Effective Modelling of Traffic Dynamics: Classification and Unification

Bo Yang and Christopher Monterola
Complex Systems Group
Institute of High Performance Computing
A*STAR, Singapore
Outline:

• Overview of the traffic system
• A master model and its controlled expansion
  • Two-phase vs. three-phase traffic models
  • Occam's razor
  • Model extensions and tuning
• Microscopic empirical data collection
  • Video-taping of the traffic flow
  • Image processing and machine learning
  • Data processing and averaging
• Empirical verification of traffic theories and models
An overview of the traffic system

- theoretical modeling

A One-Dimensional Driven System

\[ \Delta v_n = v_{n+1} - v_n \]

nearest neighbour, anisotropic non-linear interactions in a dissipative media
An overview of the traffic system

• theoretical modeling

• no symmetry
• non-identical components
• stochasticity and time dependence
An overview of the traffic system

- empirical observations

Kerner et.al. 2002
An overview of the traffic system
• empirical observations


D. Helbing et.al., An Analytical Theory of Traffic Flow, A selection of articles by Dirk Helbing reprinted from The European Physical Journal B
An overview of the traffic system

- empirical observations

<table>
<thead>
<tr>
<th>Flow-Density Diagram</th>
<th>Emergence of the wide moving jams</th>
<th>Congested traffic at an on-ramp bottleneck</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudo-linear relationship when density is low</td>
<td>The congested traffic (&quot;synchronized phase&quot;) can last up to an hour</td>
<td>A wide moving jam passes through the bottleneck unaffected</td>
</tr>
<tr>
<td>Large scattering of the congested flow-density data points</td>
<td>Wide moving jams mostly emerge from the congested traffic</td>
<td>A wide moving jam may induce or suppress congested traffic at the bottleneck</td>
</tr>
<tr>
<td>The &quot;hysteresis effect&quot;</td>
<td>&quot;Pinch effect&quot; and the merging of numerous narrow jams</td>
<td>The region of congested traffic gets smaller with greater bottleneck strength</td>
</tr>
<tr>
<td>Significant flow fluctuation at very large vehicle density</td>
<td></td>
<td>The frequency of the emergence of the moving jams increases with greater bottleneck strength</td>
</tr>
<tr>
<td>Quantitative features: $V_{max}, \rho_d, \rho_c, \rho_j, Q_c, Q_dj$</td>
<td></td>
<td>Downstream front of the congested traffic pinned at the bottleneck</td>
</tr>
</tbody>
</table>

Yang Bo et.al arXiv. 1504.01256
An overview of the traffic system
• microscopic models

\[ a_n (t + T) = cv_n (t + T)^l h_n^{-m} \Delta v_n \]

\[ a_n (t + T) = cv_n h_n^{-2} \Delta v_n \]

\[ a_n (t + T) = \lambda (h_n - c - T_1 v_n) \]

\[ a_n (t + T) = \lambda v_{n+1}^m h_n^{-l} \Delta v_n \]

\[ a_n = \lambda_1 \Delta v_n (t - T_1) + \lambda_2 (v_{n+2} - v_n) \bigg|_{t-T_2} \]

\[ a_n = A \left(1 - \frac{h_0}{h_n}\right) - \frac{Z^2 \left(-\Delta v_n\right)}{2 (h_n - D)} - kZ (v_n - v_{per}) \]

\[ Z(x) = \left(x + |x|\right) / 2 \]

\[ a_n = \frac{1 + b_1 v_n + b_2 h_n + b_3 v_n h_n + b_4 v_{n+1} + b_5 v_n v_{n+1}}{c^0 + c_1 v_n + c_2 h_n + c_3 v_n h_n + c_4 v_{n+1} + c_5 v_n v_{n+1}} \]

B.S. Kerner, Physica A 392 (2013) 5261–5282
An overview of the traffic system

- microscopic models

\[ a_n = a \left( 1 - \left( \frac{v_n}{v_0} \right) ^\delta - \left( \frac{h^* (v_n, \Delta v_n)}{h_n} \right)^2 \right) \]

\[ h^* (v, \Delta v) = s_0 + s_1 \sqrt{\frac{v}{v_0}} + Tv + \frac{v \Delta v}{2\sqrt{ab}} \]

\[ a_n = a_0 (V (h_n) - v_n) \]

\[ a_n = a_0 (V (h_n) - v_n) - \lambda \Delta v_n \Theta (\Delta v_n) e^{-\frac{h_n-h(v_n)}{R}} \]

\[ h (v_n) = d + Tv_n \]

\[ a_n = a_0 (V (h_n) - v_n + g (\Delta v_n)) \]

\[ g (\Delta v_n) = \begin{cases} 
\lambda \Theta (\Delta v_n) \Delta v_n \\
\lambda \Delta v_n \\
\lambda_1 \Theta (\Delta v_n) \Delta v_n + \lambda_2 \Theta (-\Delta v_n) \Delta v_n 
\end{cases} \]
An overview of the traffic system

- microscopic models

\[ a_n = \begin{cases} 
A^{\text{free}} (V^{\text{free}} (h_n) - v_n) + K (v_n, v_{n+1}) \Delta v_n & v_n \geq v_{\text{min}}^{\text{free}}, h_n > h_{\text{jam}} \\
A^{\text{syn}} (V^{\text{syn}} (h_n) - v_n) + K (v_n, v_{n+1}) \Delta v_n & v_n < v_{\text{min}}^{\text{free}}, h_n > h_{\text{jam}} \\
-K^{\text{jam}} v_n & h_n \leq h_{\text{jam}} 
\end{cases} \]

\[ V^{\text{syn}} (h) = \lambda (h - h_{\text{jam}}) \text{ or } \lambda_1 (\tanh (\lambda_2 (h - h_{\text{jam}}))) + \lambda_3 (h - h_{\text{jam}}) \]

\[ a_n (t + \tau) = \begin{cases} 
A (V^{\text{free}} (h_n) - v_n) + K (v_n, v_{n+1}) \Delta v_n & h_n > G \text{ and } h_n > h_{\text{jam}} \\
A \min (V^{\text{syn}} (h_n) - v_n, 0) + K (v_n, v_{n+1}) \Delta v_n & h_n \leq G \text{ and } h_n > h_{\text{jam}} \\
-K^{\text{jam}} v_n & h_n \leq h_{\text{jam}} 
\end{cases} \]

Kerner et.al 2001
An overview of the traffic system

- microscopic models

- How do we properly characterise the differences between two traffic models?
- Is there a standard way of extending an existing traffic model or construction of a new traffic model?
- Is there a standard way in selecting the best traffic model based on the experimental data?
A master model
- a renormalisation-like approach

\[ a_n = \mathcal{F}_{n,\{s_i\}} (\{t_i\}) \]
A master model

- a renormalisation-like approach

\[ a_n = \mathcal{F}_{n,s_i, \{t_i\}} \]

unimportant factors \rightarrow important factors
A master model

• a renormalisation-like approach

\[ a_n = \mathcal{F}_{n, \{s_i\}} (\{t_i\}) \]

unimportant factors \[ \bar{a}_n = \frac{1}{N_0} \sum_{\{s_i\}} \mathcal{F}_{n, \{s_i\}} (\{t_i\}) = f_n (\{t_i\}) \]

important factors
A master model

• a renormalisation-like approach

\[ a_n = \mathcal{F}_{n, \{s_i\}} (\{t_i\}) \]

\[ \bar{a}_n = \frac{1}{N_0} \sum_{\{s_i\}} \mathcal{F}_{n, \{s_i\}} (\{t_i\}) = f_n (\{t_i\}) \]

\[ \bar{a}_n = \frac{1}{N} \sum_k f_k (\{t_i\}) = f_0 (\{t_i\}) \]
A master model
• a renormalisation-like approach

\[ \bar{a}_n = \frac{1}{N} \sum_k f_k (\{t_i\}) = f_0 (\{t_i\}) \]

\[ \{t_i\} = \{h_n, \Delta v_n, v_n\} \]

deterministic, time independent identical drivers

\[ a_n = f_0 (h_n, \Delta v_n, v_n) \]
A master model

• a renormalisation-like approach

\[ a_n = f_{\{n\}}(v_n, v_{n+1}, h_n) \]

stochasticity, inhomogeneity, time dependence, vehicle/driver diversity

\[ a_n = \bar{f}(v_n, v_{n+1}, h_n), \bar{f} = \langle f_{\{n\}} \rangle_{\{n\}} \]

ensemble average
identical drivers, time independent, homogeneous traffic lanes

simplest possible approximation of \( \bar{f} \)
A master model
• a renormalisation-like approach

\[ a_n = f_0 \left( h_n, \Delta v_n, v_n \right) \quad \rightarrow \quad f_0 \left( h_n, 0, v_n \right) = 0 \]

the “ground state” of the traffic dynamics

\[ f_0 \left( h_n, 0, V_{op} \right) = 0 \]
A master model

- a controlled expansion

\[
\tilde{a}_n = \sum_{p,q} \kappa_{p,q} \left( \tilde{h}_n \right) \left( \tilde{v}_n - V_{op} \left( \tilde{h}_n \right) \right)^p \Delta \tilde{v}_n^q
\]

\[
\kappa_{p,q} \left( \tilde{h}_n \right) = \frac{1}{p!q!} \left. \frac{\partial^{p+q} \tilde{f}}{\partial^{p} \tilde{v}_n \partial^{q} \Delta \tilde{v}_n} \right| \tilde{v}_n = V_{op} \left( \tilde{h}_n \right) \quad \Delta \tilde{v}_n = 0
\]

Yang Bo et. al PRE 92, 042802 (2015)
A new perspective in modelling

- a universal mathematical structure

Yang Bo et.al PRE 92, 042802 (2015)
A new perspective in modelling

• a universal mathematical structure

Yang Bo et.al PRE 92, 042802 (2015)
A new perspective in modelling

- a universal mathematical structure
- two-phase vs. three-phase

\[ \Delta v = 0, h \]

\[ a_{\Delta V=0, h} \]

\[ 0 \quad v \]

\[ \tilde{a}_n \sim (\tilde{v}_n - V_{op}(\tilde{h}_n))^p \]

Yang Bo et.al PRE 92, 042802 (2015)
A new perspective in modelling
• a universal mathematical structure
• two-phase vs. three-phase

• All microscopic traffic models are defined by the optimal velocity (OV) and a set of expansion coefficients (EC).
• the two-phase and three-phase traffic models can be unified by a “common language”.
• The simplification of OV and ECs can be empirically verified.
A new perspective in modelling

- special cases - IDM

\[ a_n = a \left( 1 - \left( \frac{v_n}{v_0} \right)^{\delta} - \left( \frac{h^* (v_n, \Delta v_n)}{h_n} \right)^2 \right) \]

\[ a_n = \sum_{p=1}^{4} \sum_{q=0}^{2} \lambda_{p,q} (v_n - V_{op} (h_n))^p \Delta v_n^q \]

\[ a_n = \lambda_{10} (h_n) (v_n - V_{op}) + \lambda_{01} (h_n) \Delta v_n \]

Yang Bo et al. PRE 92, 042802 (2015)
A new perspective in modelling

- special cases - Occam’s razor

The best model should be as simple as possible (but not simpler)

\[ a_n = \sum_{p,q} \kappa_{p,q} (h_n) (v_n - V_{op})^p \Delta v_n^q \]
A new perspective in modelling

- special cases - Occam’s razor
- Model tuning

\[ a_n = \kappa (V_{op} (h_n) - v_n) + g (\Delta v_n) \]
\[ g (\Delta v_n) = \lambda_1 \Delta v_n + \lambda_2 |\Delta v_n| \]

Emergent quantities from non-linear interactions

Yang Bo et.al arXiv:1504.01256
Yang Bo et.al arXiv:1407.3177
A new perspective in modelling

- special cases - Occam’s razor
- Model tuning
A new perspective in modelling

- special cases - Occam’s razor
- numerical simulation
A new perspective in modelling

- special cases - Occam’s razor
- numerical simulation

Boomerang effect

Pinch effect

merging of numerous narrow jams

Yang Bo et.al arXiv. 1504.01256
A new perspective in modelling

- special cases - Occam’s razor
- numerical simulation

interplay of congested flow upstream of a bottleneck and the wide moving jams

Yang Bo et.al arXiv. 1504.01256
A new perspective in modelling

- special cases - Occam’s razor
- numerical simulation

Frequency of jam evolutions and the width of the “synchronized flow”

Robustness of the wide moving jams

Flow fluctuation of the dense traffic

Yang Bo et.al arXiv. 1504.01256
A new perspective in modelling

- special cases - Occam’s razor
- numerical simulation

arbitrary profile of the congested flow
A new perspective in modelling
• systematic extension of the model

• More realistic optimal velocity function
• Making the coefficients of expansion density dependent
• Include higher order terms
Empirical verification of microscopic traffic models from the detailed acceleration patterns

Bo Yang and Christopher Monterola
Complex Systems Group
Institute of High Performance Computing
Ji Wei Yoon
UC Berkeley
Empirical verifications

- understanding human driving behaviours

- Video-taping of the traffic flow with high frame-rate camera
- Machine learning to identify vehicles in the video
- Virtual sensors to measure the velocity, acceleration as well as headway and approach velocity
- Data aggregation and statistical averaging
Empirical verifications

- image processing and machine learning
Empirical verifications
• edge detections and quantitative measurements
Empirical verifications

- systematic errors
A master model
• a renormalisation-like approach

\[ a_n = F_{n,s_i}\{t_i\} \]

unimportant factors \[ \bar{a}_n = \frac{1}{N_0} \sum_{s_i} F_{n,s_i}\{t_i\} = f_n\{t_i\} \]

important factors \[ \bar{a}_n = \frac{1}{N} \sum_k f_k\{t_i\} = f_0\{t_i\} \]
Empirical verifications

- tentative results

Headway = 4 m, Approach velocity = 0 m/s

![Graph showing velocity and acceleration for a headway of 4 m and approach velocity of 0 m/s.]

Headway = 5 m, Approach velocity = 0 m/s

![Graph showing velocity and acceleration for a headway of 5 m and approach velocity of 0 m/s.]

Velocity (m/s)  
Acceleration (m/s/s)
Empirical verifications
• tentative results

Headway = 6 m, Approach velocity = 0 m/s

Headway = 7 m, Approach velocity = 0 m/s
Empirical verifications
• tentative results

Headway = 8 m, Approach velocity = 0 m/s

Headway = 10 m, Approach velocity = 0 m/s
Empirical verifications
• tentative results

Headway =11 m, Approach velocity = 0 m/s

Velocity (m/s)

Acceleration (m/s/s)
Empirical verifications
• tentative results - summary

• The renormalisation-like procedure does yield well-defined master microscopic model from the microscopic empirical data
• The empirical data shows evidence of the multitude of steady-states (or at least very long lasting states) that can be characterised as the states in the “synchronized phase”
• There are strong dependence of the expansion of coefficients on the density.
Thank you very much!
A new perspective in modelling

- special cases - IDM

Yang Bo et al. PRE 92, 042802 (2015)
Coexistence curve
Neutral stability line
$\lambda^{(1)}$
$\lambda^{(2)}$
$\lambda^{(3)}$