UNCERTAINTY ANALYSIS OF WATER LEVELS ON LAKE IJSSEL IN THE NETHERLANDS: A DECISION-MAKING ANALYSIS

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Abstract

In this paper, the reliability-based optimal design of the dikes along the Dutch Lake IJssel (1500 km²) is studied.

Introduction

Lake IJssel is situated in the northern part of the Netherlands (Fig. 1). It has an area of approximately 1500 km². The lake is surrounded by dikes in order to protect the low-lying polders from flooding. The required safety against inundation of the polders is 1/4000 yr⁻¹. In Westphal et al. (1997), a physical model has been developed for the water levels of Lake IJssel. It is based on WAQUA (a two-dimensional water flow model) and HISWA (a wave model). The model of Westphal has been analyzed in an uncertainty study by Vrijling et al. (1999). Uncertainties with respect to water level, wind speed, wind surge, wave height, wave steepness, wave run-up, and lake oscillations were taken into account. Two locations along the Lake IJssel were considered in detail: Enkhuizen and Rotterdamsche Hoek (Fig. 1). A short summary of the uncertainty analysis study of Vrijling et al. (1999) will be given in the next section. The goal of this paper is to analyze the influences of the uncertainties in the water levels on the reliability-based optimal design of the dikes at the locations of Enkhuizen and Rott.Hoek. This will be subject in the main part of this paper. The paper will end with conclusions and the list of references.
Uncertainty analysis of the physical model

The probability of $Z < 0$ (overtopping of the Lake IJssel dikes) was calculated by a first order reliability method (FORM) in Vrijling et.al. (1999). Given the uncertainties in water level, wind speed, wind surge, wave height, wave steepness, wave run-up, and lake oscillations, the uncertainties in the probability of overtopping at the two locations were determined.

The results of the uncertainty calculations are summarized graphically in Figs. 2 and 3 for Rotterdamsche Hoek and Enkhuizen respectively. Notice the differences between the required crest heights for the three cases: intrinsic uncertainty, intrinsic + statistical uncertainty, and intrinsic + statistical + model uncertainty. These differences can be up to 1 metre. There has also been made a distinction between additive and multiplicative uncertainty modelling (Vrijling et.al., 1999), but this distinction is not important for the next part of this paper.

![Figure 2: Crest height as a function of the probability of overtopping for Rotterdamsche Hoek.](image1)

![Figure 3: Crest height as a function of the prob. of overtopping for Enkhuizen.](image2)

Notice that the probabilities of overtopping are given by straight lines in the above semi-logarithmic figures. This means that they can be described by exponential distribution functions: $P(K<k) = 1-\exp(-(k-A)/B)$.

When the following notation is adopted:

- $P$ = probability of overtopping [1/yr]
- i.u. = intrinsic uncertainty
- s.u. = statistical uncertainty
- add. = additive model
- mult. = multiplicative model
- m.u. = model uncertainty
- $h$ = required crest height

the following table can be derived:
Table 1. Comparison of the distribution parameters and the required crest heights.

<table>
<thead>
<tr>
<th></th>
<th>A [m]</th>
<th>ΔA [m]</th>
<th>B [m]</th>
<th>ΔB [m]</th>
<th>h for F = 1/4000 [0/1yr] [m]</th>
<th>Δh [m]</th>
<th>P [1/yr] for h = 3.95 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotterdamse Hoek</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δu</td>
<td>2.303</td>
<td>- -</td>
<td>0.198</td>
<td>- -</td>
<td>NAP + 3.95</td>
<td>- -</td>
<td>1/4000</td>
</tr>
<tr>
<td>Δu + s u (add)</td>
<td>2.285</td>
<td>- 0.018</td>
<td>0.260</td>
<td>0.042</td>
<td>NAP + 4.28</td>
<td>0.33</td>
<td>1/1000</td>
</tr>
<tr>
<td>Δu + s u (mult)</td>
<td>2.113</td>
<td>- 0.150</td>
<td>0.257</td>
<td>0.069</td>
<td>NAP + 4.33</td>
<td>0.38</td>
<td>1/1000</td>
</tr>
<tr>
<td>Δu + s u (add) + m.u.</td>
<td>2.492</td>
<td>- 0.039</td>
<td>0.2645</td>
<td>0.065</td>
<td>NAP + 4.71</td>
<td>0.76</td>
<td>1/250</td>
</tr>
<tr>
<td>Δu + s u (mult) + m.u.</td>
<td>2.357</td>
<td>- 0.054</td>
<td>0.2832</td>
<td>0.0852</td>
<td>NAP + 4.70</td>
<td>0.75</td>
<td>1/275</td>
</tr>
<tr>
<td>Enkhuizen</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δu</td>
<td>1.559</td>
<td>- -</td>
<td>0.166</td>
<td>- -</td>
<td>NAP + 2.95</td>
<td>- -</td>
<td>1/4000</td>
</tr>
<tr>
<td>Δu + s u (add)</td>
<td>1.701</td>
<td>0.142</td>
<td>0.194</td>
<td>0.028</td>
<td>NAP + 3.33</td>
<td>0.38</td>
<td>1/625</td>
</tr>
<tr>
<td>Δu + s u (mult)</td>
<td>1.600</td>
<td>0.041</td>
<td>0.183</td>
<td>0.017</td>
<td>NAP + 3.13</td>
<td>0.18</td>
<td>1/1600</td>
</tr>
<tr>
<td>Δu + s u (add) + m.u.</td>
<td>1.782</td>
<td>0.223</td>
<td>0.252</td>
<td>0.036</td>
<td>NAP + 3.89</td>
<td>0.94</td>
<td>1/100</td>
</tr>
<tr>
<td>Δu + s u (mult) + m.u.</td>
<td>1.700</td>
<td>0.141</td>
<td>0.273</td>
<td>0.107</td>
<td>NAP + 3.68</td>
<td>0.73</td>
<td>1/200</td>
</tr>
</tbody>
</table>

The results of this table will serve as input for the analysis of the economic optimal dike height in the next section.

Reliability-based optimal design

In the reliability-based design of hydraulic structures, the idea is to determine the total costs function (Van Gelder et al., 1997). By assuming the exponential distribution (with parameters A and B) for the probabilities of overtopping, we can write:

\[ C_{total} = I_0 + I_1k + S r (1 - F(k)) = I_0 + I_1k + S e^{-\frac{k-A}{B}} \]

The optimal dike height follows from the minimization of the total costs function and can be expressed by the formula:

\[ k_{opt} = A - B \log \left( \frac{I_1Br}{S} \right) \]

and the optimal probability of failure is given by:

\[ p_{opt} = \frac{I_1Br}{S} \]

It is interesting to notice that the optimal probability of failure is independent of the A parameter of the exponential distribution. The ΔA values from table 1 are therefore neglected in the determination of the optimal failure probability. An increase in the
slope of the exponential distribution (i.e. $\Delta B > 0$), results in an increase in $p_{opt}$. From table 1, it was seen that more uncertainty results in a higher $\Delta B$-value. Consequently this leads to a higher optimal probability of failure.

The change in the optimal probability of failure (from $p_{opt}$ to $p_{opt}'$) caused by the increase in uncertainty (from $B$ to $B + \Delta B$) can also be expressed as follows:

$$p_{opt}' = \frac{B + \Delta B}{B} p_{opt}$$

Given an optimal probability of failure of $1/4000$ yr$^{-1}$ for Rott.Hoek, the inclusion of all uncertainties ($\Delta B = 0.0852$) leads to a new optimal probability of failure of $0.2645 \cdot \frac{1}{4000} = \frac{1}{3000}$ yr$^{-1}$. Instead of an economic optimal dike height of 4.70 m, a height of 4.60 m is the result. For the location of Enkhuizen, the proposed approach leads to a decrease in the dike height from 3.89 m to 3.77 m.

**Conclusions**

The reliability-based decision-making procedure which has been applied in this paper can successfully be used in the analysis of the optimal failure probabilities for the dikes along the Lake IJssel. The influence of the uncertainties lead to an increase in the probability of exceedance lines. When the hydraulic boundary conditions are modelled in an exponential way, analytical considerations can be given for the optimal probabilities of failure and the optimal dike heights.

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**References**

