From Smart Grid to Neighbourhood Acting as a Self-Sufficient Island

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Abstract—We are witnessing an increasing energy demand that goes hand-in-hand with declining conventional energy resources. Fortunately, more and more houses are capable to generate, for multiple hours, a considerable amount of electricity, with renewable energy resources. It is challenging to use this source of renewable energy wisely. Throughout this paper we try to use renewable energy as efficient as possible by shifting loads in time and storing energy to use it at later moments. We consider and propose a mixed-integer nonlinear program for it that is capable to react to load changes. Simulation results confirm the effect of load shifting and storing energy; not to mention, they show that a neighbourhood can act as a self-sufficient island for 24 hours a day.

Index terms— anticipatory system, centralised control, mixed-integer nonlinear programming (MINLP), smart grids, voltage/reactive power control.

I. INTRODUCTION

Due to the scarce/exhausted conventional energy resources and climate changes, there is an increasing pressure on society to come with alternative energy solutions. Photovoltaic systems are winning attention. But there is a down side of generating energy by sun: we cannot access it by night. However, to ease the load on electricity grids and increase consumption of self-generated power, the consumer has the opportunity to shift some of their load in time, to moments where there is enough self-generated energy [1]; or the energy should be somehow stored so that it can be accessed at a later time, when it is needed [2].

There exist multiple projects world-wide that are capable of being self-sufficient for multiple hours. For instance, take the micro-grid in Bronsbergen Holiday Park, near Zutphen, The Netherlands. This micro-grid consists of battery containers and 208 households, where 108 houses have a photovoltaic system installed on their roof. Tests have shown that this grid can react on power outages [3].

The aim of this work is to show how sustainable a neighbourhood needs to be to reach a maximum of hours, every day, where it could provide itself completely, with electricity. While the focus will lie on houses that own a photovoltaic system, the interest will lie on how many of these houses need a variable load and/or storage device to reach that maximum point: where a neighbourhood could be disconnected from the main grid and act as a self-sufficient island.

The result will be obtained by running multiple simulations on a model, which is based on a proposed situation. This model will be substantiated by equations, which are solved by Matlab for different percentages of houses with a variable load and/or storage device. As a result, we can determine when and where the maximum of hours with disconnection can be achieved.

The model used in this paper is based on real-word projects; nevertheless, the model and its effect on the real world is not tested. Therefore, the results can only serve as an indication of the true meaning of the effect of the percentages of houses with a variable load and/or storage device in a neighbourhood.

This paper is organized as follows. Section 2 gives an overview of the current situation of the conventional grid and a micro grid. Section 3 proposes a model for the electricity distribution and consumer usage. In Section 4 our MINLP is formulated. The simulated results are presented in Section 5. Section 6 concludes this paper with further research options.

II. SITUATION

In the Netherlands the conventional grid looks basically like Fig. 1. Here the electricity generation is done by a few power-plants that are mostly located far away from the cities. Because the power plants are not located near the cities, the electricity has to be transmitted through high-voltage transmission lines, where the voltage lies around the 100kV and up to 400kV. Otherwise, you will face a tremendous loss in the grid [4]. Most of the time, just out-side the city, the high-voltage transmission grid goes further as a medium-voltage distribution grid. Here the voltage lies around the 10kV and up to 20kV. In the city we will face our last down conversion of 10-20kV to the well-known low-voltage of 0.4kV.

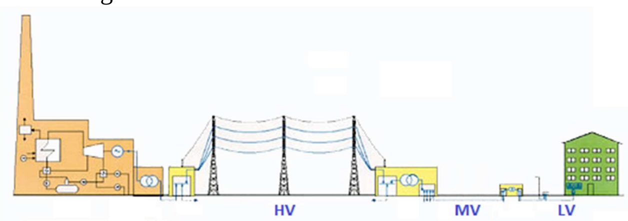

Fig. 1. Sketch of conventional grid.

The conventional power-plant is not the only generator of electricity anymore. Houses become capable of generating their own energy. The best known household generators are the photovoltaic cells and wind-turbines. The last one is mostly used by farmers, but there are many more possible generators. For instance fuel-cells and micro-turbines, like a micro-CHP [5], etc.

The generators that are located in or near the house will be further referred to, in this paper, as distributed generators.

The grid of our focus is a radial grid as displayed in Fig. 2. This proposed grid consists out of 196 houses divided over 10 areas. In our situation we label as main grid the conventional power plant, the high-voltage grid, and a part of the medium-voltage grid.

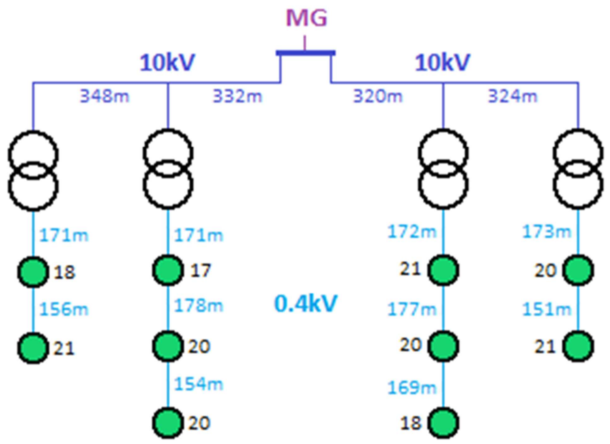

Fig. 2. Proposed grid, where MG stands for the main grid, and the united circles represent the substations MV-LV, i.e. the transformers.

Each household has its own agent in the form of a smart meter. For instance, this smart meter keeps track of when the user wants to turn his dishwasher on. When the smart meter knows this it makes contact with the central coordinator to find the best moment, considered from the perspective of the whole grid, while it still fulfils the users' needs. This architecture is shown in Fig. 3, where in this example a central coordinator controls 6 interconnected households, indirectly.

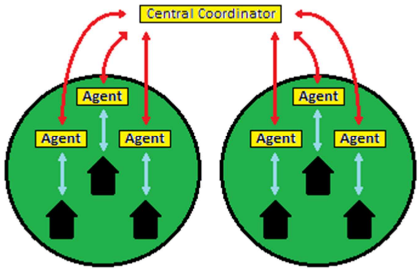

Fig. 3. Centralized control architecture for 6 households divided over 2 areas. The arrow indicate the information exchange between the houses and control units.

III. LOAD FLOW EQUATIONS

A. Electricity grid

The electric power flows are defined as nodal complex power balances that each node in the grid has to obey; otherwise, the proposed grid will be internally inconsistent with his power flows. The complex power balance is formulated as

$$S_i[k] - S_{in}[k] = 0 \tag{1}$$

where $S_i[k]$ represents the injected complex power at node i. In our situation, $S_i[k]$ can be divided in multiple terms. It can be the complex power injected by the main grid, distributed generators, storage devices, variable loads, and fixed loads. The power injected by the loads can be considered as negative terms. This is caused by the fact that the power is consumed instead of supplied. Likewise, the complex power injected by the storage devices can be negative as well. Namely it can store or deliver energy; this depends on the current state of the device.

The complex power flow injected to node i by all connected nodes N_n is denoted as $S_{in}[k]$. Therefore

$$S_{in}[k] = V_i[k]e^{j\theta_i[k]} \sum_{n=1}^{N_n} Y_{in}^* V_n[k] e^{-j\theta_n[k]} \tag{2}$$

where $V[k]$ and $\theta[k]$ represent the magnitude and angle of the voltage. The admittance between the nodes i and n is expressed by Y_{in}; the superscript * states that the complex conjugate has to be taken of Y_{in}.

B. Storage device

We consider basic storage facilities in our grid. In fact, we see it as a black box where we can store electricity, although the storage device can be any kind of different things, as an electric car (it has to be connected to the grid), a Micro-CHP, a freezer, or even a hydrogen tank, etc. To

simplify the equations, we have made two assumptions: its lifetime is endless and so are the charge/discharge cycles.
We will only consider storage devices that can inject/demand real power. Therefore the injected complex power to the grid will be

$$S_s[k] = P_{sd}[k]\eta_{eff} - \frac{P_{sc}[k]}{\eta_{eff}} \quad (3)$$

where $P_{sd}[k]$ and $P_{sc}[k]$ represent the power necessary to discharge and charge the storage device, respectively. We use two variables instead of just one, because of the complications that arise when the efficiency factor η_{eff} of the storage device is not stated as one-hundred per cent, i.e. the function would become discontinuous.
We have divided the power in two variables, one for charging $P_{sc}[k]$, the demand of electricity from the grid, and the other for discharging $P_{sd}[k]$, the supply of electricity to the grid. Where the power flows have the following constraints:

$$0 \leq P_{sd}[k] \leq P_{sdMax} \quad (4)$$

$$0 \leq P_{sc}[k] \leq P_{scMax} \quad (5)$$

where P_{sdMax} and P_{scMax} are the maximum bounds for discharging and charging of the storage device. This will make certain things clear like we do not want the storage device to be filled/emptied in a split second, since this would be quite unrealistic. Similarly, we do not allow a device to discharge and charge at the same time. Hence

$$P_{sd}[k]P_{sc}[k] = 0 \quad (6)$$

Depending on the current capacity level of the storage device we have to decide what is still possible, i.e. the storage device has a limitation on the amount of energy that can be stored. The current stored energy $E_s[k]$ and the power flows $P_{sd}[k]$ and $P_{sc}[k]$ are at every time step, k, restricted as follows:

$$E_{sMax} \geq E_s[k] + P_{sc}[k]\Delta t \quad (7)$$

$$E_{sMin} \leq E_s[k] - P_{sd}[k]\Delta t \quad (8)$$

where E_{sMax} and E_{sMin} denote the maximum and the minimum allowed energy to be stored. Because we are working in discrete time, we have to multiply the power flows with Δt, the time between each time step, to get the energy for the current time step to the next time step.
The relation between the power exchange $P_{sd}[k]$ / $P_{sc}[k]$ and the effectively stored energy $E_s[k+1]$ at time step k+1 is defined by the following equation:

$$E_s[k+1] = (E_s[k] - P_{sd}[k]\Delta t + P_{sc}[k]\Delta t)(1 - \eta_{lkg}) \quad (9)$$

where $1 - \eta_{lkg}$ is the efficiency factor of the stored energy in a storage device, i.e. η_{lkg} is the leakage factor that you lose of energy with the passage of each time step.

C. Variable load

Depending on how sustainable a neighbourhood is, households could have a variable-load. We can classify multiple devices under variable-load. For instance, it could be a washing machine, a dryer or even a dishwasher. This device will be under control of a smart meter, which on its turn is controlled by the electricity companies (in our case the central coordinator). The reason for implementing this smart meter in combination with a specific device is that the electricity companies have the opportunity to flatten out the electricity demands in time. The consequence is that a user has to give away some of his control of that particular device. As a matter of fact, the only thing that is left for the user is that he has to give the boundaries, in time, where the device may and eventually has to operate in. The profit for the user by giving away some of his control is that the variable load can be activated on times when the price of the electricity is low, i.e. the user will be out cheaper in his electricity demands.
We define the complex power that the variable load injects in the grid at time step k as follows:

$$S_v[k] = -(P_v + jQ_v)\sum_{n=0}^{N_L-1} L_v[k-n] \quad (10)$$

where P_v and Q_v denotes the demand of real and reactive power, respectively, and L_v identifies the first time-step where the variable-load is/was activated. The first time-step, L_v, is formulated as: activated $L_v = 1$; still has to be or already was activated $L_v = 0$. How long the device has to be active so that it can complete its task completely is denoted by N_L. An example is shown in Fig. 4.

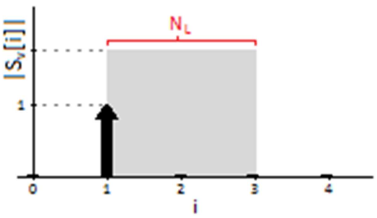

Fig. 4. Example of a variable load. The moment where the variable load is activated is stated with an arrow (in this case it is when i is 1); this is also the only moment where L_v is 1. The rectangular box denotes the energy used by the variable load. Here the operating time where the variable load is active is from i=1 to i=3.

Since we want to make sure that the variable load has completed its task before the last supplied operating time step k_{LN} we introduce the following constraint:

$$\sum_{n=k_{L1}}^{k_{LN}-N_L+1} L_v[n] = 1 \quad (11)$$

where k_{L1} represent the first time-step of the operating range of the device.

Because we want to make sure that the device has completed its task on time, we have to force it $N_L - 1$ time steps before the end. Therefore, in addition to constraint (11), we have to define a constraint for every time step k as

$$\mathbb{I}_{\{k=k_{LN}-N_L+1\}} \leq \sum_{n=k_{L1}}^{k} L_v[n] \quad (12)$$

with

$$\mathbb{I}_{\{k=k_{LN}-N_L+1\}} = \begin{cases} 1, & \text{if k} = \text{k}_{LN} - N_L + 1 \\ 0, & else \end{cases} \quad (13)$$

IV. PROBLEM FORMULATION

As stated in the introduction, we are aiming to come up with a solution where we can disconnect a grid from the main grid as long as possible. To achieve this aim we make use of Tomlab-MINLP (mixed-integer nonlinear programming) [6]. Tomlab is an optimization-based solver that tries to minimize an objective function, F, while satisfying the constraints at each time step.

To reach our goal we will predict the best result of our grid by making use of an *N* period receding horizon; this means that we can anticipate on events in the future. Therefore, at each new time step we make new predictions of actions each household could take depending on the new load profiles.

In our particular case we want to minimize the power taken from the main grid at time step k. Hence, to come up with the best predicted results for time step $k+1$ at time step k, we make use of the objective function

$$F[k] = \sum_{n=k+1}^{k+N} [aF_{CP}[n] + bF_{VL}[n] + cF_{SD}[n]] \quad (14)$$

where F_{CP} is the objective function for the power taken from the main grid; F_{VL} and F_{SD} are denoting the objective function for the variable load and storage device, respectively. The last two functions are introduced to choose the right moment for activating the variable load and storing or delivering energy to the grid. To make clear where our main goal lies we make use of penalty coefficients. These penalty coefficients are denoted by a, b, and c. Since we want to minimize F, the higher a coefficient is, the more important it becomes to fulfil this objective with the best result.

As can be seen from the Section IV, we are in the situation that we need integers to evaluate the constraints of the variable load. Tomlab is handling these integers with a Branch and Bound method [7]. 'Branch and Bound is very reliable but can be extremely slow' [8]. Therefore to lower the simulation time, it is desirable to have a function where we can insert a continuous variable with an outcome that is also continuous, but tends to react as an integer.

A. From integer to continuous

If we highlight a sine function as displayed in Fig. 5 then it can easily be seen that the sine function always

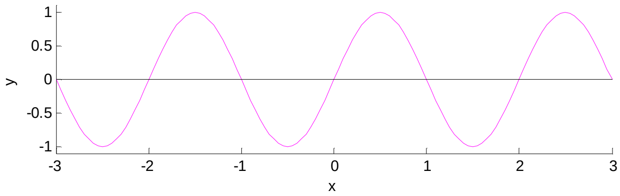

Fig. 5. y=sin(πx)

hits the value zero for the input πi where i is an element of $\mathbb{Z}$ (i.e. i is an integer). But if we take a closer look to this sine function with bounds from 0 to 1, as shown in Fig. 6, then we can manipulate the sine function as follows

$$y = min(sin(\pi x)) \quad (15)$$

where x is an element of $\mathbb{R}$, i.e. x can take up any real value, and so x is continuous. The min stands for the minimization of $sin(\pi x)$, and in our particular case this will result in a zero. And it will only occur when x is a zero or one.

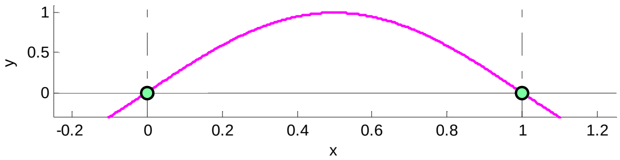

Fig. 6. y=sin(πx), where the dash dotted lines represent the bounds of x, and the circles show the two points where our interest lays.

So if we minimize $sin(\pi x)$ as much as possible, than this will always result in x is 0 or 1. In conclusion we have achieved with (15) that we can go from a continuous variable, with the bounds zero to one, to an integer with the value 0 or 1.

The discussed method above will only be used for the variable loads. Therefore, in addition to (14) we have to introduce, for all households that have a variable load (this amount is stated as N_v), an extra objective function defined as

$$F_{IC}[k] = \sum_{i \in N_v} \sin(\pi L_{v_i}[k]) \quad (16)$$

Hence, we rewrite (14), the objective function $F[k]$, as

$$F[k] = \sum_{n=k}^{k+N-1} [aF_{CP}[n] + bF_{VL}[n] + cF_{SD}[n] + dF_{IC}[n]] \quad (17)$$

B. Main grid

To fulfil our main goal we have to set up an optimization function that tries to reduce the amount of power taken or given to the main grid as much as possible. Since, if the power is reduced to "zero" then we are capable of disconnecting our grid from the main grid.

For convenience we will enclose the main grid by a central power plant. This central power plant will function as a generator or load, depending on the situation. Because we are only interested in the moments were we can disconnect the main grid from our grid, it is unwanted that we inject or demand electricity from this central power plant. Therefore, the individual powers of our power plant are squared in the optimization function F_{CP} as

$$F_{CP}[n] = \left(P_{cp}[n]\right)^2 + \left(Q_{cp}[n]\right)^2 \quad (18)$$

where P_{cp} and Q_{cp} are representing the active and reactive power produced by the central power plant.

C. Reducing risk

There is a saying "a safe dollar is worth more than a risky dollar", and with this expression we will approach our next objective function. Because we can never be one-hundred per cent certain about what a later time step will bring, we want to activate the variable load as quick as possible. Therefore, we introduce $-\sum_{i\in N_v} L_{v_i}[k]\, e^{k-n}$ as a possible objective function. Since *n* is greater than or equal to *k*, the exponent is smaller than or equal to zero. Therefore, this optimization function wants to activate the variable load as soon as possible. But if we have a heap of variable loads, this function could interfere with our main optimization function (18). Therefore, this function has to be bounded such that

$$F_{VL}[n] = -\frac{\sum_{i\in N_v} L_{v_i}[n]}{length(N_v)} e^{\frac{k-n}{\varepsilon}} \quad (19)$$

Now our objective function F_{VL} is bounded as $-1 \leq F_{VL}[n] \leq 0$. The ε restrains the rapidness of activation of variable loads. The total amount of houses with a variable load is formulated as $length(N_v)$. And the higher ε is the less pressure lies on activating the variable load as soon as possible.

To make sure that we can handle unexpected situations, it is useful that the storage devices are all charged up. Therefore, we need an objective function who tries to fill-up the storage devices as much as possible, but does not interfere with our main target. Therefore, it has to avoid situations where the demand of energy is higher than what the distributed generators can produce. Hence, at these moments the storage devices have to stand in for the main grid, and deliver the necessary power, by discharging. To make this possible we have to make sure that the objective function never exceeds a specified value. Therefore, we divide it by the maximum value that can occur. In our case this is the maximum amount of energy that can be stored. Hence, the resulting objective function for the storage devices F_{SD} will be

$$F_{SD}[n] = \frac{\sum_{i\epsilon N_s}\left[E_{sMax_i} - E_{s_i}[n] + P_{sd_i}[n]\Delta t - P_{sc_i}[n]\Delta t\right]}{\sum_{i\epsilon N_s} E_{sMax_i}} \quad (20)$$

V. RESULTS

The simulations are done on our proposed grid in Fig. 2. Next the setup of the simulation is given. Then, the simulation results derived from our centralized approach, in combination with our optimization function, discussed above. What should be taken into consideration is that the optimization problem is non-convex (and a non-concave) function. Therefore, the global optimum cannot be guaranteed when this numerical method is applied. Nevertheless, the simulation results can serve as a reference for the true optimality.

A. Simulation setup

Each household has typical daily profiles, which are represented in Fig. 7. These profiles are based on forecasts of the average weather and electricity demands. However, in true-life there are always forecast errors and the prediction is never 100% certain, but as a first study we assume *N* hour period perfect forecasts. Since we are dealing with a few hundred of houses, we believe that the results are also representative for small forecast errors.

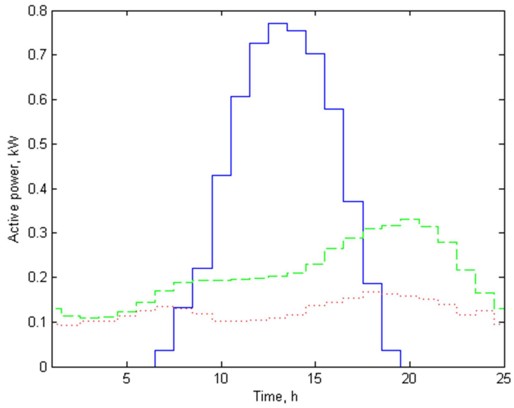

Fig. 7. Average demand and supply of active power for a single household. The blue solid line displays the maximum possible generation of the PV cells; the green dashed line is the average consumption of a household with only fixed-loads, and the red dotted line is when it also has a variable load (a 1 kW dishwasher with a running time of 2 hours [9]) which is not displayed.

The prediction horizon used is 3 hours, where every hour corresponds to 1 time-step.

A common technique in solving these types of electricity network optimization problems is to choose one node as a slack node, i.e. the voltage angle and magnitude are fixed (common values are an angle of 0° and a magnitude of 1 p.u.). However, since we are interested in the moment where we can disconnect the grid completely, and forcing a node to a fixed voltage can be quite expensive, we disregard the use of a slack node. The voltage parameters, including all the other coefficients, are listed in Table 1.

Since it is almost impossible to be 100% certain about what should be the correct initial values as a starting point, the

first 48 hours will be left out. These two days are considered as settlement days.
Because we are working with numerical results, we will count an hour where we are capable in disconnecting our grid from the main grid only when the absolute complex power is lower than 2kVA. The official value should be zero. But out of our optimization point, we will consider this as a valid assumption. Moreover, 2kVA can be easily delivered, e.g. by a small generator.

Table 1. Used parameters. Remark: to improve the readability most parameters are not shown in the right unity; the values of these parameters need to be converted before usage.

Parameter	Value	Unity
Δt	1	h
N	3	h
$V_{Min}(LV)$	0.9	p.u.
$V_{Min}(MV)$	0.6	p.u.
$V_{Max}(LV)$	1.1	p.u.
$V_{Max}(MV)$	1.4	p.u.
$Y_{in}^{-1}(LV)$	0.713 + i0.089	Ω/km
$Y_{in}^{-1}(MV)$	0.229 + i0.078	Ω/km
P_{scMax}	0.33	kW
P_{sdMax}	2	kW
η_{eff}	95	%
E_{sMin}	0	kWh
E_{sMax}	3.4	kWh
η_{lkg}	1	%
N_L	2	h
ε	3	
k_{L1}	1;25;49	h
k_{LN}	24;48;72	h
a	1,000,000	
b	5	
c	5	
d	500	
base value	100	kVA
power factor	82	%

B. Simulation results

All the households in the proposed grid (Fig. 2) have a photovoltaic system installed on their roof. This photovoltaic system is connected with an efficiency of 90% to a controllable inverter. Moreover, if a household also has a storage device than this storage device is connected in combination with the photovoltaic system to the inverter [2].
To show the influence of a neighbourhood with households which possess a variable load and/or storage device we let the amount of houses with a variable load and/or storage device per area vary.

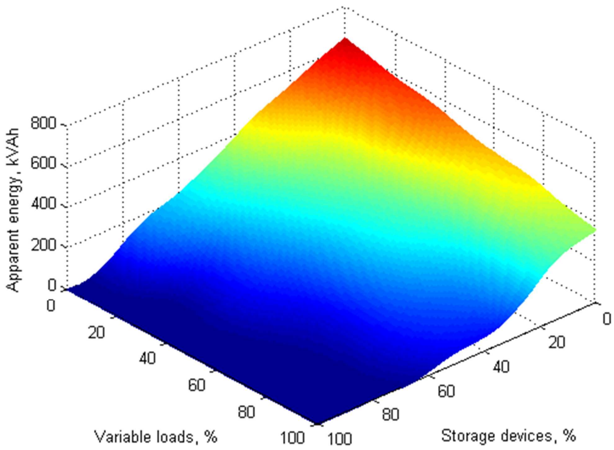

Fig. 8. The apparent energy demand from the main grid.

To let a household vary in the usage of a variable load the demand of power changes as shown in Fig. 7.
The direct influence of the apparent energy demand from the main grid is shown in Fig. 8. Here it can be observed that a 43% reduction of kVAh is achieved by using only a variable load; not to mention, a 100% reduction can be achieved by a neighbourhood where every house only possesses a storage device. Therefore it is quite likely that this neighbourhood could be an island for 24 hours.
The indirect impact of the apparent energy demand from the main grid to the possible hours that the neighbourhood can be self-sufficient is displayed in Fig. 9. Despite the fact that a 43% reduction can be obtained by a neighbourhood with only houses which have a variable load, only a few more hours in disconnection can be achieved. The reason for this lies in the fact that by using only one variable load it is not possible to reach zero kVA (see Fig. 7); while by using a storage device it is possible.

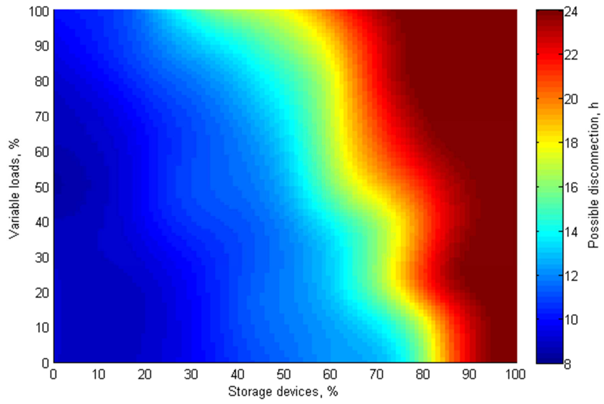

Fig. 9. Possible disconnection, in hours, from the main grid.

VI. FURTHER RESEARCH

Further research should point out what the effect is of a household that owns multiple variable loads instead of only one. After all, we have shown in this paper that we could achieve 43% reduction in apparent energy with households possessing only one variable load; although, there was only a minor increase in hours where the grid could be a self-sufficient island.

Despite the minor increase in disconnection time with one variable load per household, it could still be interesting to have a few houses with a variable load. For instance, due to stress on a storage device, it could be more economically compelling if a grid could act as an island for a few hours instead of the full 24 hours a day.

In addition, further research should indicate the effect of the amount of households in a neighbourhood. While in this paper we have made the assumption that it was licit to apply average load profiles, it does not mean that it is valid to apply these average profiles without any incorporation of possible profile errors. Therefore, further research should point out whether the possibility in forecast errors on 196 houses, in our proposed radial micro-grid, is negligible or if further efforts should be undertaken.

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