An optimization model of automated taxis in trip assignment under elastic demand for the last mile problem

First Research Seminar of the hEAT Lab

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We consider automated vehicles using as taxis and aim at identifying their role in urban mobility systems.
We study an AT system that provides accessibility to and from train stations. We aim to get a clear insight into the system optimization from the system planner’s point of view.
What is the system we want to study?

Mode choice

AT is better than bike

People choose AT

Traffic congestion generated

Bike is better than AT

People choose bike

Traffic congestion released

Equilibrium
What is the system we want to study?

What we want to know from this model?

Trip assignment
What is the system we want to study?

Key words

- Shared mobility
- Choice modelling
- First/last mile problem

Assumption

- Two travel modes are considered: by AT and by bike.
- Total demand for AT and bike is fixed.
- Travel time of AT is dynamic based on the traffic flow.

Output

- Optimize the fleet size
- Design the ATs’ routes
Model formulation

System setting

Demand node
Train station
Demand node

Objective function: minimize the total cost by AT and by bike

- AT depreciation cost
- Travel time cost
- AT fuel cost
Model formulation

Constraints: choice modelling

Utility

\[ U_{ijt}^{AT} = -r^{AT} \cdot d_{ij} - T_{ijt}^{AT} \cdot a, \quad \forall (i,j) \in K, t \in M \]

\[ U_{ij}^{B} = -t_{ij}^{B} \cdot a, \quad \forall (i,j) \in K \]

Travel time

Probability

\[ P_{ijt}^{AT} = \frac{\exp(U_{ijt}^{AT})}{\exp(U_{ijt}^{AT}) + \exp(U_{ij}^{B})}, \quad \forall (i,j) \in K, t \in M \]

Constraints: dynamic travel time

Travel time

\[ T_{ijt_1} = t_{ij}^{min} \cdot \left(1 + 0.15 \cdot \left(\frac{S_{ijt_1}t_2 + v_{ijt_1}t_2}{cap_{ij}}\right)^4\right), \quad \forall (i,j) \in K, t \in M \]
Solving algorithm

**Customized gradient algorithm**

Initialize each $P = 0.5, \forall (i,j) \in K, t \in M$

- **Calculate the value**
  - $Q$: served demand by AT
  - $U$: utility
  - $T$: travel time

- **Solve the optimization model**
  - $Obj.$: objective function value
  - $S$: traffic flow
  - $F$: fleet size

Stopping criteria?

- **Calculate the partial derivative of each $P$ to $Obj.$ (descent direction)**
- **Calculate the step size**
- **Update each $P$**
  
  $$P = P - \text{step size} \times \text{descent direction}$$
### Case study

**Small case example**

- Demand zones: 5
- Time length: 5 hours
- Requests: 132

<table>
<thead>
<tr>
<th>Zone 1</th>
<th>Zone 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train station</td>
<td>Zone 3</td>
</tr>
<tr>
<td>Zone 4</td>
<td>Zone 5</td>
</tr>
</tbody>
</table>
## Results

### Initial value variation

Price rate: 0.5 €/min

<table>
<thead>
<tr>
<th>Initial value of P</th>
<th>Obj. value (€)</th>
<th>Number of iterations</th>
<th>Fleet size</th>
<th>Total ticket cost (€)</th>
<th>Total travel time (hour)</th>
<th>Share of AT</th>
</tr>
</thead>
<tbody>
<tr>
<td>30%</td>
<td>744.0</td>
<td>5</td>
<td>20</td>
<td>120.0</td>
<td>35.3</td>
<td>36%</td>
</tr>
<tr>
<td>50%</td>
<td>658.5</td>
<td>7</td>
<td>19</td>
<td>167.5</td>
<td>26.7</td>
<td>51%</td>
</tr>
<tr>
<td>70%</td>
<td>695.0</td>
<td>3</td>
<td>23</td>
<td>175.0</td>
<td>24.4</td>
<td>53%</td>
</tr>
</tbody>
</table>
## Results

### Price rate variation

P initial value: 50%

<table>
<thead>
<tr>
<th>Price rate (€/min)</th>
<th>Obj. value (€)</th>
<th>Fleet size</th>
<th>Total ticket cost (€)</th>
<th>Total travel time by AT (hour)</th>
<th>Total travel time by bike (hour)</th>
<th>Satisfied requests by AT</th>
<th>Share of AT</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>658.5</td>
<td>19</td>
<td>167.5</td>
<td>12.5</td>
<td>14.2</td>
<td>67</td>
<td>51%</td>
</tr>
<tr>
<td>0.4</td>
<td>761.0</td>
<td>15</td>
<td>90.0</td>
<td>10.9</td>
<td>32.5</td>
<td>45</td>
<td>34%</td>
</tr>
<tr>
<td>0.3</td>
<td>703.5</td>
<td>18</td>
<td>85.5</td>
<td>14.0</td>
<td>22.5</td>
<td>57</td>
<td>43%</td>
</tr>
</tbody>
</table>
This customized gradient algorithm is able to **find a good solution** to the original non-linear model.

**Initial values of** $P$ **is a critical control parameter** with respect to the final best solution.

**AT service price rate** is a key issue that **affects** the utility of AT and the result of mode split.
Thank you!

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