difficulties of a probabilistic design.

Introduction

At the German North Sea Coast the hinterland is protected from flooding by a series of coastal protection elements. The design of the artificial coastal protection elements, like brush wood fences, summer dikes and main dikes, is traditionally deterministic. This deterministic design approach is today complemented with probabilistic design methods in order to derive the recurrence interval of failure of the coastal protection system [1]. Taking into account the damage due to failure, e.g. inundation of coastal areas in consequence of a dike breach, the probabilistic design concept may be extended to a risk analysis [2, 3]. Besides the calculation of today’s safety standard the probabilistic method provides a means to estimate the effect of raising water-levels and intensifying winds, e.g. caused by climate change [4]. In this paper special emphasis is put on the quantification of the uncertainty of the calculated failure probabilities due to the extrapolation of the statistics of water-levels and wind derived from long-term data-sets [5, 6]. The analysis of failure probabilities is worked out for the protection system of the coastal zone between the ports of Wilhelmshaven and Bremerhaven. A detailed description of the protection system and the protected hinterland can be found in [7].

Theoretical Background

Risk analysis uses the idea of a failure function, Z, describing the difference between the resistance of the coastal protection element, R, and the stress acting on the element, S:
\[ Z(\mathbf{u}, \mathbf{v}) = R(\mathbf{u}) - S(\mathbf{v}) \]  

(1)

where \( \mathbf{u}, \mathbf{v} \) are the parameter vector describing the resistance and stress. Failure is denoted to all combinations of variables \( \mathbf{u} \) and \( \mathbf{v} \) leading to negative values of the failure function, i.e.

\[ Z(\mathbf{u}, \mathbf{v}) < 0 \]  

(2)

The parameter \( \mathbf{u} \) and \( \mathbf{v} \) are generally probability distributed and are described by using the joint probability density function \( p_{\mathbf{u}, \mathbf{v}}(\mathbf{u}, \mathbf{v}) \). The probability of failure \( p_{Z<0} \) is determined by integrating the joint probability density function over the failure region:

\[ p_{Z<0} = \int \cdots \int_{Z<0} p_{\mathbf{u}, \mathbf{v}}(\mathbf{u}, \mathbf{v}) \, d\mathbf{u} \, d\mathbf{v} \]  

(3)

For the sea dike, the most important coastal protection element at the German North Sea Coast, wave overtopping is considered to be the most important failure mode (Fig. 1) [2]. The failure function is defined using two different criteria, one calculating the run-up level [8] and the other calculating the overtopping rate [5]:

\[ Z = h_D - Thw - R \]  

(4a)

\[ Z = Q_{tol} - Q \]  

(4b)

with
- \( h_D \): height of the dike
- \( Thw \): tidal high water-level
- \( R \): wave run-up
- \( Q_{tol} \): tolerable overtopping rate [9]
- \( Q \): overtopping rate

![Fig. 1 Failure mechanism – wave overtopping](image)

The wave run-up is given by [10]:

\[ R = 0.75 \cdot \gamma \cdot \frac{1}{n_D} \sqrt{g \cdot H_s \cdot T_m} \]  

(5)

where \( H_s, T_m \) are the significant wave height and mean wave period, \( 1/n_D \) the slope of the dike, \( g \) the acceleration of gravity and \( \gamma \) a reduction factor describing the effect of slope roughness, shallow fore shore, berm and oblique wave attack.
Two different approaches describing the overtopping rate introduced in equation (4b) can be distinguished – one using the peak overtopping rate [5] the other using the mean overtopping rate [10]:

\[
Q_{\text{peak}} = \gamma_{p,1} \cdot g \cdot H_s \cdot T_m \cdot e^{-\frac{1}{\tau_{p,2} T_m \sqrt{g H_s}} \left( h_D - Thw \right)}
\]

(6)

\[
Q_{\text{mean}} = \gamma_{m,1} \sqrt{g \cdot H_s^3} \cdot e^{-\frac{1}{\gamma_{m,2} H_s} \left( h_D - Thw \right)}
\]

(7)

with the dimensionless parameter \( \gamma_{p,1}, \gamma_{p,2}, \gamma_{m,1} \) and \( \gamma_{m,2} \).

The wave parameter \( H_s \) and \( T_m \) at the coastal defence structure can be estimated using numerical wave models, e.g. SHALLOW WAVES NEARSHORE SWAN [11]. In mathematical terms the wave parameter are a function of water-level \( Thw \) and wind conditions \( (u_{\text{wind}}, \alpha_{\text{wind}}) \):

\[
H_s = H_s (Thw, u_{\text{wind}}, \alpha_{\text{wind}})
\]

(8)

\[
T_m = T_m (Thw, u_{\text{wind}}, \alpha_{\text{wind}})
\]

(9)

For the failure mechanism of wave overtopping the parameter vector \( \tilde{v} \) describing the stress is given by

\[
\tilde{v} = (Thw, u_{\text{wind}}, \alpha_{\text{wind}})
\]

(10)

while the parameter vector \( \tilde{u} \) describing the resistance is given by

\[
\tilde{u} = (h_D)
\]

(11a)

\[
\tilde{u} = (Q_{\text{tol}})
\]

(11b)

depending on the chosen criteria (equation 4a/b). Within this study the height of the dike respectively the tolerable overtopping rate at a certain position is assumed to be exactly determined, i.e. not probability distributed. The load vector \( \tilde{v} \) is represented by a joint probability function of water-level and wind, derived from a 50 years data set.

**Probability of Dike Failure in the Event of Climate Changes**

An example of the probability distribution of the tidal high water-level is given in figure 2 (left) using data of a gauge at Wilhelmshaven. The solid black line represents today’s probability distribution while the coloured lines are scenarios of increased tidal high water-level in case of climate change. The scenarios correspond to estimates of sea-level rise at the German North Sea coast within the next 50 respectively 100 years derived by downscaling global climate change estimates to regional scales [11].
Fig. 2 Scenarios of probability density functions of water-levels and wind

The wind speed $u_{\text{wind}}$ is Weibull distributed [12]:

$$p(u_{\text{wind}}) = \frac{k}{A} \left( \frac{u_{\text{wind}}}{A} \right)^{k-1} \exp\left( -\frac{u_{\text{wind}}}{A} \right)^k$$  \hspace{1cm} (12)

with scaling parameter $A$ and $k$. An example of a probability distribution of wind speed for winds from 300° is given in figure 2 (right). Again the solid black line gives today’s situation and the coloured lines give scenarios of increased wind speed.

Fig. 3 Numerical simulation of wave propagation within an estuary
For various sets of the parameter vector $\vec{v}$ the wave climate is calculated using the wave model \textsc{Shallow Wave Nearshore} (SWAN) in order to derive the transfer functions stated in equation (8) and (9). An example of the wave propagation calculated with SWAN is given in figure 3. The calibration of the model is described in [13]. Both wave parameter – significant wave height and mean wave period – reduce significantly for waves propagating towards the coast because of the reduction in water depth. Extracting the wave parameter at different locations of the numerical model (fig. 3) for all sets of the parameter vector $\vec{v}$ gives the recurrence time of wave parameter. An example of the recurrence time of the significant wave height within the model area is given in figure 4. At the model boundary the 100-year significant wave height is 3.5 m decreasing to 1.4 m - 2.0 m over the tidal flats and to 0.4 m - 0.8 m over the foreland in front of the main dike.

![Wave propagation diagram](image)

**Fig. 4** Recurrence interval of significant wave heights at different locations within the coastal protection system

With these wave parameter the recurrence interval of wave overtopping at the main dike is calculated for various scenarios of climate change applying equations 3, 4a and 5. Figure 5 presents the influence of water-level rise on the recurrence interval of wave overtopping for the dikes near the cities Wilhelmshaven and Bremerhaven. Today’s recurrence interval of wave overtopping is approximately 1000 years. It decreases to about 350 years in case of a water-level rise of 0.50 m and to 100 years in case of a water-level rise of 1.00 m.
The effect of an enhancement of the wind speed on the recurrence interval is minor, as shown in figure 6. An increase of wind speed of 10% reduces the recurrence interval only from approximately 1000 years to 900 years. An increase of wind speed of 3.8%, begin a more probable estimate of possible climate change within the next 100 years, only leads to a reduction of 20 years in recurrence interval.

![Figure 5](image1.png)
**Fig. 5** Recurrence interval of wave overtopping at the main dike in case of sea-level rise

![Figure 6](image2.png)
**Fig. 6** Recurrence interval of wave overtopping at the main dike in case of increasing winds

**Uncertainty of Calculated Failure Probabilities**

Besides the intrinsic uncertainty, which is a consequence of tidal high water-level and wind being stochastic processes [6], statistical uncertainties and model uncertainties should be taken into account calculating the probability of overtopping, e.g. [14] states that these additional uncertainties might change the probability by a factor of 10. In the following statistical uncertainties respectively distribution type uncertainties and their effect on the probability of wave overtopping are analysed exemplary for tidal high
water-levels. Figure 2 exemplifies the problem of distribution type uncertainties giving four different extrapolations of the probability density function of the tidal high water-level at the coast near Wilhelmshaven.

The extrapolations are calculated using non-linear least-square method assuming different types of probability distributions:

- Normal distribution (Gauss)
- Gumbel distribution
- Weibull distribution
- Exponential distribution

The fits of exponential and Gumbel distribution lead to approximately the same probability density for extreme water-levels while especially the normal distribution results in lower values of the probability density function. The tidal high water-level with a recurrence interval of 100 years respectively 1000 years is listed for the different types of probability distributions in table 1.

<table>
<thead>
<tr>
<th>recurrence interval</th>
<th>type of probability distribution of tidal high water-level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>normal</td>
</tr>
<tr>
<td>100</td>
<td>5.17</td>
</tr>
<tr>
<td>1000</td>
<td>5.82</td>
</tr>
</tbody>
</table>

Tab. 1 100-year / 1000-year tidal high water-level at the gauge Wilhelmshaven
The introduction of the different distributions into equation (3) gives the recurrence interval. Figure 8 shows the recurrence interval of wave overtopping at the dike profile P2 located at the Jade estuary for the different types of probability distribution and the scenarios of sea-level rise. For today’s situation the recurrence interval ranges from 500 years (Weibull) to 4000 years (Gauss). The influence of distribution type uncertainty on the recurrence interval decreases with decreasing recurrence intervals. In the event of a sea-level rise of 1 m the recurrence interval varies from 60 years (Weibull) to 90 years (Gauss).

![Figure 8: Recurrence interval of wave overtopping at the main dike using different extrapolations of the statistics of tidal high water-levels](image)

**Conclusion**

A probabilistic analysis of the safety of dikes is worked out within this paper. Today’s recurrence intervals of wave overtopping will reduce to 10 % in case of a rise in tidal high water-level of 1.00 m. The probabilities of overtopping are not identical with the probability of inundation but can be transferred by multiplication with a so-called transition probability describing the probability that a dike fails under the condition of wave overtopping. Beyond intrinsic uncertainties statistical uncertainties have a large influence on the calculated recurrence interval leading to variations of an order of magnitude.

**References**


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