The work of professor Jan van Noortwijk (1961–2008): an overview

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Abstract. We give an overview of the research and publications by professor Jan van Noortwijk starting from his graduation at the Delft University of Technology in 1989 up to his death on September 16, 2008. The goal of this overview is to list all of his scientific publications and to put these in a historical perspective. We show how his Ph.D. thesis was a stepping stone to the two primary fields in which he did most of his later work: maintenance optimization and the management of risks due to flooding.

1 THE FORMATIVE YEARS: 1988 TO 1995

In 1988 Jan was an undergraduate student at the Delft University of Technology. At that time, he was majoring in applied mathematics at the faculty of Mathematics and Computer Science and working on his Master's thesis under the supervision of Roger Cooke. Rommert Dekker, now a professor at the Erasmus University in Rotterdam but at that time working in the department of Mathematics and Systems Engineering at the research laboratory of Royal Dutch/Shell in Amsterdam, approached Roger Cooke with a problem they were having with a decision support system for maintenance optimization called PROMPT-II [1].

The PROMPT system was designed for optimal opportunity-based preventive maintenance. One problem was that the system used lifetime distributions requiring an amount of data which was unavailable at that time. Their attempts at elicitation of this data using expert opinion among their engineers resulted in many inconsistencies between estimates. During his internship at Royal Dutch/Shell, where he was supervised by Rommert Dekker and Thomas Mazzuchi, Jan van Noortwijk developed methods to elicit expert opinion on reliability data in a structured manner and to combine these estimates into a consensus distribution for the lifetime of a component. This research resulted in his Master's thesis [2] with which he graduated from the university in 1989. It also resulted in his first and most cited publication in a scientific journal [3]. Another student of Roger Cooke,
René van Dorp, continued Jan's work at Shell and implemented the elicitation procedure suggested by Jan. He also developed feedback for the elicitation procedure, which included feedback for evaluating the optimal maintenance interval given the elicited lifetime distribution [4].

Both Roger Cooke and Rommert Dekker suggested to Jan that he should pursue a doctoral degree at the university, but Jan went to work for the Dr. Neherlab in Leidschendam, which was the research laboratory of the Dutch national telecommunications company. During the short period that he worked there (up to August 1990), he co-authored one conference paper [5]. In September 1990, Jan returned to the university in Delft and became a graduate student, initially with professor Freek Lootsma in the Operations Research chair, but later with Roger Cooke whom became a professor in the Risk Analysis and Decision Theory chair. Around this time, Matthijs Kok of hydraulic structures. After a meeting with Roger Cooke and Jan van Noortwijk, Matthijs appointed Jan as a contractor. Jan held his position in the university until June 1995 and obtained his doctoral degree on the 28th of May in 1996 with his thesis Optimal maintenance decisions for hydraulic structures under isotropic deterioration; see [6] and Figure 1.

![Figure 1](https://example.com/image1.png)

**Figure 1.** The front cover of Jan van Noortwijk's Ph.D. thesis also known as the 'little yellow book' due to the bright yellow color of the cover.

The contract work for Delft Hydraulics provided a unique opportunity for Jan to work on real life problems and almost every chapter from his thesis was later published in a scientific journal. The four primary applications discussed in his thesis are: optimal sand nourishment decisions for the Dutch coastline [7], optimal maintenance decisions for dykes [8], for berm breakwaters [9], and for the sea-bed protection of the Eastern-Scheldt storm surge barrier [10]. The problem of optimally inspecting the block-mats of the Eastern-Scheldt barrier was also the topic of a chapter in a book published in conjunction with a workshop which was organized to celebrate his Ph.D. thesis; see [11]. These mats prevent possible instability of the piers in the barrier due to erosion and must be inspected periodically to check for the presence of scour holes. Jan proposed a Poisson process for the random occurrence of these scour holes and a gamma process for the stochastic expansion of the size of the holes once they have appeared.

From a theoretical point of view, his most important contribution by his use of the gamma process to model uncertain deterioration over time. His motivation for this was not only the fact that the increments of this particular stochastic process are non-negative, which makes the process of deterioration monotonically increasing, but also that it could be characterized by the only (subjective) information which is commonly available, namely the limiting average rate of deterioration. This feature makes the gamma process fit within the operational Bayesian approach advocated by Max Mendel and Richard Barlow; see [12] and [13]. The basic thought behind this approach is that any model should be designed such that prior information need only be given over parameters with an operational meaning. Jan visited Max and Dick as a visiting scholar at the University of California at Berkeley in 1992 and this ultimately gave direction to the mathematical aspects of his research [14]. These aspects are the topics of the second and third chapter in Jan's Ph.D. thesis.

In the second chapter of his Ph.D. thesis, Jan discusses a Bayesian isotropic failure model which is based on two assumptions: (1) the order in which the increments appear is irrelevant (i.e., they are exchangeable) and (2) given the average amount of deterioration per unit time, the decision maker is indifferent to the way this average is obtained (i.e., the amounts of deterioration are $\ell_1$-isotropic, which implies exchangeability). The latter may also be stated as follows: all combinations leading to the same average have the same degree of belief for the decision-maker. This chapter was later published in the European Journal of Operations Research [15]. Note that the assumption of $\ell_1$-isotropic deterioration implies that the expected amount of deterioration is linear in time. The third chapter in his Ph.D. thesis characterizes the general gamma process in terms of sufficiency and isotropy. This work was done together with Jolanta Misiewicz from the University of Zielona Góra in Poland, which was later published in the Journal of Mathematical Sciences [16]. The ninth and last chapter of his thesis contains the results of a follow-up on the research he did for his M.Sc. thesis and which was reported in his first journal publication [3]. This chapter, which was later published in the Journal of Quality in Maintenance Engineering [17], proposes the use of the Dirichlet distribution as a discrete lifetime distribution, which can be used when experts give estimates of lifetimes in the form of a histogram.
Jan van Noortwijk also wrote several reports for Delft Hydraulics. The first was an inventory of problems for his research [18]. In 1991, Leo Klat­

ter from the Ministry of Transport, Public Works and Water Management asked Matthijs Kok to research how to optimally maintain parts of hydraulic structures which were located below the waterline [19]. The report included an

analysis of the Eastern-Scheldt barrier, which would become one of the real life applications in Jan’s thesis. In [20], Jan used the method of paired comparisons to rank, amongst other variables, the various designs of the bridge now known as the Erasmus bridge in Rotterdam. For each layout, ship pilots (i.e., the experts), were asked whether it would be easier or more
difficult to navigate relative to the other layouts. However, his most im­
portant work for Delft Hydraulics would become the work he and Matthijs
Kok did in 1994 for the Committee Flood Disaster Meuse, also known as “Boertien-II”, which will be discussed in Section 5.

2 THE PROFESSIONAL CAREER: 1995 TO 2008

On September 1, 1995, Hans Hartong, Matthijs Kok and Kees Vermeer founded HKV Lijn in water B.V. (English: HKV Consultants) in the city of Lelystad in the Netherlands. One month later, Jan van Noortwijk joined them as their first employee. From this point on, his work would focus on roughly two areas: maintenance optimization of man-made structures and systems, and the assessment and management of risks related to natural hazards such as coastal and fluvial flooding. This is best expressed by the longstanding relationship with people at two divisions of the Directorate-

General for Public Works and Water Management, namely the Centre for
Public Works and by his position as professor at the university in Delft. Jan’s work in deterioration modeling and maintenance optimization was largely influenced by his work for Leo Klatter and Jaap Bakker at the Centre for Public Works and by his position as professor at the university in Delft. The Centre for Public Works is essentially a knowledge centre for issues regarding the management of important civil infrastructures, such as the national roads, bridges, sluices, storm-surge barriers, etcetera. The two subjects that Jan was most involved with, were the management of road bridges and the maintenance of coating systems on steel structures. He would complete several projects for the Centre, but he was also hired as a contractor for a long period of time during which he spent about one day a week at the offices of the Centre in Utrecht.

3 UNCERTAINTY AND SENSITIVITY ANALYSIS

Around the time that Jan was finishing his Ph.D. thesis and starting his
work at HKV Consultants, he worked on general developments in the theory of uncertainty and sensitivity analysis. In particular, he was involved with the development of a software tool called Uncertainty analysis with Cor­
relations (UNICORN) together with Roger Cooke at the Delft University of Technology. He co-authored several papers related to this tool together with Roger Cooke. In [21, 22] they discuss graphical methods for use in uncertainty and sensitivity analyses. One of these methods is the use of so-called cobweb plots for the visual display of correlated random variables. These were used in their uncertainty analysis of the reliability of dike-ring areas in the Netherlands [23].

4 MAINTENANCE OPTIMIZATION

Jan’s work in deterioration modeling and maintenance optimization was
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4.1 Lifetime-extending maintenance model

Together with Jaap Bakker and Andreas Heutink at the Centre for Public Works and several colleagues at HKV, Jan developed the lifetime-extending maintenance (LEM) model [24] and the inspection-validation model [25, 26]. The LEM model originated from a spreadsheet module for the calculation of the net present value of future expenditures, which was made together with Harry van der Graaf. Given the available information on the rate of deteri­
oration and the uncertainty in the expected lifetime of the object, the LEM model can be used to balance the costs of lifetime-extending maintenance versus complete replacements. It does so by comparing the life-cycle costs of two maintenance policies: one with periodic (imperfect) repairs which extend the lifetime of the object and one with only periodic replacements which bring the object back to an as-good-as-new state. The inspection­
validation module can be used to update the deterioration process in the LEM model, which is based on the gamma process, with information gained by inspection of the state of the object.
4.2 Bridge management and life-cycle costing

Most of his work for the Centre concerned the topic of bridge management. Many of the bridges in the national road network of the Netherlands were built in the late 1960's and early 1970's. As many of these structures require significant maintenance and retrofitting after approximately 40 to 50 years, the Centre expects that a large number of bridges will have to be maintained in the near future and that this would put severe pressure on the already shrinking budget for the management of national infrastructures. The challenge is therefore to prioritize maintenance actions and to communicate the necessity of large-scale repairs to policy makers and to the public. Prioritization should be based on the principle of life-cycle costing (LCC), as current decisions affect future maintenance requirements. In order to apply this principle, it is necessary to have an estimate of the uncertain lifetime of bridges. For this, Jan and Leo Klatter proposed to use a Weibull distribution fitted to observed lifetimes of demolished bridges and censored lifetimes of existing bridges [27, 28] (later published in *Computers & Structures* [29]). The censored observations of the lifetimes were incorporated by using the left-truncated Weibull distribution.

With the information on the estimated lifetimes of bridges and the costs of various types of repairs, they defined a decision-theoretic approach to bridge management in the Netherlands [30, 31, 32, 33] (later published in *Structure and Infrastructure Engineering* [34]). Jan also looked into the application of the Life-Quality Index (LQI) for objectively assessing the increase in the quality of life in the Netherlands as a result of bridge maintenance [35]. Although this approach looked promising, it didn’t really catch on in the bridge management community.

It is through his work for the Centre of Public Works that Jan met professor Dan Frangopol at the *International Conference on Structural Faults and Repairs* held in London in 1999, where they agreed to collaborate on research in the area of maintenance modeling. In particular, they compared the LEM model with the time-dependent reliability models developed by Dan Frangopol and his co-workers; see [36], which was later published in *Probabilistic Engineering Mechanics* [37]. In 2004, they published an invited paper, together with Maarten-Jan Kallen, with a review of probabilistic models for structural performance [38].

Maarten-Jan Kallen started his Ph.D. research under the supervision of Jan van Noortwijk in April 2003. Jan arranged for him to be an employee at HKV Consultants and to be hired as a consultant by the Centre for Public Works. It is a typical example of his ability to connect scientific research with business and it is reminiscent of the collaboration between the university in Delft and Delft Hydraulics during his own Ph.D. research. Before this time, Jan had already supervised Maarten-Jan during his M.Sc. project, which applied the gamma process for modeling the deterioration in pressure vessels used by the oil and gas industry. Companies which operate these types of vessels are increasingly turning to a more probabilistic approach, known as ‘Risk-Based Inspections’ (RBI), for planning their inspections. The main results of his M.Sc. thesis were published in a paper at the ESREL conference in Maastricht, the Netherlands in 2003 [39], which later appeared in a special issue of the journal *Reliability Engineering and System Safety* [40]. Whereas these concerns the updating of the gamma process with information obtained using imperfect inspections, they also presented a paper, which considered multiple failure modes in the maintenance optimization of these pressure vessels, at the joint ESREL and PSAM conference in Berlin in 2004 [41]. This was not done by considering a bivariate deterioration process, but by reformulating the probabilities of preventive and corrective replacements due to at least one of these failure modes. This particular approach assumes that both degradation processes are independent.

The original idea for Maarten-Jan’s Ph.D. project was to apply the gamma process for modeling bridge deterioration, but it soon became clear that insufficient data was available for this purpose. The Centre did have a database with data from visual inspections performed over a period of more than 20 years. It is therefore that the focus of the research shifted to fitting finite-state Markov processes to this data by use of appropriate methods for estimating the rate of transitions between condition states. The results of this research were published in papers at the ESREL conference held in Poland in 2005 [42], the IABMAS conference held in Portugal in 2006 [43], and in a special issue of the *International Journal of Pressure Vessels and Piping* [44] for which the model was reformulated to fit into the context of pressure vessels.

4.3 Sewer system management

Jan’s first Ph.D. student was Hans Korving, who performed his research at HKV and at the section of Sanitary Engineering at the faculty of Civil Engineering and Geosciences of the Delft University of Technology. His supervisor there was prof. François Clement. Hans did his research towards the probabilistic modeling of the hydraulic performance and the management of the operational and structural condition of sewer systems. The overall aim was to include uncertainties of various types when making decision concerning the design, operation and maintenance of sewer systems [45, 46]. They used Bayesian statistics to determine the return period of combined sewer overflow (CSO) volumes, which is information that can be used for the risk-based design of such systems [47, 48]. For the maintenance and reliability modeling of sewer systems, they analysed failure data of sewage pumps assuming a non-homogeneous Poisson process for the occurrence of failures [49]. They also proposed a Bayesian model for updating prior knowledge on the condition state of sewer systems with the results of visual inspections [50]. The work presented in this paper is related to the work that Jan did for his M.Sc. thesis [2]. In the Netherlands, the condition of sewer systems is classified in one of five states according to the provisions
by the European norm NEN-EN-13508-2. If the likelihood of being in one
these states is represented by a multinomial distribution, then the Dirichlet
distribution may be used as a conjugate prior.

4.4 Corrosion modeling
In the publications [39, 40, 41] with Maarten-Jan Kallen, Jan van Noortwijk
considered the thinning of steel walls due to corrosion and the process of
stress-corrosion cracking. Using a gamma process to model the uncertain
rate of thinning and cracking, they proposed a model which is updated
with the results of imperfect (i.e., inaccurate) inspections. At the Centre
for Public Works, Jan also considered problems related to corrosion of steel
structures. Using the LEM model, he compared different strategies for the
maintenance of the coating on the steel doors in the ‘Haringvliet’ storm-
surge barrier [51]. He also co-authored a survey on deterioration models for
maintenance of the coating on the steel doors in the ‘Haringvliet’ storm-
surge barrier [51]. He also co-authored a survey on deterioration models for
corrosion modeling [52] together with Robin Nicolai and his Ph.D. supervi-
sor at the time, Rommert Dekker.

Jan also published a few papers together with another Ph.D. student,
Sebastian Kuniewski, whose research is sponsored by Shell Global Solutions
in Amsterdam. His research is primarily focused on corrosion modeling of
steel pipelines and vessels. In particular, they consider a form of sampling
inspection, which is performed when a complete inspection of the whole
surface of an object is not feasible. The information obtained from this
partial inspection is then used to estimate the distribution of the largest
defects in those areas which were not inspected [53, 54]. In a paper together
with a former M.Sc. student of Jan, Juliana López de la Cruz, they looked at
identifying clusters of pit corrosion in steel [55], based on a method to
assess the goodness-of-fit of a non-homogeneous Poisson point process.

4.5 Gamma processes and renewal theory
Jan van Noortwijk is possibly best known for his work on the use of gamma
processes for the stochastic modeling of deterioration. Starting with his
Ph.D. thesis and ending with a survey of the application of gamma pro-
cesses in maintenance [56] (published in Reliability Engineering and System
Safety after his death in 2008), he published many papers in various subject
areas in which the gamma process was used to model continuous and mono-
tonically increasing processes of deterioration. Some variations included the
combined probability of failure due to wear and randomly occurring shocks
[57] (later published in a special issue of Reliability Engineering and System
Safety [58]) and a bivariate gamma process to model two dependent
deterioration processes [59].

Many of these publications were co-authored by prof. Mahesh Pandey
from the University of Waterloo in Canada. Together with Hans van der
Weide from the Delft University of Technology, he travelled to Canada for
extended periods of time on several occasions, and they were in the process
of writing a book together. Together with Mahesh, Jan published several
papers which were aimed at ‘promoting’ the use of the gamma process in
the area of civil engineering. At three different conferences, they presented
similar papers which highlighted the benefits of using the gamma process:
an IFIP working conference in 2003 [60] (Jan has also written a general
paper on the use of the gamma process for condition-based maintenance
optimization at an earlier IFIP conference [61]), the IABMAS conference in
2004 [62], and the ICOSSAR conference in 2005 [63]. Finally, the contents
of these papers were also published in Structure and Infrastructure Engineer-
ing in 2009 [64].

Another topic Jan worked on together with Hans and Mahesh, is the use
of renewal theory in maintenance and reliability. In particular, they worked
on various forms of monetary discounting for comparing future streams of
expenditures based on their present value [65, 66, 67, 68]. This research fol-
lowed Jan’s work on cost-based criteria for maintenance decisions, in which
he also considered the variance of costs [69] (later published in Reliability
Engineering and System Safety [70]). In most cases, the policy with the low-
est expected costs is chosen, but these papers show that the costs of these
policies have the highest uncertainty (i.e., the largest variance) associated
with them. In [71] (later published in Reliability Engineering and System
Safety [72] and used in [73]).

During his professional career, Jan van Noortwijk became a respected
consultant and researcher in the area of maintenance optimization and re-
liability modeling. His authority in these subject areas is confirmed by his
position as professor at the Delft University of Technology, by his posi-
tion as lecturer at courses organized by the Foundation for Post Graduate
Education in Delft, and the numerous invited papers and articles for jour-
nals and encyclopedia. For the Wiley Encyclopedia of Statistics in Quality
and Reliability, he co-authored two articles: one on models for stochastic
deterioration [74] and one on maintenance optimization [75].

5 FLOOD RISK MANAGEMENT
Jan started his research in flood risk management in 1994 with an uncer-
tainty analysis of strategies to reduce the risk of flooding in the river Meuse.
This research was carried out in a Delft Hydraulics project for the Commit-
tee Flood Disaster Meuse [76]. It became the topic of the eighth chapter in
his Ph.D. thesis and it later also became a chapter in the book The prac-
tice of Bayesian Analysis [77]. The new idea of his approach was to use a
Bayesian approach for the assessment of the uncertainties in the expected
flood damage and the costs of the strategies. The most important uncer-
tainties were the river discharge, the flood damage given the discharge, the
downstream water levels along the Meuse given the discharge, and the costs
and benefits of decisions.

In one of his first projects at HKV, Jan derived the generalised gamma
distribution for modelling the uncertain size of peak discharges in the Rhine
when the uncertainty is imprecise. This work was published in [78]. In a
series of papers, he studied the benefits of improved forecasting methods
for flood management [79]. In the following years, Jan and his research
group at HKV worked on many other topics in flood risk management.

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This particular probability distribution has the advantage of fitting well with the stage-discharge curve being an approximate power law between water level and discharge [79].

In 1996, Jan made a big contribution in a study on the modeling of the roughness of submerged vegetation [80]. A new, analytical, physics-based model of the vertical flow velocity profile and the hydraulic roughness of submerged vegetation was developed. Jan found an analytical solution to the differential equation of the model, which was not known in the literature at that time and which opened a wide range of applications. Another contribution in this area is the calibration of hydraulic models. Calibration of these mathematical models is a time consuming process. This process can be automated by function minimisation with the simplex algorithm. In [81] it is described how Jan, together with two colleagues (Matthijs Duits and Anne Wijbenga), contributed to this problem with an application to one of the Dutch rivers.

The contributions of Jan in the field of flood risk management were remarkable and included an amazing number of topics. In particular, he covered both aspects of risk, namely the probability of occurrence and the consequences of a flood event. In the following sections, an overview of his contributions to both aspects is given.

5.1 The probability of occurrence of a flood

The main contribution of Jan van Noortwijk in flood risk management has been the use of Bayesian statistics. Jan has written nine papers about this topic [79, 82, 83, 84, 85, 86, 87, 88, 89] and he also initiated a common research program between HKV Consultants and the Ministry of Transport, Public Works and Water Management, from 2000 to 2008. Program leader on behalf of the Ministry was mr. Houwne Chibab. This research program resulted in new Bayesian methods and a software program to apply these methods in practice. One of the applications is the assessment of 'design' discharges of rivers, which represent the discharges with a given return period (i.e., the reciprocal of the probability of exceedance). In the classical approach, statistical uncertainties are not taken into account. In the Bayesian approach, the prior distribution represents information about the uncertainty of the statistical parameters, and, using Bayes' theorem, it can be updated with the available data. So, rather than choosing one particular probability distribution a priori, Jan proposed to fit various probability distributions to the observations and to attach weights to these distributions according to how well they fit this data. So-called Bayes factors are used to determine these weights. Another major contribution is his derivation of non-informative Jeffrey's priors for a large number of probability distributions. Data from many rivers (for example, the Rhine and Oder rivers) and results of the Bayesian approach are included in the papers. An important conclusion is that the design discharges increase when taking the statistical uncertainties into account properly [88].
a monotone stochastic process with expected decline being linear of non-linear in time. For a particular unit of time, the increments are distributed according to mixtures of exponentials. In a case study, the maintenance decision model has been applied to the problem of heightening the Dutch ‘Oostmolen’ [1]. In [57, 58], Jan addressed the time dependent reliability of the Den Helder sea defence as stochastic processes of deteriorating resistance and hydraulic load. Recently, Jan also addressed the cost-benefit method of flood protection as a non-stationary control problem, as suggested by mr. Carel Eigenraam of the Central Planning Office. Here, the benefits of a decision are modeled as the present value of expected flood damage. Jan has written two HKV reports about this optimization problem, and also guided the Den Helder sea defence as stochastic processes of deteriorating resistance context. Any omissions or inaccuracies of facts presented in this overview help in putting many of the publications by Jan van Noortwijk in a historical context. Many people have given valuable input during the writing of this overview and for which we are very grateful. In particular, we would like to thank Roger Cooke, Rommert Dekker, René van Dorp, Pieter van Gelder, Jaap Bakker, Hans Korving, Thomas Mazzuchi and Hans van der Weide for their help in putting many of the publications by Jan van Noortwijk in a historical context. Any omissions or inaccuracies of facts presented in this overview are solely due to the authors. Acknowledgments

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On some elicitation procedures for distributions with bounded support – with applications in PERT

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Abstract. The introduction of the Project Evaluation and Review Technique (PERT) dates back to the 1960’s and has found wide application since then in the planning of construction projects. Difficulties with the interpretation of the parameters of the beta distribution led Malcolm et al. [1] to suggest the classical expressions for the PERT mean and variance for activity completion that follow from lower and upper bound estimates a and b and a most likely estimate \( \theta \) thereof. The parameters of the beta distribution are next estimated via the method of moments technique. Despite more recent papers still questioning the PERT mean and variance approach, their use is still prevalent in operations research and industrial engineering text books that discuss these methods. In this paper an overview is presented of some alternative approaches that have been suggested, including a recent approach that allows for a direct model range estimation combined with an indirect elicitation of bound and tail parameters of generalized trapezoidal uniform distributions describing activity uncertainty. Utilizing an illustrative Monte Carlo analysis for the completion time of an 18 node activity network, we shall demonstrate a difference between project completion times that could result when requiring experts to specify a single most likely estimate rather than allowing for a modal range specification.

1 INTRODUCTION

The three parameter triangular distribution \( \text{Triang}(a, \theta, b) \), with lower and upper bounds \( a \) and \( b \) and most likely value \( \theta \), is one of the first distributions on the bounded range proposed back in 1755 by English mathematician Thomas Simpson [2, 3]. It received special attention as late as in the 1960’s, in the context of the PERT (see, e.g., Winston [4]) as an alternative to the four-parameter beta distribution:

\[
\begin{align*}
    f_T(t|a, b; \alpha, \beta) &= \frac{1}{\Gamma(\alpha + \beta)} \frac{(t-a)^{\alpha-1}(b-t)^{\beta-1}}{(b-a)^{\alpha+\beta-1}},
\end{align*}
\]