Assessing Uncertainty When Predicting Extreme Flood Processes

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SUMMARY
This paper offers a review of techniques for expressing and presenting uncertainty, along with consideration of how these approaches might be applied within the IMPACT project.

IMPACT Project Workshop
Mo I Rana, Norway 12/13th September, 2002
1 IMPACT ~ QUANTIFYING & INVESTIGATING UNCERTAINTY

1.1 A Definition of Uncertainty
There are many definitions of uncertainty. Perhaps the simplest and most complete is that “Uncertainty is a general concept that reflects our lack of sureness about something or someone, ranging from just short of complete sureness to an almost complete lack of conviction about an outcome” (NRC, 2000). It is the complete range of influences on uncertainty there will need to be reported by each partner within the IMPACT project.

1.2 Introduction
Understanding the uncertainty within our predictions and decisions is at the heart of understanding risk and hence an important issue to be appropriately and consistently addressed within IMPACT. In recognising uncertainty we are able to acknowledge our lack of knowledge of breach or sediment or flow behaviour (knowledge uncertainty) and the inherent variability in load (natural variability). Consideration of uncertainty within IMPACT will therefore attempt to quantify our lack of sureness, and thereby provide the decision maker with additional information on which to base a decision.

Through investigation of the sources of uncertainty, this type of analysis enables the decision-maker to identify the uncertainties that most influence the final outcome and focus resources efficiently.

2 EXPRESSING AND PRESENTING UNCERTAINTIES
Uncertainties can be expressed in a number of different ways, both qualitative and quantitative (HR Wallingford (1997):

- **Deliberate vagueness** – ‘There is a high chance of breaching’
- **Ranking without quantifying** – ‘Option A is safer than Option B’
- **Stating possible outcomes without stating likelihoods** – ‘It is possible the embankment will breach’
- **Probabilities of events or outcomes** – ‘There is a 10% chance of breaching’
- **Range of variables and parameters** – ‘The design flow rate is 100 cumecs +/- 10%’
- **Confidence intervals** – ‘There is a 95% chance that the design flow rate lies between 90 and 110 cumecs’ – See Box 2.1
- **Probability distributions** – See Box 2.1
Two of the most widely used quantitative expressions of uncertainty are confidence intervals and probability distributions. These are discussed below.

**Confidence Intervals** - A confidence interval specifies the probability that a variable falls within a range of values. For example, there is a 95% (this is the confidence level) chance that the design flow rate lies between the confidence limits of 90 and 110 cumecs. Confidence intervals can be formally calculated for some forms of uncertainty - statistical inference uncertainty (see Section 3) for example. However, expert judgement can also be applied to specify confidence intervals. For example, an experienced wave modeller may judge the output from his model to provide results that are accurate to within +/-10%. The modeller may be able express this accuracy with a probability (90% for example) that reflects his strength of belief in the model results, based on the quality of the calibration procedure.

It is important to note that a confidence interval does not provide any information regarding how the probability of achieving different values within the range may vary. Using the wave modelling example above, although the interval has been specified as being symmetrical, the modeller may know from experience that the model is more likely to under predict than over predict. A symmetrical confidence interval does not contain this information and an asymmetrical description may be provided.

**Probability distributions** – a probability distribution describes the probability of obtaining different values of a variable or parameter and hence the associated uncertainty. Probability distributions can be discrete or continuous. A frequently used continuous probability distribution is the Normal, or Gaussian Distribution. Many natural phenomena conform well to the Normal Distribution, which makes it particularly useful.

The figure below uses the Normal Distribution (shown as a probability density function) to illustrate the uncertainty of two Benefit Cost Ratios (BCR’s). The expected BCR for Option 2 is higher than Option 1 and, based only on this information, would make Option 2 the obvious choice. However, the additional information regarding uncertainty, provided by the probability distribution, shows Option 2 to have a higher chance of achieving a BCR of less than 1 (indicated by the hatched area under the Option 2 curve). If the decision maker places greater importance on achieving a BCR of greater than 1, as opposed to the highest expected BCR, Option 1 is preferred.
3 SOURCES OF UNCERTAINTY
Implicit within any risk analysis are many different types of uncertainty, the majority of which can be conveniently categorised under two simple headings (Box 3.1 and Figure 3.1):

- Natural variability
- Knowledge uncertainty

To help understand the relative importance of uncertainty within the decision-making process, an overview of how these uncertainties arise and how we can deal with them is given in the following sections.

Box 3.1 Uncertainty definitions

**Natural variability** - refers to the randomness observed in nature (standard term to be used).

also referred to as

- Aleatory uncertainty (meaning to ‘gamble’)
- External uncertainty
- Inherent uncertainty
- Objective uncertainty
- Random uncertainty
- Stochastic uncertainty
- Irreducible uncertainty
- Fundamental uncertainty
- Real world uncertainty

**Knowledge uncertainty** - refers to the state of knowledge of a physical system and our ability to measure and model it (standard term to be used).

also referred to as

- Epistemic uncertainty (meaning ‘knowledge’)
- Functional uncertainty
- Internal uncertainty
- Subjective uncertainty
- Incompleteness

Through investigation of the sources of uncertainty, this type of analysis enables the decision-maker to identify the uncertainties that most influence the final outcome and focus resources efficiently. For example, consider the problem of managing a shingle barrier beach prone to breaching. There is little benefit in spending significant time reducing the uncertainty associated with a wave model (so, say, that it is accurate to within ± 10%) if this uncertainty results in a 1% change in breach probability, when the process model representing the breach mechanisms is only reliable to ± 20%.

Understanding the sources and importance of uncertainty within the decisions we make is a key driver in making more informed choices. However, uncertainties arise at every stage in the decision process. The nature and form of these uncertainties are explored in this chapter together with a discussion on how uncertainty can be categorised and handled.
3.1 Natural Variability – Understanding Its Sources
Temporal variations in nature’s forces are well known and, in general, it is not possible to reduce the uncertainty related to the temporal natural variability of our environment. For example, it is, at present, not possible to say when a 100-year return period river discharge will next be observed at any given location on a river. A time period of 400 years could pass without observing a 100-year event, but then two could arrive within a year of each other.

3.2 Knowledge Uncertainty – Understanding Its Sources
The concept and importance of knowledge uncertainty is less commonly considered and formally assessed. For example, a numerical model of wave transformation may not include an accurate mathematical description of all the relevant physical processes. For example, wave breaking aspects may be parameterised to compensate for the lack of knowledge regarding the physics. The model is thus subject to a form of knowledge uncertainty. Unlike the uncertainties associated with natural variability it is possible to reduce knowledge uncertainty. For example if the IMPACT research results in a better mathematical description of erosion of cohesive material and this is included in a model of breach growth, or the field data is utilised to better represent the physical conditions present then the knowledge uncertainty may be reduced.

Under the generic heading of knowledge uncertainty a number of specific forms of uncertainty will need to be identified and formally calculated. An overview of these different sources is provided below:

**Statistical Uncertainty** can be sub-divided into Statistical Inference Uncertainty and Statistical Model Uncertainty:

- **Statistical Inference Uncertainty** (sometimes referred to as parameter uncertainty) refers to the uncertainty resulting from the need to extrapolate short datasets to provide more extreme estimates.

Statistical Inference Uncertainty is perhaps the most well recognised form of statistical uncertainty encountered by the flood and coastal defence community. Statistical Inference Uncertainty results when fitting a statistical model to a sample of data rather than a full population. The uncertainty is therefore related to the size and variability of the data sample and the degree to which it is representative of the full population. For example, a 200-year return period estimate of an environmental variable derived from a data source that has been collated over 80 years, will clearly be subject to less uncertainty than the same estimate based on 20 years of data.

- **Statistical Model Uncertainty** (sometimes referred to as distribution uncertainty) refers to the uncertainty that results from the selection of a particular statistical model to extrapolate a particular set of data.

For example, once selected, it is assumed that the statistical model is correct for the purposes of data extrapolation. However, it is quite conceivable that an alternative statistical model may provide an equally valid fit to the data but yield a significantly
different extrapolation. The difference in the extrapolation of the two models gives an indication of the Statistical Model Uncertainty (there may be other models that can also contribute to the overall statistical model uncertainty or the actual distribution may not conform well to any of the extreme value models). To minimise Statistical Model Uncertainty it is important to use judgement in the selection of the model and compare different fitting techniques.

Box 3.3 Statistical Inference Uncertainty

It is the Statistical Inference Uncertainty that gives rise to one of the most frequently asked questions when designing flood defences: “What is the most extreme event that can be predicted from, say, a 10 year data set?”. To answer this question a number of simple ‘rules’ are often quoted. For example ‘it is only possible to derive return period estimates up to 2.5 times the length of the data set’. Such ‘rules’ are somewhat arbitrary and are essentially an attempt to recognise that the Statistical Inference Uncertainty may become ‘unacceptable’ outside of this range. Theoretically it is possible to derive any extreme return period estimates from any data length. However, it is important to recognise that the Statistical Inference Uncertainty may be significant.

It should also be recognised that the application of stringent ‘rules’ to the length of return period that can be estimated from any given data set due to the Statistical Inference Uncertainty can be, at best, misleading as other sources of uncertainty may influence the dependability of the result far greater (e.g. Statistical Model Uncertainty or Process Model Uncertainty).

Process Model Uncertainty (standard term to be used – note sometimes referred to as model uncertainty and data uncertainty) describes the uncertainty associated with using a process model based on incomplete process knowledge, or data, to represent reality. Numerical models of physical processes are incomplete. Likewise, physical models are subject to uncertainties regarding scale effects.

For example, our knowledge of the processes that drive breach are incomplete and rapidly evolving through IMPACT. As improved representations of physical processes are imbedded within breach models our predictions change; however, it is unknown just how many important processes remain missing. Therefore, an important outcome of the IMPACT studies will be to demonstrate in a quantitative manner how uncertainty has been reduced and/or better understood.

4 HANDLING UNCERTAINTY

Some or all of the types of uncertainty described above are present in some form or another in all breach models. Some types of uncertainty are explicitly considered. It is more common, however, for these uncertainties to be implicitly accounted for through the intuition of the decision-maker. HR Wallingford (1997) identified a number of sources of uncertainty in flood and coastal defence. These are detailed below together with discussion of how these types of uncertainty are dealt with in current practice.
• **Natural variability (temporal) - Associated with random hydraulic conditions**
  (e.g. river flows, waves and surges) – as discussed above, it is not possible to reduce
  the uncertainty due to the natural variability of our environment.

  Generally this aspect is dealt with through the use of probability distributions. Data
  are gathered on the variable of interest (this may be from measurements or
  numerical models). A probability distribution is then fitted to the data to provide
  estimates of the likelihood of occurrences of events that are outside the range of the
  data (i.e. the data are extrapolated). This information is then used to assess the
  probability of occurrence of extreme events in a specified period of time. For
  example, an embankment with a required design life of 50 years may be constructed
  to prevent overflow during a 200 year return period water level event. The
  probability of encountering one or more 200 year return period event/s in a 50 year
  period is approximately 0.25 (25%) which, when considered with the impact of the
  event arising, may be considered an acceptable level of risk.

  When fitting probability distributions it is standard practice to derive estimates of
  the **Statistical Inference Uncertainty** (i.e. confidence limits), as this is formally
  quantifiable with standard statistical techniques. **Statistical Model Uncertainty** is
  rarely considered explicitly and this can lead to confusion. More specifically, there
  is a danger that the confidence limits quantified from the **Statistical Inference
  Uncertainty** will be considered as representative of the total uncertainty. This is
  clearly an inaccurate assumption if **Statistical Model** and **Process Model Uncertainty**
  have not been considered as these sources of uncertainty maybe considerably
  greater.

• **Knowledge Uncertainty - Associated with environmental variables**
  Data on environmental variables such as rainfall, river flow, wave conditions and
  wind speeds form the basis for much of the decision making in coastal and fluvial
  engineering. These data can generally be considered in two forms; measured or
  output from process models (numerical or physical). The uncertainty on these two
  types of data is often termed **data uncertainty**. However, here, the specification of
  **data uncertainty** as a separate source of uncertainty is not made, as data
  uncertainties essentially arise due to **Statistical or Process Model Uncertainties**.

  Uncertainty from measurements can come in different forms. For example, water
  level is often interpreted from pressure measurements. The measured pressure
  signal is converted to a water level through an equation that incorporates knowledge
  on the density of water. In estuaries, where the salinity (hence density) of the water
  is constantly changing, there will be (some small) **Process Model Uncertainty** (the
  equation is a simplification of reality) on the water level measurements. Additionally,
  the pressure record may be sampled over a short period of time (e.g. 1 minute) at 15 minute intervals. These sampled data may then be used as
  representative of a continuous record and a statistical model used to reconstruct a
  continuous record, in which case the continuous record will be subject to **Statistical
  Inference Uncertainty**. When providing such data, the data provider should, through
  appropriate metadata, record the expected confidence limits associated with the data.
Where no confidence intervals are provided, data users have to make estimates. These estimates are likely to be founded on the dependability of the data. The user confidence in the data will depend upon the presence and adequacy of the metadata and quality assurance history. If, for example, calibration and verification data are provided and well documented the associated uncertainty will be small and it is likely to be assumed that the measurement accuracy is as stated.

It is important, however, to distinguish inaccuracies due to calibration and instrument limitation errors from those that can arise from neglect or bad practice. For example, a water gauge may be functioning well, but set up at an incorrect datum as a result of a levelling mistake. This type of mistake is often termed a gross error. Data providers and data users guard against gross errors by checking against other sources of data, where available. The possibility of a gross error occurring would not normally be considered in the analysis of uncertainty, but the possibility of their occurrence should be identified through the risk assessment process of identifying hazards and mitigating risk, associated with the data collection exercise.

Process Model Uncertainty (described in Section 3 above) is, where possible, minimised by utilising measured data to calibrate and validate the selected process model. However, often, measured data are not available and the reliability of output from process models becomes more uncertain. In such circumstances, judgement based on the experience of the model user is applied. It is general practice to apply an arbitrary element of ‘conservatism’ when there is little or no calibration information. Model parameters will be adjusted in a way to ensure that the model output errs on the ‘safe’ side of what the model user considers to be the best estimate. This type of arbitrary conservatism is rarely detailed and consequently rarely considered in subsequent consideration of the model output.

A methodology has been developed that assesses the uncertainty of process models by considering the uncertainty of parameters within the model set up. The methodology is called GLUE (Generalised Likelihood Uncertainty Estimation) and is discussed briefly below.

- **Knowledge uncertainty - Future changes in the physical climate.** It is widely recognised that environmental parameters exhibit non-stationary behaviour (i.e. wave and rainfall patterns may be changing). In these circumstances uncertainties are handled through consideration of scenarios and scenario testing. A scenario can be described simply as statement of a possible outcome (e.g. Carbon dioxide will double by 2050) without a corresponding statement of the likelihood of occurrence. However, the selection of the scenario relevant to a particular decision should be considered through the strength of belief the decision-maker has in each scenario.

Another aspect of climate change is sea level rise. Trends in sea levels have been measured at many locations around the UK (although it is recognised that difficulty exists in dividing land movement and water level). In current practice, the trend is removed from the data before fitting a probability distribution. Once the
distribution has been fitted and extreme values estimated the trend can then be accounted for within subsequent calculations if required.

- **Knowledge uncertainty** - Responses of defences (structural damage / deterioration / overtopping / breaching / landslide). Knowledge uncertainties dominate our ability to design and manage flood and coastal defences. Often data on the present condition of defences (e.g. ground condition properties) are sparse and our ability to predict behaviour (assuming complete knowledge of defence materials) is imperfect. The gaps in knowledge can be treated as probability distributions and used to describe the likely variation (for example this is the assumption utilised in the Dutch PC-Ring software used to investigate the likelihood of failure within a dyke ring (Vrijling and van Gelder (2000))).

5 COMBINING UNCERTAINTIES

When carrying out an analysis of uncertainty inevitably there will be a requirement to combine uncertainties from a variety of different sources. Depending on the circumstances and specific uncertainties, this procedure can range from a straightforward calculation to more complex and involved computations. The nature of the uncertainties to be combined may be, where appropriate, estimated, or they may be formally quantified. Discussed below are three different approaches. Further discussion will be required to determined which one(s) will be adopted by each IMPACT Partner.

5.1 General Approach

This approach is a simple and general method, which forms the basis for more complex methods.

For example, let R equal the response of interest, and x, y, and z the variables upon which R depends, then R can be said to be a function of x, y and z or

\[ R = R(x,y,z) \]

Where the input variables and their uncertainties are independent, the uncertainty (denoted by \( \text{unc} \)) of R is related to the uncertainty of the input variables by the following general equation for the propagation of uncertainty:

\[
R_{\text{unc}} = \sqrt{\left( \frac{\partial R}{\partial x} \right)^2 x_{\text{unc}}^2 + \left( \frac{\partial R}{\partial y} \right)^2 y_{\text{unc}}^2 + \left( \frac{\partial R}{\partial z} \right)^2 z_{\text{unc}}^2}
\]

The partial derivatives (\( \frac{\partial R}{\partial x} \) for example) reflect the relative importance of each of the input variables on the response variable, whilst the ‘unc’ terms reflect the relative uncertainties in the input variables.
When the partial derivatives are one (i.e. a change in the input variable gives an equivalent change in the response function - for example, the cost of construction of a breakwater equals the sum of the cost of the rock plus the cost of the concrete wave wall plus contractor fees, i.e. \( R = x + y + z \). The general equation for calculating uncertainty simplifies to:

\[
R_{\text{unc}} = (x_{\text{unc}}^2 + y_{\text{unc}}^2 + z_{\text{unc}}^2)^{1/2}
\]

NB: In applying these relationships it is important to have the level of confidence (estimated or calculated) equal for each of the input variable uncertainties. The uncertainty on the response will then be of the same confidence level. Typically the uncertainty will reflect the 90 or 95% confidence levels.

### 5.2 Simulation Approach

The simulation approach involves representing uncertainties by probability distributions. These probability distributions are then combined to provide a probability distribution of the response variable, which incorporates the uncertainties. Where uncertainties are expressed as confidence intervals, as opposed to probability distributions, it is necessary to make an assumption regarding the type of probability distribution to be used in the simulation. If there are many different types of uncertainty, involving many different parameters and variables, this approach can become complex. This is particularly so where there are dependencies between separate parameters and variables. To avoid over complicating the process, it is worthwhile considering the sensitivity of the response variable to each of the parameters, together with the associated uncertainty. If a parameter has a narrow confidence interval (small uncertainty) and has a minor effective on the response it is feasible to consider it as known. Additionally, it may be necessary to consider the different sources of uncertainty as separate elements and structure the analysis to calculate specific uncertainty types before combining these analyses in an overall simulation.

To establish the response variable as a probability distribution some method of integration of the input probability distributions is required. Where the distributions are continuous, often Monte Carlo simulation techniques are used to sample the input probability distributions. This approach avoids analytical integration, which can be complex. There is a range of commercially available software tools and packages that can facilitate this process.

### 5.3 Sensitivity Testing

Sensitivity testing enables the robustness of a decision to be tested. It involves examining a number of scenarios without attaching probabilities to them. Nonetheless, it does enable preliminary exploration of the potential consequences of uncertainty in future performance.

Sensitivity testing can be used to identify by how much key variables can change before a different preferred option is identified. There will then follow some judgement of the likelihood of that change actually taking place. Sensitivity testing usually involves varying each parameter in turn with other parameters at their ‘best estimate’ value. It is
often appropriate to conduct some sensitivity tests before embarking on more thorough probabilistic methods discussed above.

6 CONCLUSIONS

“Uncertainty is a general concept that reflects our lack of sureness about something or someone, ranging from just short of complete sureness to an almost complete lack of conviction about an outcome” (NRC, 2000).

- Consideration of uncertainty provides the decision maker with additional information on which to base a decision. Consideration of uncertainty can therefore lead to different and more justifiable decisions than studies that do not include uncertainty.

- Uncertainty can stem from a variety of different sources. These sources can be generally categorised under two headings:
  - Natural Variability
  - Knowledge Uncertainty

These two categories are known by a variety of different names.

- Uncertainty can be presented or expressed and handled in a variety of different ways. To facilitate incorporating uncertainty within the IMPACT project, the following practices are proposed:

  - Consistent terminology be adopted when considering uncertainty, using the terms and definitions detailed above, for example, clear identification of the source of uncertainty: Natural Variability or Knowledge Uncertainty.

  - Improved articulation of sources of uncertainty should accompany all results derived from national, regional and local studies, as well as data measurement activities.

  - Statistical Model, Statistical Inference and Process Model should all be identified and considered separately.

  - Each partner should identify the sources of uncertainty, appropriately classify them and proposed an approach for combining based on either analysis, simulation of sensitivity testing.

  - HR Wallingford, to develop an uncertainty standard based on partner inputs for adoption in IMPACT. This will explicitly outline the methodology to be adopted for handling/analysing uncertainty within the evidence/model results.
7 REFERENCES


