Vehicle Trajectory Planning Considering Traffic Signals on Urban Roads

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I. Platoon trajectory planning under fixed traffic signal
Motivation

1) Scalable to multiple intersections with queues;
2) Capture queue discharging, platoon splitting and merging performances;
3) Multi-criteria in the objective function, jointly optimizing fuel efficiency and travel delay of the whole platoon.
Control objectives

1) driving comfort
2) throughput in green phase
3) travel delay of passing vehicles (speeds)
4) fuel consumption of vehicles that stop

Controller Constraints

1) Admissible acceleration: \( a_{\text{min}} \leq a_i(t) \leq a_{\text{max}} \)
2) Limited speed: \( 0 \leq v_i(t) \leq v_{\text{max}} \)
3) No-collision requirements: \( x_i(t) - x_{i+1}(t) \geq v_{i+1}(t)t_{\text{min}} + x_0 + l \)
4) Red phase position constraint: 
   \[
   x_{g_j}(t = g_j) \geq L_j \\
   x_i(g_j \leq t \leq g_j + r_j) \leq L_j \quad i \in \{Q_{j-1}, Q_j\}
   \]

\[
\min_u J = \min_u \int_0^T \left( \sum_{i=1}^{Q_2} a_i^2(t) \right) - \beta_2q - \sum_{i=1}^{Q_2+q} v_i(t) \\
+ \beta_4 \sum_{i=Q_2+q+1}^{N} f(v_i(t), a_i(t)) \right) \, dt
\]

\( u \) -- control input variable;
\( a_i(t) \) -- accelerations at time step \( t \);
\( i \) -- vehicle sequence number;
\( q \) -- throughput in green phase;
\( N \) -- the number of controlled vehicles;
\( i=1 \) to \( i=Q_2 \) -- queues on downstream intersections;
\( i=Q_2+1 \) to \( i=Q_2+q \) -- \( q \) passing vehicles at the most upstream intersection.
Simulation results: Scenario 1

**Scenario design**
- **Scenario 1**: An isolated intersection without downstream queue

**Settings**
- 10 vehicles

**Experiment objectives**
- To test the validity of position constraint during red phase and the flexibility of the control framework under multiple objectives, and to tune the cost weights under eco-driving objective function
Scenario design Settings Experiment objectives

Scenario 2 An isolated intersection with downstream queue 15 vehicles; 4 queueing vehicles To evaluate the effectiveness of downstream queue constraints and the scalability of control system
Simulation results: Scenario 3

I. Fixed-timing
II. Actuated signal
III. Adaptive (ongoing)

(a) Acceleration  (b) Speed  (c) Position  (d) Fuel consumption

<table>
<thead>
<tr>
<th>Scenario design</th>
<th>Settings</th>
<th>Experiment objectives</th>
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<tbody>
<tr>
<td>Scenario 3</td>
<td>10 vehicles; Queue: 2 and 2 separately; 400 m lane length</td>
<td>To examine the application of the control framework on urban corridors and validate possible deceleration maneuvers</td>
</tr>
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</table>
I. Fixed-timing

II. Actuated signal

III. Adaptive (ongoing)

Scenario 1

Optimal position

Scenario 2

IDM position

Scenario 3

Save 18.4509 ml
II. Platoon trajectory planning under actuated traffic signal
Red phase: Position constraint → penalty term
Control approach: Feedforward (open loop) → feedback (closed loop)
Signal control approach: fixed timing → (semi-) actuated signal

Control objectives
- Optimizing multi-criteria of the whole platoon (e.g. driving comfort, travel delay, throughput);
- Guarantee no-collision and safe driving requirements;
- Stop/decelerate before the stop-line during the red phase;
- Computational load.
Controller formulization

- **Control variable**: acceleration $u$
- **State variables**: longitudinal position $x$, speed $v$
- **System dynamics model**: 
  \[
  \frac{dx}{dt} = \frac{d}{dt} \begin{pmatrix} x(t) \\ v(t) \end{pmatrix} = f(x, u)
  \]
- **Initial condition**: $x(0) = x_0$
- **Constraints**: $x(t) \in X, u(t) \in U, t \in [0, t_f]$
- **Cost function**: 
  \[
  \min_{u,q} J(x, u, t, q) = \min_{u,q} \int_0^{t_f} L(x, u, t, q) + G(x(t_f)) dt
  \]

$i$ -- vehicle sequence number
$q$ -- the maximal throughput
$L$ -- running cost
$G$ -- terminal cost
1) Driving comfort term: $u_i^2(t)$

2) Travel delay term: $-v_i(t)$

3) Safe following term: $(v_{i-1}(t) - v_i(t))^2 / \left( x_{i-1}(t) - x_i(t) - l \right)$

4) Desired time gap term: $(x_{i-1}(t) - x_i(t) - v_i(t) t_{min} - s_0 - l)^2$

5) Fuel consumption term: $f_{eco} = b_0 + b_1 v_i(t) + b_2 (v_i(t) - v^2(t))$

6) Virtual vehicle term: $\left( v_{virtual}(t) - v_i(t) \right)^2 / x_{virtual}(t) - x_i(t)$

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Running cost specification

I. Fixed-timing
II. Actuated signal
III. Adaptive (ongoing)
**I. Fixed-timing**

**II. Actuated signal**

**III. Adaptive (ongoing)**

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**Running cost specification**

\[ L(x, u, t, q_j) = \sum_{i=1}^{N} L_i(x, u, t, q_j) \]

**Leading mode:** Leader within the signal cycle  \( i = 1, t \in [0, t_f] \)

1. Driving comfort
2. Travel delay
5. Fuel saving

\[ L_i = \beta_1 u_i^2(t) - \beta_2 v_i(t) + \beta_5 f_{eco}(u_i, v_i) \]

**First-stopping mode:** First-stopping vehicle during the red phase  \( i = q + 1, t \in [g, t_f] \)

1. Driving comfort
2. Travel delay
5. Fuel saving
6. Virtual vehicle

\[ L_i = \beta_1 u_i^2(t) - \beta_2 v_i(t) + \beta_5 f_{eco}(u_i, v_i) + \beta_6 \left( \frac{v_{virtual}(t) - v_i(t)}{x_{virtual}(t) - x_i(t)} \right)^2 \]

**Following mode:** Followers within the signal cycle; First-stopping vehicle during the green phase

5. Others

1. Driving comfort
2. Travel delay
3. Safe following
4. Desired time gap
5. Fuel saving

\[ L_i = \beta_1 u_i^2(t) - \beta_2 v_i(t) + \beta_3 \frac{(v_{i-1}(t) - v_i(t))^2}{x_{i-1}(t) - x_i(t)} + \beta_4 \left( x_{i-1}(t) - x_i(t) - v_i(t) t_{min} - s_0 - l \right)^2 + \beta_5 f_{eco}(u_i, v_i) \]

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**TU Delft**
The control problem is solved based on Pontryagin’s principle.

Define Hamiltonian and introduce co-state $\lambda$:

$$H_i(x,u,\lambda,t,q_j) = L_i(x,u,t,q_j) + \lambda f_i(x,u,t)$$

$$= \beta_1 u_i^2 - \beta_2 v_i + \beta_3 \left(\frac{v_{i-1} - v_i}{x_{i-1} - x_i - l_i}\right)^2 + \beta_4 \left(x_{i-1} - x_i - v_i t_{min} - s_0 - l_i\right)^2 + \beta_5 f_{eco}(u_i,v_i) + \beta_6 \left(\frac{v_{i,virtual} - v_{q_{j+1}}}{x_{i,virtual} - x_{q_{j+1}}}\right)^2$$

$$+ \lambda_i v_i + \lambda_2^i u_i$$

Thus, the optimal control law is: $u_i^* = \begin{cases} \frac{-\lambda_i^j}{2\beta_1} & \lambda_2^i + \beta_3 (c_0 + c_1 v_i + c_2 v_i^2) \geq 0 \\ \lambda_i^j + \beta_3 (c_0 + c_1 v_i + c_2 v_i^2) & \frac{-\lambda_i^j}{2\beta_1} < 0 \end{cases}$
Controller Constraints

- Admissible acceleration is bounded:
  \[ a_{\min} \leq u_i(t) \leq a_{\max} \]

- Speed should be lower than the limit speed but nonnegative:
  \[ 0 \leq v_i(t) \leq v_{\max} \]
Solution approach

- Discretization
- iPMP algorithm
- MPC framework
- Constrain control variables
- Computational time

**Model predictive control closed-loop**

- Anticipate signals: update cost weights ahead of the beginning of the red phase
- Implement actuated signal: adjust the signal settings and update cost weights

**Optimal control open-loop**

- Solve the state (co-state) dynamic equation forward (backward) in time
- Update the co-state

- Implement the solution of the first time step from optimal control
- Update the system state
- Finish the simulation horizon
### Scenario design

#### I. Fixed-timing
- **Position**: $x(t)$
- **Downstream intersection**: 30 s - 30 s - 30 s
- **Upstream intersection**: 30 s - 30 s - 30 s

#### II. Actuated signal
- **Position**: $x(t)$
- **Downstream intersection**: 20 s - 30 s - 20 s - 10 s
- **Upstream intersection**: 30 s - 20 s - 30 s

#### III. Adaptive (ongoing)

### Signal setting and Anticipation time of the red phase

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<th>Signal setting</th>
<th>Anticipation time of the red phase</th>
<th>Objectives</th>
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<td><strong>Baseline scenario</strong> (pre-timed)</td>
<td>No</td>
<td>Test the validity of the red phase (virtual vehicle) term</td>
</tr>
<tr>
<td>Scenario 1</td>
<td>Opposite</td>
<td>Compare and explore the benefits of anticipation</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>Overlapped</td>
<td>Prove the workings of the adjustment in signal settings</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>Actuated</td>
<td>Investigate the workings under the actuated signal plan</td>
</tr>
</tbody>
</table>
Figure. Optimal trajectories of longitudinal position

Simulation results

- green phase: 20 s, 30 s, red phase: 20 s, 30 s
- prediction horizon: 10 s, simulation time step: 1 s
- limit speed: 20 m/s

I. Fixed-timing
II. Actuated signal
III. Adaptive (ongoing)

Baseline scenario (no anticipation)
Scenario 1 (anticipating 10s)
Scenario 2 (anticipating 10s)
Scenario 3 (Actuated)

Opposite
Overlapped

1.52 ml/m
1.50 ml/m
1.44 ml/m
1.40 ml/m
A receding horizon control framework is proposed at signalized intersection, aiming at optimizing throughput, driving comfort, travel delay, fuel consumption and safety.

The red phase is represented by keeping the safe gap with a virtual vehicle standstill at the stop bar during the red duration with certain anticipation time.

Simulation under four scenarios verified the performance of the approach.

- The red phase term with anticipation works better;
- The flexibility is demonstrated (i.e. changes in signal parameters under pre-timed and actuated signal plan)
III. Platoon trajectory planning under adaptive traffic signal
The integrated optimization of traffic signals and vehicle trajectories at full intersections

- Upper layer: optimize signal
- Lower layer: optimize acceleration  ➡️  Phase II
First-stopping vehicles
- optimize acceleration:
  \[ a_1(k_1, b_1) = k_1 t + b_1 \]
  \[ a_2(k_2, b_2) = k_2 t + b_2 \]
- running cost:
  \[ L(x, u, t) = f_{cco}(u(k, b), v(k, b)) \]
- terminal cost: (at the end of red phase)
  \[ G(x(t_f)) \int dt \]
  to stimulate the first-stopping vehicle to reach the stop-line with the maximal speed; under the collision-free position constraints

Followers
- apply car-following model

Update signal plan
- based on enumeration
- the length of green and red time
- signal phase sequence

Simplification

I. Fixed-timing
II. Actuated signal
III. Adaptive (ongoing)
Thank you!

First hEAT Lab Seminar

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