Long-term discharge prediction for the Turnu Severin station (the Danube) using a linear autoregressive model

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Abstract:
The long-term trends of yearly discharge time series and runoff variability at seven stations along the River Danube are identified. The results of statistical analysis of discharge time series indicate the period around the year 1860 was the driest decade in central and eastern Europe since 1840. In these years, the mean annual air temperature in central Europe was lower by about 1°C compared with the 1990s. It is important to notice that the two driest decades (around 1860s and 1990s) of the instrumental era occurred in very different temperature conditions. The 28–31 years; 20–21 years; 14 years, as well as 4-2, 3-6, and 2-4 years fluctuations of annual discharge in the River Danube were found.

Also, the long-term streamflow prediction based on stochastic modelling methods is treated. Harmonic models and the Box–Jenkins methods were used. The predictions of yearly River Danube discharge time series were made for two decades ahead. From the stochastic models it follows that the annual discharge in the Danube at Turnu Severin station should reach its local maximum within the years 2004–06. The period 2015–19 should be dry. Copyright © 2005 John Wiley & Sons, Ltd.

KEY WORDS long-term discharge prediction; discharge fluctuation; stochastic models; River Danube

INTRODUCTION
One of the most difficult issues of hydrology is how to appreciate the long-term variability of runoff, i.e. how to appreciate future discharges. A detailed comprehensive analysis of the past and an identification of both dry and wet periods followed by a prediction of future dry and wet periods are two possible approaches.

About 40 years ago, Williams (1961) investigated the nature and causes of cyclical changes in hydrological data of the world. He attempted a correlation between hydrological data and sunspot activity with varying success. The most frequently studied cycles in connection with precipitation, temperature and runoff variability are the 11-year (22-year) Hale cycles and the 88-year Gleissberg cycle of solar activity. Another cycle studied in connection with hydrological and climatic data is the 18–6-year cycle lunar–solar tidal period. This period, together with solar cycles, is analysed in detail by Currie (1996). Interesting results were obtained by Charvatova and Strestik (1995, 2004), who employed the inertial motion of the Sun around the barycentre of the Solar System as the base in searching for the possible influence of the Solar System as a whole on climatic processes, especially on changes in the surface air temperature. Charvatova (2000) explained a solar activity cycle of about 2400 years by solar inertial motion. She described a 178-7-year basic cycle of solar motion. Similarly, Esper et al. (2002), Vasiliev and Dergachev (2002), and Liritzis and Fairbridge (2003) showed that
multiannual cycles probably have their origin in motion of the Earth in space. Solanki et al. (2004) report a reconstruction of the sunspot number covering the past 11,400 years. According to their reconstruction, the level of solar activity during the past 70 years is exceptional, and the previous period of equally high activity occurred more than 8,000 years ago. These studies underline the theory of the dependence of climate variability of the Earth on solar activity.

As the series of measured hydrological and meteorological data become longer and easier to access worldwide it is possible to deal with a large amount of complex historical data. For example, Probst and Tardy (1987) and Labat et al. (2004) studied mean annual discharge fluctuations of major rivers distributed around the world. Probst and Tardy (1987) showed that North American and European runoffs fluctuate in opposition, whereas South American and African runoffs present synchronous fluctuations. Kane (1997) predicted the occurrence of droughts in northeast Brazil. He found that the forecast of droughts based on the appearance of El Niño alone would be wrong half the time. Instead, predictions based on significant periodicities (~13 and ~26 years) give reasonably good results. Brazdíl and Tam (1990), Walanus and Soja (1995), Sosedko (1997), Pekarova et al. (2003) and Rao and Hamed (2003) found several different dry and wet periods (2–6, 3–5, 5, 20–21, 29–30 years) in the precipitation, temperature and discharge time series in the whole world.

It is clear that predicting discharge for several years ahead based only on deterministic models does not result in meaningful data. This is why the use of stochastic models proceeding from the stochastic characteristics of the measured discharge time series are required. During the 1990s, rapid progress in long-term time-series modelling was made. This progress was enabled due to the development of several stochastic models of hydrological time series using a random sampling method (the Monte Carlo method), classical time series analysis, spectral analysis, or the Box–Jenkins methodology (van Gelder et al., 2000; Popa and Bosce, 2002; Brockwell and Davis, 2003; Lohre et al., 2003; Rao and Hamed, 2003).

This paper deals with:

1. an analysis of natural fluctuations and long-term trends in annual discharge time series of the River Danube;
2. a stochastic prediction of the River Danube discharge at the Turnu Severin station for the next 20 years using linear autoregressive models.

**ANALYSIS OF THE DANUBE DISCHARGE TIME SERIES VARIABILITY**

*Spatial and temporal runoff development along the River Danube*

We assembled the mean annual runoff series of seven stations along the River Danube (Figure 1) from the Global Runoff Data Center, Koblenz, database with the aim to analyse the long-term variability of the mean discharge.

![Figure 1. Scheme of River Danube basin; location of water gauging stations considered](image-url)
LONG-TERM DISCHARGE PREDICTION

Table I. Basic hydrological characteristics of discharge time series for the Danube River

<table>
<thead>
<tr>
<th>Year of record</th>
<th>Hofkirchen</th>
<th>Achleiten</th>
<th>Kienstock</th>
<th>Bratislava</th>
<th>Nagymaros</th>
<th>Turnu Severin</th>
<th>Ceatal Izmail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area (km²)</td>
<td>47 496</td>
<td>76 650</td>
<td>95 970</td>
<td>131 329</td>
<td>183 533</td>
<td>576 232</td>
<td>807 000</td>
</tr>
<tr>
<td>( Q_a ) (m³ s⁻¹)</td>
<td>639</td>
<td>1423</td>
<td>1821</td>
<td>2033</td>
<td>2341</td>
<td>5609</td>
<td>6419</td>
</tr>
<tr>
<td>Median</td>
<td>629</td>
<td>1407</td>
<td>1817</td>
<td>1999</td>
<td>2335</td>
<td>5498</td>
<td>6355</td>
</tr>
<tr>
<td>( q_a ) (l s⁻¹ km²⁻¹)</td>
<td>13 46</td>
<td>18 56</td>
<td>18 97</td>
<td>15 48</td>
<td>12 75</td>
<td>9 73</td>
<td>7 95</td>
</tr>
<tr>
<td>( c_s )</td>
<td>0 14</td>
<td>0 11</td>
<td>0 23</td>
<td>0 24</td>
<td>0 34</td>
<td>0 61</td>
<td>0 41</td>
</tr>
<tr>
<td>( c_v )</td>
<td>0 20</td>
<td>0 16</td>
<td>0 16</td>
<td>0 17</td>
<td>0 17</td>
<td>0 18</td>
<td>0 18</td>
</tr>
<tr>
<td>( Q_{am,min} )</td>
<td>343</td>
<td>983</td>
<td>1285</td>
<td>1420</td>
<td>1630</td>
<td>3782</td>
<td>4024</td>
</tr>
<tr>
<td>( Q_{am,max} )</td>
<td>925</td>
<td>1869</td>
<td>2525</td>
<td>2856</td>
<td>3331</td>
<td>8265</td>
<td>9370</td>
</tr>
</tbody>
</table>

¹ \( Q_a \): average multiannual discharge; \( q_a \): mean annual specific yield; \( c_s \): coefficient of asymmetry; \( c_v \): coefficient of variation.

annual runoff along the River Danube: (1) Hofkirchen; (2) Achleiten; (3) Kienstock (Krems–Stein till 1970); (4) Bratislava; (5) Nagymaros; (6) Turnu Severin (Orsova till 1970); (7) Ceatal Izmail. The basic statistical characteristics of annual discharge time series for the period 1901–2000 are presented in Table I. Details on monitoring of Danube runoff are given in Svoboda et al. (2000). To detect the non-homogeneities in discharge series we used the AnClim software developed by Stepanek (2003). The homogeneity of discharge series was tested by the Alexandersson standard homogeneity test (Alexandersson, 1986).

The data from the Turnu Severin station are very important for the trend analysis, as they have been collected from a rocky profile since 1840 and we can trust them. The absolute minimum annual discharge in Orsova (Turnu Severin today) was observed in 1863, \( Q_{am\,min} = 3471 \text{ m}^3 \text{ s}^{-1} \), whereas the absolute maximum discharge was in 1915, \( Q_{am\,max} = 8265 \text{ m}^3 \text{ s}^{-1} \) (Figure 2a). The analysis of the Orsova data shows that the wettest decade was 1910–19, and the driest decade was 1857–66, when the absolute maximum and minimum annual discharges were observed respectively. The wet and dry periods, as well as the long-term trends, are easy to show on the plots of the filtered values. We used several (low-pass, band-pass, high-pass) filters for visual identification of the cyclic component. In Figure 2b, the course of filtered discharge values using a Hodrick–Prescott filter with parameter \( \alpha = 10, 50, 500, \text{ and } 5000 \) are presented (Maravall and Del Rio, 2001; Pekarova and Miklanek, 2004b). The wet and dry periods are easy to identify in the graph.

To analyse the possible existence of a long-term trend in the discharge data (at the Bratislava and Turnu Severin stations) we used the CTPA (Change and Trend Problem Analysis) software (Prochazka et al. 2001), which is aimed at detecting point changes in time series. We applied two tests: (1) a test of trend existence and (2) a test of trend appearance. The tests did not reject the null hypotheses (the series fluctuates along its constant mean) at a significance level 0.05 in the Bratislava and Turnu Severin discharge series.

The course of the individual dry and wet periods is similar along the whole river stretch. Figure 2c shows the plot of double 5-year moving averages of the Danube discharge at seven stations. These results indicate that the period around the year 1860 was the driest period in central and eastern Europe since 1840. It is interesting to note that in the period around the year 1860 the mean annual air temperature in the upper Danube basin was lower by about 1 °C compared with the 1990s. Figure 3a plots the yearly temperature series from Budapest, Bratislava, Prague, Wien and Hohenpeissenberg stations smoothed by a Hodrick–Prescott filter. The relative homogeneity of the temperature series for the period 1780–2004 was tested by the procedure suggested by Alexandersson and Moberg (1997) and by a CUSUM test of change in linear regression (with Wien as the stated reference station).

The two driest periods of the instrumental era occurred in different temperature conditions. For the local cold periods in 1874–84 and 1935–45 the annual Danube discharges at Turnu station were above the normal level, whereas during the local warm periods in 1860–70 and 1945–55 the discharges were below the normal level.
A weak negative correlation was found between the annual Danube runoff and the annual surface temperature at the Wien station for the period 1840–2000. According to the CUSUM test, the linear regression between the annual Danube discharge $Q$ and that of surface temperature $T$ at the Wien station in the year 1953 changed from $Q = 10345 - 512T$ in 1841–1953 to $Q = 10800 - 525T$ in 1954–2000 (Figure 3b). A $1^\circ$C temperature growth should have led to an annual discharge decrease of about 500 m$^3$ s$^{-1}$, i.e. about 10%.
From the long-term point of view, this has not happened yet, and we have not calculated any significant decrease in the River Danube’s discharge at the Turnu station for the whole period 1840–2000.

The variability of streamflow results from the global system of oceanic streams, the global circulation of the atmosphere, and the transport of moisture (precipitation). In recent years, many scientists have studied relationships between atmospheric phenomena (e.g. Arctic oscillation (AO), Southern oscillation (SO), Pacific decadal oscillation (PDO) and North Atlantic oscillation (NAO)) and some hydrological and climatic characteristics (e.g. total precipitation, air temperature, discharge, snow and ice cover, flood risk, sea-level series, or coral oxygen isotope records, dendrochronological series). For example, Jevrejeva and Moore (2001; Jevrejeva et al., 2003) studied variability in time series of ice conditions in the Baltic Sea within the context of NAO and AO winter indices using a singular spectrum analysis and wavelet approach. According to these authors, the cross-wavelet power for the time series indicates that the times of largest variance in ice conditions are in excellent agreement with significant power in the AO at 2.2–3.5, 5.7–7.8, and 12–20 year periods; similar patterns are also seen with the SO index (SOI) and Niño-3 sea-surface temperature series. Compagnucci et al. (2000), in order to analyse for other wavelength phenomena and to examine the influence of the El Niño–SO (ENSO) events, employed a wavelet filter for removing the strong annual wave in the Atuel river streamflow data. Anctil and Coulibaly (2003) described the local interannual variability in southern Qu´ebec streamflow based on wavelet analysis, and identified plausible climatic teleconnections that could explain...
these local variations. The span of available observations, 1938–2000, allows the depiction of the variance for periods up to about 12 years. The most striking feature in the 2–3- and 3–6-year bands (the 6–12-year band was dominated by white noise and was not considered further) is a net distinction between the timing of the interannual variability in local western and eastern streamflows that may be linked to the local climatology. Tardif et al. (2003) studied variations in periodicities of the radial growth response of black ash exposed to yearly spring flooding in relation to hydrological fluctuations at Lake Duperquet in northwestern Quebec. They detected periodicities of about 3.5, 3.75, and 7.5 years in all the dendrochronological series, with the 3.75- and 7.5-year components being harmonics of a 15-year periodicity.

The NAO refers to swings in the atmospheric sea-level pressure difference between the Arctic and subtropical Atlantic that are associated with changes in the mean wind speed and direction (Hurrell et al., 2003). Whereas runoff in western and northern Europe increases with positive values of the NAO and AO indices during the period 1901–2000 (Arpe et al., 2000; Turkes and Erlat, 2003; Lapin, 2004; Lindström and Bergström, 2004; Pekarova and Miklanek, 2004a,b; Pekarova and Pekar, 2004), in the middle and lower parts of the Danube basin the annual precipitation totals and runoff decrease with positive NAO values.

The 29-year runoff variability can be clearly identified in the Neva River series. The Neva River drains the territory of the Finnish and Russian lakes, which accumulate large volumes of water and, thus, multianually regulate and smooth the runoff. The driest year on the Neva was the year 1940, with a mean discharge of 1340 m³ s⁻¹; the wettest year was 1924, with a mean discharge of 3670 m³ s⁻¹. The driest 5-year period on the Neva occurred in 1938–42, which is about 7 years earlier than in the Danube basin. We should note that the period 1938–42 was quite wet in the Danube basin. Such analysis supports the hypothesis that dry periods do not occur simultaneously in European rivers, but they are shifted a few years depending on the basin location. The time lag of the dry periods between the Danube and Neva runoff is about 11–12 years (Figure 4).

Spectral analysis of the discharge

From Figure 2 it is clear that the annual Danube runoff fluctuates around the long-term average. To predict the annual discharge for several years ahead, the identification of the cyclical component of the runoff as accurate as possible is necessary (Shmagin and Trizna, 1992).

Significant frequencies \( \lambda_j \) (periods \( T_j = 2\pi/\lambda_j \)) of discharge time series for all Danube stations were identified using the combined periodogram method; see Pekarova (2003; Pekarova et al., 2003). The combined periodograms obtained of the Danube runoff at Achleiten, Bratislava, and Turnu Severin stations are drawn in Figure 5. In the Danube discharge time series the following significant periods were found: about 29–31, 20–21, 14, 5, 4.2, 3.64, and 2.4 years. The significance of the appropriate periods was tested by the Fisher–Whittle test (Pekarova et al., 2003).

The periods of 2.1–2.4, 3.6, 5–6, 7, 10–11, 14, 20–22, and 28–30 years were found in almost all world discharge series (as well as in precipitation and temperature series) analysed within different geographical zones, and they can be considered as the general regularity. The regularity is related to general oceanic and atmospheric circulation, part of which are also the quasi-biennial oscillation, ENSO, AO, PDO and NAO

![Figure 4. Course of filtered annual discharge for the Danube at Turnu Severin and Neva. HP filter for \( \alpha = 400 \)](image)
phenomena. The long-term runoff variability has its own (today unknown), possibly extraterrestrial, origin (Currie, 1996; Charvatova and Strestik, 2004).

RESULTS OF DISCHARGE PREDICTION

When modelling the annual Danube discharge, several linear autoregressive models were used. These models can be described in the following general form:

\[ X_t = A_0 + \sum_{j=1}^{m} [A_j \cos(\lambda_j t) + B_j \sin(\lambda_j t)] + ARIMA + C R + \epsilon_t \quad t = 1, \ldots, N \quad (1) \]

where the second term on the right-hand side is a harmonic component and the third term is a Box–Jenkins component; \( X_t \) is the mean annual discharge, \( \lambda_j \) are the significant frequencies given by the periodogram, \( A_j \) and \( B_j \) \((j = 1, 2, \ldots, m)\) are parameters of the harmonic component \((A_0\) is the mean of the time series), ARIMA is the autoregressive component \((\text{Box–Jenkins model}), C \) is the regressive coefficient of an exogenous impact \( R \) on the discharge \((\text{e.g. NAO}), \) and \( \epsilon_t \) is the stochastic component \((\text{white noise})\).

The following model types were tested successfully (Pekarova, 2003):

1. a model based on harmonic functions \((\text{hidden periods})\);
2. linear Box–Jenkins autoregressive models AR, MA, ARMA, and SARIMA \((\text{Box and Jenkins, 1976})\);
3. a model involving a harmonic \((\text{deterministic})\) component and a Box–Jenkins autoregressive component;
4. a model involving a harmonic \((\text{deterministic})\) component, a Box–Jenkins autoregressive component, and a regressive component modelling the impact of the winter NAO phenomenon.

Model 4 can predict 1 year only. Therefore, we focus on model 3.

**Autoregressive model with the harmonic component for Danube: Turnu Severin**

In order to obtain the long-term annual discharge prediction, we proceed according to the following steps:
1. take the time series of logarithms of annual discharge and centre it;
2. remove the harmonic (deterministic, wavelet) component from the time series using the model PYTHIA (the sum on the right-hand side of Equation (2))—the time series obtained is called the residuals;
3. remove the autoregressive component from the residuals (we found the appropriate Box–Jenkins autoregressive model)—the time series obtained is called the innovations (shocks);
4. test the correctness of the model specification (the autoregressive part of the Box–Jenkins model must be stationary, the moving average part must be invertible, and the innovations must be Gaussian white noise);
5. predict dynamically the annual discharge, to specify the appropriate confidence intervals.

The harmonic component specification. The deterministic (harmonic) component of the model is identified in the form

\[
X_t = A_0 + \sum_{j=1}^{m} \left[ A_j \cos(\lambda_j t) + B_j \sin(\lambda_j t) \right] + \eta_t, \quad t = 1, \ldots, N
\]  

(2)

Here, the parameters \( A_j \) and \( B_j \) \((j = 1, 2, \ldots, m)\) in Equation (2) are estimated by

\[
A_j = \frac{2}{N} \sum_{t=1}^{N} x_t \cos(\lambda_j t)
\]

(3)

\[
B_j = \frac{2}{N} \sum_{t=1}^{N} x_t \sin(\lambda_j t)
\]

(4)

Then, a simple prediction of the stochastic process at time \( z \) can be given as

\[
x_{N+z} = A_0 + \sum_{j=1}^{m} \left\{ A_j \cos \left( \frac{2\pi}{T_j} (N + z) \right) + B_j \sin \left( \frac{2\pi}{T_j} (N + z) \right) \right\}
\]

(5)

On the basis of Equations (2)–(5), a model for predicting the harmonic component of the discharge time series (model PYTHIA) was developed (Pekarova, 2003). Parameters for the model are given in Table II.

We performed verification of the PYTHIA model by calibrating on the period 1840–1980, and simulated values for the period 1981–2000 were compared with the measured discharge. The results obtained by the model depend on the proper cycle length’s estimation. For example, when the period length of 3-64 was estimated with an error of 0-2 years, the error of the prediction at year 10 can be up to 2 years.

After removing the cyclical component from the residuals, we look for the autoregressive component.

The autoregressive component specification. The autoregressive component in residuals is identified by the Box–Jenkins methodology. Before doing this, it is necessary to verify whether the residuals are (at least

<table>
<thead>
<tr>
<th>j</th>
<th>( T(j) ) (years)</th>
<th>( A(j) )</th>
<th>( B(j) )</th>
<th>j</th>
<th>( T(j) ) (years)</th>
<th>( A(j) )</th>
<th>( B(j) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>140</td>
<td>-0.00966</td>
<td>-0.02017</td>
<td>10</td>
<td>12.5</td>
<td>0.01911</td>
<td>0.02576</td>
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<tr>
<td>2</td>
<td>105</td>
<td>0.01722</td>
<td>-0.01637</td>
<td>11</td>
<td>10.7</td>
<td>-0.02496</td>
<td>-0.02411</td>
</tr>
<tr>
<td>3</td>
<td>75</td>
<td>0.01364</td>
<td>-0.00828</td>
<td>12</td>
<td>8</td>
<td>-0.01261</td>
<td>-0.03019</td>
</tr>
<tr>
<td>4</td>
<td>57</td>
<td>0.00404</td>
<td>0.02246</td>
<td>13</td>
<td>4.99</td>
<td>-0.02647</td>
<td>0.04409</td>
</tr>
<tr>
<td>5</td>
<td>47</td>
<td>0.02174</td>
<td>-0.02835</td>
<td>14</td>
<td>3.64</td>
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</tr>
<tr>
<td>6</td>
<td>42</td>
<td>0.04426</td>
<td>0.02443</td>
<td>15</td>
<td>4.3</td>
<td>-0.04078</td>
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</tr>
<tr>
<td>7</td>
<td>31</td>
<td>-0.00653</td>
<td>0.0695</td>
<td>16</td>
<td>2.4</td>
<td>-0.05727</td>
<td>-0.00443</td>
</tr>
<tr>
<td>8</td>
<td>21</td>
<td>-0.04212</td>
<td>-0.05034</td>
<td>17</td>
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</tr>
<tr>
<td>9</td>
<td>14</td>
<td>0.01168</td>
<td>0.04632</td>
<td>18</td>
<td>1.2</td>
<td>-0.01942</td>
<td>-0.01105</td>
</tr>
</tbody>
</table>
weakly) stationary, i.e. whether the methodology is applicable. In order to do this, we use the augmented Dickey–Fuller test. Since the test statistic \((-5.2760)\) is less than the 5% critical value of the test \((-1.9419)\), the hypothesis that the time series considered is non-stationary is rejected at a significance level of 0.05.

The Box–Jenkins model consists of two parts, i.e. that of autoregression and that of moving averages. The partial autocorrelation function (PACF) tells us that for residuals the autoregressive part is at most of the order 30, whereas the autocorrelation function (ACF) gives us that the moving average part is at most of the order 26. Here, the limit of the significance of the coefficients at the 5% level is ±0.158. The values of the autocorrelation function and the partial autocorrelation function are shown in Figure 6.

In order to choose the appropriate model, the following statistics and criteria were taken into account: adjusted $R^2$, sum of squared residuals, Akaike information criterion, stationarity and invertibility of the model. Statistical properties of all possible ARMA models within a given range show that the most appropriate model among them is ARMA(30, 26) with non-zero parameters as given in Table III. The table also contains the statistical characteristics of the model and its parameters.

![Figure 6. Autocorrelation function (ACF) and partial autocorrelation function (PACF) of the residuals (the annual discharge time series after removing the deterministic part of the discharge)](image)

Table III. Statistical properties of the ARMA model and its parameters for the Danube:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard error</th>
<th>t-statistic</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(15)</td>
<td>-0.206551</td>
<td>0.073183</td>
<td>-2.822376</td>
<td>0.0056</td>
</tr>
<tr>
<td>AR(18)</td>
<td>0.242639</td>
<td>0.069500</td>
<td>3.491226</td>
<td>0.0007</td>
</tr>
<tr>
<td>AR(30)</td>
<td>-0.172386</td>
<td>0.064778</td>
<td>-2.661200</td>
<td>0.0088</td>
</tr>
<tr>
<td>MA(4)</td>
<td>0.135307</td>
<td>0.050743</td>
<td>2.666549</td>
<td>0.0087</td>
</tr>
<tr>
<td>MA(5)</td>
<td>-0.149948</td>
<td>0.060415</td>
<td>-2.481961</td>
<td>0.0144</td>
</tr>
<tr>
<td>MA(7)</td>
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<td>0.0000</td>
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<tr>
<td>MA(10)</td>
<td>0.186058</td>
<td>0.052844</td>
<td>3.520883</td>
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<tr>
<td>MA(21)</td>
<td>-0.401109</td>
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<tr>
<td>MA(26)</td>
<td>-0.156082</td>
<td>0.053861</td>
<td>-2.897875</td>
<td>0.0045</td>
</tr>
</tbody>
</table>

$R^2$ 0.374663
Adjusted $R^2$ 0.333658
Standard error of regression 0.097554
Sum of squared residuals 1.161057
Log likelihood 123.663
Mean dependent variance 0.006263
Standard deviation dependent variance 0.119508
Akaike information criterion -1.750585
Schwarz criterion -1.553053
Durbin–Watson statistic 1.968596
Now we verify whether the ARMA component of the model is specified correctly. The moduli of all inverted roots of the characteristic equation of the autoregressive part are less than unity (i.e. all roots lie outside the unit circle); hence, the specified autoregressive process is stationary. Similarly, the module of all inverted roots of the characteristic equation of the moving average part are less than unity; hence, the specified moving average process is invertible.

Finally, we verified that innovations are Gaussian white noise; namely, we show that innovations satisfy the following two properties:

1. they are independent (there is no correlation between any two terms);
2. they are identically distributed from $N(0, \sigma^2)$.

All statistical tests resulted in the conclusion that the model was built correctly.

The model was used in order to predict the annual Danube discharge at Turnu Severin station. The predicted data for 20 years ahead are presented in Figure 7. The dry period around 1990 should be followed by a wet period peaking at around year 2005. At Turnu Severin, the year 2006 should be the wettest.
CONCLUSIONS

The long-term Danube discharge time series and the stochastic prediction analysis at Turnu Severin led to the following results:

- The minimum annual discharge in Orsova was in 1863, i.e. $Q_{amin} = 3471 \text{ m}^3 \text{ s}^{-1}$, and the maximum annual discharge was in 1915, i.e. $Q_{amax} = 8265 \text{ m}^3 \text{ s}^{-1}$. The analysis of the Orsova data shows that the wettest decade was 1910–19, and the driest decade was 1857–66, when the maximum and minimum runoffs were observed respectively.

- The results of the statistical analysis of discharge time series indicate that the period around the year 1860 was the driest period in central and eastern Europe since 1840. It is interesting to note that in the period around the year 1860 the mean annual surface temperature in the upper and middle Danube basin was lower by about 1°C compared with the 1990s. It is important to note that the two driest periods of the instrumental era occurred under different temperature conditions.

- Discharge fluctuations of 28–31, 20–21, 14, 4–2 and 3–6 years were found in River Danube.

- In order to provide long-term predictions, four different models were developed: harmonic models, linear autoregressive Box–Jenkins models, autoregressive models with a harmonic component, combined autoregressive models with a harmonic component and a regressive component expressing the impact of the NAO.

- According to the model results, the dry period around 1990 in the Danube basin should be followed by a wet period peaking at around year 2005 and 2010. For the Danube at Turnu Severin, the year 2006 should be the wettest.

Finally, we should comment on the occurrence of the uncertainty in the results obtained by stochastic models. These results cannot be considered as definitive. The time series has to be completed by new data; it is necessary to analyse not only annual discharge time series, but also those of extremes. It should be profitable to analyse time series observed over shorter time steps (monthly) than those of the annual time step.

ACKNOWLEDGEMENTS

This work was supported by the Science and Technology Assistance Agency (Slovakia) under contract no. APVT-51-006502 and by the Science Granting Agency (Slovakia) under contract nos. VEGA-5056 and VEGA-2032.

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