A FORCE-BASED MODEL TO REPRODUCE
STOP-AND-GO WAVES
IN PEDESTRIAN DYNAMICS

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MOTIVATION
Typically 1D phenomenon
Unlike in traffic there is no extensive database
- Cellular automata: Discrete in space

- First-order models (velocity models):
  - Rules defining the velocity
    \[ x'_n = V(t) \]

- Second-order models (force-based models):
  - Newtonian mechanics: Definition of the acceleration
    \[ x''_n = g(x'_n) + f(x'_n, \Delta x'_n, \Delta x_n, \cdots) \]
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  - Lane formation
  - Stop-and-go waves
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Investigation of stop-and-go waves with force-based models
STABILITY ANALYSIS
- One-dimensional system with closed boundary conditions

- Normal distribution of $N$-agents with zero-speed.
- Position of one agent (e.g. the first) is slightly perturbed

\[ x_0 \leftarrow x_0 + \epsilon. \]
Find out under which conditions can we hope to reproduce stop-and-go waves

How does the perturbation $\epsilon$ behave with time? (decays to zero? increases to some limit?)

Define analytic conditions that guarantees $^1$ reproduction of stop-and-go waves

$^1$in theory at least!
Stability region for a (well-known) force-based model \(^2\)

The generic equation of motion

\[ x''_n = \frac{v_0 - x'_n}{\tau} + \overbrace{f(x'_n, \Delta x'_n, \Delta x_n, \ldots)}^{\text{repulsive term}} \]

can be rewritten as

\[ x''_n = \frac{v'_0 - x'_n}{\tau}, \]

with \( v'_0 = \tau f + v_0 \leq v_0. \)
We are looking for a function $f$ such that by contact the "desired" speed is zero

$$f(0, 0, 0) = -v_0/\tau$$
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$$f(0, 0, 0) = -\frac{v_0}{\tau}$$
MODEL DEFINITION

\[ f(\Delta x_n, x'_n, x'_{n+1}) = -\frac{v_0}{\tau} \log \left( c \cdot R_n + 1 \right), \]  

(1)

with

\[ R_n = r \varepsilon \left( \frac{\Delta x_n}{a_n + a_{n+1}} - 1 \right), \quad c = e - 1. \]  

(2)
\[ f(\Delta x_n, x'_n, x'_{n+1}) = -\frac{v_0}{\tau} \log \left( c \cdot R_n + 1 \right), \quad (1) \]

with

\[ R_n = r \varepsilon \left( \frac{\Delta x_n}{a_n + a_{n+1}} - 1 \right), \quad c = e - 1. \quad (2) \]
The model has two parameters:
- Desired speed
- Velocity dependence of the "space requirement"
RESULT
Speed of pedestrians at different time steps

$t = 300 \text{ s}$

$t = 2000 \text{ s}$
\[ \sigma(x_n') \]

\[ t \text{ (s)} \]

\[ 0 \quad 500 \quad 1000 \quad 1500 \quad 2000 \quad 2500 \quad 3000 \]

The figure shows a graph plotting \( \sigma(x_n') \) against time \( t \) in seconds. The standard deviation of \( x_n' \) increases rapidly initially and then levels off after a certain point.
SIMULATIONS - TRAJECTORIES

2000
1800
1600
1400
1200
1000
0
50
100
150
200

$t$ (s)

$x_n'$
LIMITATIONS OF THE MODEL
As a force-based model with some *intrinsic* problems that are quite difficult to solve in a *satisfactory* manner.
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- Immediate stopping of pedestrians is not instantaneous: Stopping after delay lead to collisions
- Additional components are necessary to “stabilize” the model (e.g. relative velocities, physical forces, collision-avoidance algorithms, …)
■ Maybe a shift to first-order models gives the answer?
■ Models are easy to control
■ See talk by Antoine Tordeux in Session 8A (Friday, 09.50)

*Collision-free first order model for pedestrian dynamics*
THANK YOU FOR YOUR ATTENTION!