Data assimilation in fluid dynamics

Background
Experiments and simulations have complementary strengths and weaknesses. By combining them we want to create powerful flow analysis tools. The goal of the research is to develop practical data assimilation methods for fluid dynamic problems. Since there are uncertainties associated with both worlds, we are motivated to use a statistical framework. To be more specific, we use the Bayesian framework.

The Bayesian framework
Data assimilation is the approach of fusing data (observations) with prior knowledge (e.g., mathematical representations of physical laws; model output) to obtain an estimate of the distributions of the true state \( f \) of a process (Wikle and Berliner, 2007). In the Bayesian framework, beliefs are encoded in probability. The knowledge one has in a state can be separated into a past belief, a.k.a. the prior \( p(f) \), and new observations \( y \). Bayes’ Theorem relates the prior belief to the posterior (updated) belief, i.e. the probability of the true state given the observations. Assuming Gaussian Processes, the updated mean is given by the following expression

\[
E(f | y) = \mu + PH^T(R + HPH^T)^{-1}(y - Hy)
\]

where \( \mu \) is the prior mean, \( P \) the prior covariance matrix, \( H \) the observation operator, and \( R \) the measurement uncertainty matrix.

Experiments
- + true physics
- + cheap topology changes
- - measurement noise
- - limited data

Simulations
- + complete data
- + cheap shape changes
- - discretization error
- - model inadequacy

Vorticity magnitude of a jet in water, measured with tomographic PIV (top) and simulated with CFD (bottom).

Solenoidal filtering of volumetric PIV data
Many flows investigated with volumetric PIV are essentially incompressible. According to the mass conservation equation, their velocity fields should be solenoidal (divergence-free). However, measurement errors will cause spurious divergence. This can be reduced through increased smoothing of the data, but a more physically consistent approach is to include the mass conservation equation in the reconstruction.

A spatial filter, SGPR (solenoidal gaussian process regression), was developed to this end. It is based on the notion that a solenoidal field can be obtained by taking the curl of a vector potential. This information is included by defining the state vector to contain the vector potential components and their first partial derivatives. The prior covariance matrix is defined accordingly (Constantinides an Anisescu, 2013) and the observation operator such that it produces the curl of the vector potential. For the efficient solution of the problem for large data sets on (almost) regular grids, we exploit the multilevel Toeplitz structure of the gain matrix \( R^{-1}HPH^T \).

Spatio-temporal filtering of time-resolved volumetric PIV data
Though solenoidal filters improve the spatial coherence of the measured flows, they will not necessarily improve the temporal coherence as well. This is because the mass conservation equation for incompressible flows does not contain the temporal derivative. Temporal physical information can be included by adding the vorticity transport equation, which can be obtained by taking the curl of the Navier-Stokes equations.

A spatio-temporal filter, VFT (vorticity transport filter), was developed to this end. It extends SWR, the solenoidal filter introduced by Schiavazzi et al. (2014), by including the vorticity transport equation. This results in a non-linear system of equations, which is solved in a least-squares sense. The advantages of VFT are:
1. Localized features are conserved
2. Approximately half of the noise is eliminated
3. Boundary conditions like solid walls can easily be included
4. The solenoidal waveforms are the smallest possible compatible with the measurement grid, so large local errors are limited to propagate to velocities at neighboring grid points
5. The reconstructed field is divergence-free from its construction
6. The compact support of the waveforms allows significant memory and computational savings

PhD Candidate: Iliass Azijli
Department: AWEP
Section: Aerodynamics
Supervisor: R.P. Dwight
Promoter: H. Bijl
Start date: 01-01-2012
Funding: TNO
Cooperations: TNO & IIT Madras

Aerospace Engineering

TU Delft
Delft University of Technology

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Progress and Objectives
- Two physics-based filters were developed: the solenoidal filter SGPR and the vorticity transport equation based solenoidal filter VFT. Compared to existing methods, they have shown to improve the reconstruction of measured velocity and derived fields.
- VFT will be formulated from a Bayesian perspective to allow a natural inclusion of measurement uncertainty
- The usefulness of SGPR will be expanded by propagating its confidence intervals through the Navier-Stokes equations. This will enable us to get uncertainties for pressures.

Publications