FAILURE MECHANISMS OF SEA DIKES- INVENTORY AND SENSITIVITY ANALYSIS

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This paper aims at probabilistic investigation of various possible failure mechanisms which often experience by coastal flood defence structures under the impact of sea loads. The factors that affect structure performance are usually varied. In order to undertake an effective assessment of overall reliability of the coastal flood defences it is essential to have a thorough knowledge and understanding of all possible failure modes and their contribution to the total failure probability of the whole system. All failure mechanism of these structures often relate with various stochastic variables. Sensitivity analysis is therefore necessary to perform in order to see the importance of these variables for each certain failure mode. Application of the method is made for a case study of coastal sea dikes in Vietnam.

INTRODUCTION

Coastal flood defence structures react differently under the impact of sea loads. The factors that affect structure performance are usually varied. Over recent years considerable effort has been devoted to improving our knowledge of how the sea flood defences fail. However, gaps in knowledge still remain. The appropriate characterisation of failure mechanisms of coastal flood defences is a key component in effective reliability analysis and flood risk assessment. The quantification of these failure mechanisms is facilitated by a number of methods ranging from indicative equations to more physical process-based models. Currently, there is an increasing interest to quantify the reliability of coastal flood defences using probabilistic approach (see e.g. the concept, method and application in Bakker & Vrijling 1980; Vrijling et al., 2001; Oumeraci et al., 2001 and Voortman 2002). The probabilistic method allows designers to take into account uncertainties of the input parameters, treat them as the random variables and aims at determination of the true probability of flooding of a polder. A central concept in probabilistic design approach is the limit state function (LSF). It links the performance target to the processes that lead to failure to fulfil that target. On this basis, the defensive structures are considered to be failed if its performance does not meet its design functional requirements.

In practice, the ways that the structures fail are normally referred to failure modes. The process at which the dikes are failed by a certain failure mode is called failure mechanism. The failure mechanism is described probabilistically by the LSF, which is often based on the critical state of the physical process of the failure. The LSF is therefore represented by a combination between the characteristic strength of the structure, $R$, and the load, $S$, acting on the structure,
\[ Z = R - S \]

in which \( R \) and \( S \) comprise a number of stochastic variables. At a system level, for instance a dike ring, the system can be decomposed by various components. Each of those components can be failed in various failure modes. Main interests are the overall system failure probability, \( P_f^{sys} \), and how much each failure mode contributes to \( P_f^{sys} \). In addition, finding the weakest component of the system is necessary for the design, maintenance and management purposes. Influences of load and strength variables to the system/component failure are necessary to see which loads are important. In this paper, with the uses of probabilistic design approach at level III and level II (CUR/TAW 141, 1990) in combination of fault tree analysis by Monte Carlo simulation, efforts is made to give more appropriated answers for these previous issues with an application to the case study of coastal dike rings in Vietnam.

**PROBABILISTIC APPROACH IN COASTAL FLOOD DEFENCES**

In the low-lying coastal regions the coastal flood defence system is important in protecting hinter lands from flooding. Typically this system contains sea dikes, dunes, estuarine levees, sluices, pumping stations, etc. Sea dikes are usually the most common and important components of the system. In general, design of these coastal structures is still based on a deterministic approach which often not accounts for uncertainties of sea boundary and the resistance. Normally the dike crest exceeds the design water level by some measure, thus the probability of overtopping is smaller than the design frequency. However, as lessons learnt from practices, some parts of the dike may already be critically loaded before the design water level is reached. There are more failure mechanisms that can lead to flooding of the polder than overtopping. Another danger is that dike crossing structures (e.g. sluices, gates, ship logs...) do not function well in time before the high water moment (i.e. see Katrina case in US by W. Kanning et al., 2007; Damrey case in Vietnam by C. Mai Van et al., 2006a, ...). Furthermore the length of a dike ring has a considerable influence e.g. with a multiple section dike system of more than 90km long, the probability of dike failure may increase by factor 3 to 10 (see C. Mai Van et al., 2006b). A chain is as strong as the weakest link. Hence a single weak spot determines the actual safety of an entire dike ring (see Vrijling, J.K., 2001).

Due to the action of sea boundaries and their uncertainties the failure of sea dikes may presented in various mechanisms. Some of the most possible failure mechanisms are: overtopping, instability of slope protected element, sliding of outer and/or inner slope, piping, erosion of outer and/or inner slopes, dike’s toe instability, etc. These failure mechanisms of a dike section can be schematised with a fault-tree as in Figure 1.a. On the other hand these failures of sea dikes can be presented in relation to their functional elements as in Figure 1.b. The failure is considered to occur if the functional component does not fulfill its pre-defined functions.
Failure mechanisms can also be described by a “failure matrix” which shows clearly relations between types of structures, functional components and types of driven loads leading to failures. Simplify version can be derived for coastal flood defence structures as in Figure 2. By filling all failure probabilities of corresponding failure modes in the failure matrix the weakest point/link of the system can be identified.

Quantification of the probability of system failure starts with the definition of reliability functions for all potential failure modes of all system elements. The general form of reliability function can be written by:

$$g(z,X)=R(z,X)-S(z,X)$$  \hspace{1cm} (1)$$

where $R$ is the resistance of the component, $S$ is the loading on the component, $z$ is a vector of design variables describing among others the structural geometry of the component and $X$ is a vector of load of random variables.
If the joint probability density function \( f_{R,S}(R, S) \) of the strength \( R \) and the load \( S \) is known, the probability of failure can be calculated by means of integration:

\[
P_f = P(Z < 0) = \int_{Z < 0} f_{R,S}(R, S) \, dR \, dS
\]

Because \( Z < 0 \) if \( R < S \), if \( R \) and \( S \) are statistically independent, the following applies:

\[
P_f = P(Z < 0) = \int_{Z < 0} f_{R,S}(R, S) \, dR \, dS = \int_{R < S} f_R(R) f_S(S) \, dR \, dS = \int_{R < S} f_R(R) \, dR \int_{S > R} f_S(S) \, dS
\]

(3)

Usually, the strength and the load are functions of one or more random variables. In such a case the reliability function can be rewritten as \( Z = g(X_1, X_2, \ldots, X_n) \). If the variables \( X_1, X_2, \ldots, X_n \) are statistically independent, the equation can be simplified to:

\[
P_f = \int_{Z < 0} f_{X_1}(X_1) f_{X_2}(X_2) \cdots f_{X_n}(X_n) \, dX_1 \, dX_2 \cdots dX_n
\]

(4)

This integral can seldom be determined analytically. The solution is therefore usually calculated with numerical methods. The two well-known of these which usually be used, are numerical integration and solutions based on the Monte Carlo method. In order to perform a level II calculation, the first or second order reliability methods (FORM & SORM, see CUR/TAW 141, 1990). In this paper the level III method is used to calculate the failure probability, level II method is applied for sensitivity analysis of stochastic variables.

The overall failure probability of a system component is then given by combination of failure probability of all considered failure modes:

\[
P_{\text{failure comp}} = P(Z_1 < 0; Z_2 < 0; \ldots; Z_i < 0; \ldots Z_m < 0)
\]

(5)

where \( (Z_1 < 0; Z_2 < 0; \ldots; Z_i < 0; \ldots Z_m < 0) \) denotes at least one of \( m \) failure mechanisms occurs.

The overall system failure probability is determined in a similar way as that of the system component takes in to account the correlation between components. Several methods are available in calculating exactly the system failure probability such as methods of fault-tree analysis with numerical integration and/ or Monte Carlo simulation. For civil engineering practice one may prefer simpler but still reliable way by using fundamental lower and upper bounds. For narrower bounds with better approximation it is suggested to use Ditlevsen bound (see Ditlevsen, 1979).

**PHYSICAL-BASED PROBABILISTIC DESCRIPTION OF FAILURE MODES**

Description of failure mechanisms is an important task in the design and risk analysis of coastal flood defence system. In this section number of possible failure modes for coastal flood defence structures will be analysed and described. The failure mechanisms are discussed in this paper including: Overflowing, excessive wave overtopping, instability of amour protected
elements, macro instability of dike slopes, instability of toe structures and piping.

1. Overflowing of a sea dike: This failure mechanism occurs when the actual water level of the sea exceeds the crest level of the dikes. The dominant load which drives this failure mode are sea maximum possible high sea water levels which often are a combination of high astronomical tides, storm surge level, gust pump, increased water level due to long waves such as seiches, and wave run-up due to short waves acting on slope of the dikes.

   Limit state equation:
   \[ Z = Z_{dc} - Z_{wl} \]  
   where \( Z_{dc} \) = actual dike crest level [m]; and \( Z_{wl} \) = actual high sea water level [m]

   Loading equation: \( Z_{wl} = MHWL + Z_{Surge} + Z_{Seiches} + Z_{gust} + Z_{run-up} \%

   Resistance (strength) equation: Actual dike crest level \( Z_{dc} \) can be obtained from monitoring measurement data or design dike crest level.

2. Excessive wave overtopping at a dike: Sea dikes is considered functional failure when total wave overtopped discharge exceeds the design value in which by excessive wave overtopping, either by reference to the receiving area or by virtue of failing to deliver adequate resistance to hazards.

   Reliability equation: Wave overtopping rate exceeds admissible rate of water behind the sea defence. The “load” is the actual wave overtopping rate; the “strength” is a critical rate which is higher than the limit for inundation. The reliability function is expressed by:
   \[ Z = q_{adm} - q \]  
   where: \( q_{adm} \) = admissible wave overtopping rate [l/m/s]; \( q \) = actual wave overtopping rate [l/m/s]

   Loading equations: Overtopping discharge (Van der Meer et al., 1991):
   \[ \frac{q}{q_{adm}} = 0.038 \left( \frac{0.160 - 0.096 \xi_d}{\xi_d} \right) \exp\left( -5.5 \frac{R}{\sigma_d} \right) \]  
   where:
   \[ q_{adm} = \begin{cases} 
   0.038 \xi_d & \text{for } \xi_d < 2.0 \\
   0.096 - 0.160 \xi_d & \text{for } \xi_d \geq 2.0 
   \end{cases} \]

   Resistance equations: Admissible wave overtopping rate e.g. \( q_{adm} = 10 \text{ l/s/m for well protected inner side or 1 l/s/m for partly-protected inner side. More detail information can be found in CUR 169. \}

   Parameter definitions: \( z_{98} \) is wave run-up height at slope (with 2% probability of exceedence) [m]; \( H_c \) is significant wave height at toe of the dike [m]; \( T_m \) is mean wave period of incident waves [s]; \( \alpha \) is outer slope [°]; \( L_c \) is wave length at toe of dike relating to \( T_m \), in [m]; \( R_c \) is crest freeboard of the dike [m].
(3) Instability of amour unit: Under the attack of waves and current the amour layer can be failure due to instability of protected elements. For coastal structures, e.g. revetment to protect outer parts of sea dikes, this failure mechanism often occur under storm condition which mainly induced by action of waves.

Reliability function:

\[ Z = (H_s/\Delta D)_K (H_s/\Delta D)_S \]  

Loading equation: \((H_s/\Delta D)_K\) is the required stability number which depends upon the boundary condition and the applied formula. For sea dike revetment, this can be determined based on Van der Meer’s and/or Pilarczyk’s formulae.

Van der Meer’s: \[
\frac{H_s}{\Delta D_{h50}} = A + p H_f \left( \frac{0.2}{\sqrt{N}} \right) \eta - 0.5 \]

Pilarczyk’s: \[
\frac{H_s}{\Delta D_{h50}} = \frac{P}{\psi_{u}} \eta - 0.5 \]

Resistance equation: \((H_s/\Delta D)_R\), is design stability number corresponds to the actual/design size of protected elements (e.g. characteristic diameter of protected element or thickness of amour layer).

Parameter definition: \(H_f\) is significant wave height at local condition; \(\varphi\) is stability factor; \(\psi_u\) is system parameter; \(P\) is notional permeability factor; \(N\) is number of waves; \(S\) is damage level; \(A, B, b\) are model factors.

(4) Toe foot instability due to local scour and beach erosion: Erosion of toe of sea dike protection leading to toe failure which may be due to a) localized scour along the toe of the revetment, b) general erosion of the bed or, c) long-term degradation of the bed (see Figure 3).

\[ Z = d_{toe} - d_{scour} \]

\(d_{toe}\): Protected depth of toe, determine from original bed level to the lowest point of the toe structure.

\(d_{scour}\): Total scour depth in front of the toe structure, it includes general erosion, long term degradation of the bed and localized scour along the toe structure.
Loading equations: 
\[ d_{\text{scour}} = d_{\text{gen}} + d_{\text{loc}} \]

Resistance (strength) equations: 
\[ d_{\text{toe}} = Z_0 - Z_{\text{toe}} \]

Parameter definitions: 
- \( d_{\text{gen}} \) is general depth erosion rate, can be determined by using numerical models; 
- \( d_{\text{loc}} \) is localised scour along the toe structure can be determined by method of Sumer & Fredsoe, 2001; 
- \( Z_0 \) is determined from design profile or actual measurement; 
- \( Z_{\text{toe}} \) is the possible lowest point of the toe structure; 
- \( Z_{\text{toe}} \) is level of toe of protection, taking into account the deployment of any falling or launching apron (the lowest point of the toe structure).

(5) Piping: Piping occurs under the dike due to the erosive action of seepage flow which causes the continuous transport of soil particles. The physical based failure mechanism of piping was described in Figure 4, (see also CUR/TAW 1990).

Limit state function: The failure mechanism of piping occurs when two conditions must be satisfied:
- The clay layer under the dike must be ruptured (1)
- Continuous transport of sand must take place (2)

Reliability function 1: The rupture of clay layer occurs when the water pressure caused by high water level is higher than the wet density of the clay layer. So the reliability function that follows from the first condition is:
\[ Z_1 = \rho_c g d - \rho_w g \Delta H \]

Reliability function 2: Based on the Bligh's criterion in the reliability function of piping is:
\[ Z_2 = m \frac{L' c}{c + B} - \Delta H \]

Parameter definitions: 
- \( \rho_c \) is density of the wet clay; \( \rho_w \) is density of water; 
- \( g \) is gravity acceleration; 
- \( d \) is the thickness of the clay layer between bottom of the dike and sand layer; 
- \( \Delta H \) is the difference in water levels between sea side and inland; 
- \( L' = L_1 + L_2 + B + d; \ c = c_B \) (constant depending on soil type, according to Bligh); 
- \( \Delta H \) is the difference in water levels between sea side and inland; 
- \( L' \); \( L_2 \); \( B \) defined as shown in Figure 4; 
- \( m \) is a model factor, taking into account the scatter in empirical observations.

(6) Macro instability of dike’s slopes: Macro instability in outer face of the dike may be initiated by (rapid) draw-down of water level on outer face of the dike, when material properties may altered in time/ space. While macro instability in inner face may be driven by high water level and infiltration over a
long period, or by water penetration into tension cracks at crest or into surface of inner face.

**Reliability equation:** The reliability function is expressed by:

\[ Z = \frac{\sum R_M}{\sum S_M} \]  \hspace{1cm} (15)

where: \( \sum R_M \) = sum of resisting moments of single segments [kNm]; \( \sum S_M \) = sum of driving moments of single segments [kNm]. These terms can be determined by applying Bishop method (see Bishop, 1955).

By applying similar way, other limit state function can be build up for various different failure mechanisms.

**RELIABILITY BASED SENSITIVITY ANALYSIS**

**Brief Introduction Of Study Case And Input Data**

A selected case study is Nam Dinh sea dikes, which belong to Nam Dinh coastal flood defence system in Namdinh province, Vietnam. The Nam Dinh sea defence system includes around 90 kilometres of sea dikes and 36 dike crossing structures i.e. sluices and pumping stations. Recently due to Damrey Typhoon 2005 several sections of Namdinh dikes were breached (C.Mai Van et al., 2006a). In attempt of rehabilitation of the sea dike system a new design cross section was approved and Nam Dinh is selected as a pilot location. This study will focus on analysis of possible failure mechanisms of the new sea dike section which is actually constructed in Nam Dinh.

**Possible Failure Modes And Failure Probability**

Follows the method given above, the reliability of Namdinh sea dike system is conducted from reliability analysis. All possible failure mechanisms in previous section will be analysed for Nam Dinh case based on given above limit state functions. Using level III method Monte Carlo simulation the failure probability of possible failure modes are tabulated in Table 1, third column.

**Fault Tree Analysis**

**Basic concept of fault trees analysis:** A fault tree is formed of events often described by Boolean variables (the event occurs or not) and related by logical functions, popularly OR and AND. Graphically, these logical functions are represented by Boolean “gates”. The output of a gate (event represented immediately above the gate) is exactly equal to the Boolean function of the inputs of the gate (events represented immediately below). Each input event can be the result of a logical function of a set of events down to the point where all inputs are primary events that cannot be practically analysed any further.

Actually two methods often use in fault trees analysis are numerical probability calculations and Monte Carlo Simulations. The earlier uses a method of direct computation from the probabilities of the primary events. The method depends upon the logically reduced tree, but the precision of the result does not. This relies on the number of terms calculated in the expression for the top level probability. The complexity of this calculation arises because
the same primary event may occur in several places in a fault tree or, in other words, the component probabilities in the tree are not independent.

In this paper Monte Carlo simulation method is used in fault tree analysis. The principle behind the Monte Carlo method is to simulate occurrences of the primary events (component failures), using a random number generator. For each trial, each primary event is simulated by generating a random real number in the range 0 to 1 inclusive. If this number is less than or equal to the probability of the primary event, the event is deemed to have occurred and its value is set to TRUE. Otherwise it is deemed not to have occurred and its value is set to FALSE. The fault tree is then evaluated with these values for the primary events to see if the top event occurs (system failure). The number of top event occurrences is stored, together with the corresponding failure mode (the list of primary events which occurred to cause the top event). The data is then used to obtain both the top-level probability and the probabilities of individual cut sets. Further details on principal knowledge on fault trees can be found in Modarres (1993), Stewart & Melchers (1997) and Barlow (1998).

Application of fault-trees analysis: Given information of failure modes and its mechanisms the fault trees can be constructed for Nam Dinh sea dikes is shown in Figure 5. The primary event is drawn by a circle associated with its failure probability. Top event and intermediate top event are described by rectangular shape. Gate “AND” and “OR” is used to link between related events.

![Figure 5. Fault trees and assigning failure probabilities of basic events](image)

Using Monte Carlo method with $10^7$ simulations provided results as summary in Table 1, two last columns.

<table>
<thead>
<tr>
<th>F.M Description</th>
<th>Failure probability [year⁻¹]</th>
<th>Absolute contribution [year⁻¹]</th>
<th>Important [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>a Overflowing</td>
<td>3.81E-03</td>
<td>3.81E-03</td>
<td>12.80%</td>
</tr>
<tr>
<td>b Excessive wave overtopping</td>
<td>1.30E-02</td>
<td>1.30E-02</td>
<td>43.69%</td>
</tr>
</tbody>
</table>
As can be seen from the results: Number of failure of each primary event and its corresponding contribution to the total failure probability was well indicated. Within $10^7$ simulations the primary event label a, overtopping mode, occurs 224483 times and this mode contribute the most (43.69%) to the failure of the top event. Instability of armour layer (event label c) also happens relatively high which has 21% of the influences. Overflowing and geotechnical instability of inner slope contribute less but considerable amount. Piping, geotechnical instability of outer slope and toe foot instability modes are likely not to occur ($5.91 \times 10^{-15}$ and $1.9 \times 10^{-4}$ per year respectively). The absolute failure probability of piping and toe instability (intermediate top events) are $5.91 \times 10^{-15}$ and $1.9 \times 10^{-4}$ per year respectively. The overall failure probability of top event, dike section failure, is $2.98 \times 10^{-2}$ per year with lower and upper bound of $-/+4.1 \times 10^{-5}$.

### Sensitivity Analysis Of Stochastic Variables

Sensitivity analysis of related variables is necessary to see which variables contributing the most influent to the failure probability of each individual failure mode. With use of level II method, values of influenced factor, $\alpha_i$, of related stochastic variables of each failure mode are determined. Influences of each stochastic variable to the total variance of reliability function of given failure modes are presented graphically from Figure 6 a-h.
In combination with result from fault trees analysis, it is possible to determine the contribution of main loads to the overall failure probability of a dike section is given an assumption that the occurrence of failure modes is statistically independent. Finding is presented on Figure 7. Wave height seems to be the most dominant loads (41% of contribution relatively) which cause sea dike failure.

**DISCUSSIONS AND RECOMMENDATIONS**

Probabilistic design approach is a powerful tool in reliability analysis of coastal flood defences. This allows us to determine the true probability of the component failures and of the whole system. By introducing simple failure matrix the weakest point/link of the system can be easily identified as the first insight by the component which has the largest failure probability. More insight into how importance of failure mechanisms contributing to the total failure probabilities of every system component and to the overall failure probabilities of the whole system can be found by use of fault tree analysis with Monte-Carlo simulation. Influence of all related stochastic variables to the corresponding failure mechanisms has been archived by sensitivity analysis based on the influence factor $\alpha$ from FORM analysis.
Because of high occurrences of wave overtopping, well protection of upper part of outer slope, dike crest, and inner dike slopes must be provided. Transition between these parts of the dikes should be well treated.

Design wave height and design water level are the most important loading parameters in design of the sea dikes which provide 41% and 38% of influences, respectively, to the total failure probability. Subsequently the design water level contributes of influence. Therefore attention should be paid carefully in determination of these parameters in the sea dike design.

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REFERENCES

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