Dynamical queueing, dynamical route choice, responsive traffic control and control systems which maximise network throughput

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Universities:
- York, Kobe, Aberdeen, Luxembourg
Dynamical queueing
Dynamical queueing

Route 1

Route 2

ORIGIN

BOTTLENECK

DESTINATION

O

B

D

QUEUE
Dynamical queueing
Dynamical queueing

ORIGIN  O  B  D  DESTINATION

Route 1

Route 2

BOTTLENECK

QUEUE
Dynamical queueing
Dynamical queueing

ORIGIN

Route 2

Route 1

BOTTLENECK

QUEUE

DESTINATION
Dynamical queueing

Route 2

Route 1

ORIGIN

BOTTLENECK

QUEUE

DESTINATION
Dynamical queueing

Route 2

Route 1

ORIGIN

BOTTLENECK

QUEUE

DESTINATION
Dynamical queueing
**SMALL Network**

Stage 1

- Signal: $(g_1, g_2)$
- Sat flow $s_1 = 1$
- Bottleneck delay $b_1$

Stage 2

- Sat flow $s_2 = 2$
- Bottleneck delay $b_2$

$X_2 = \text{route 2 flow}$

$X_1 = \text{route 1 flow}$

Origin

Destination
SMALL Network is not unrealistic
SMALL Network *is not unrealistic*
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Summary

The talk considers:

- A spatial queueing model (representing the space taken up by queues)
- Traffic signal control and route choice.
- Throughput maximising control when demand exceeds capacity
- P0 control and pricing results for the City of York.
Summary

The talk considers:
- A spatial queueing model (representing the space taken up by queues)  
  DONE
- Traffic signal control and route choice.
- Throughput maximising control when demand exceeds capacity
- P0 control and pricing results for the City of York.
AIM

TO REDUCE CONGESTION / POLLUTION IN CITIES
AIM

TO REDUCE CONGESTION / POLLUTION IN CITIES
IN PART AUTOMATICALLY
Previous Work

Modelling Signal Control and Route Choice:
Allsop, Dickson, Gartner, Akcelik, Maher, Van Vliet, Van Vuren, Smith, Van Zuylen, Meneguzzozer, Gentile, Noekel, Taale, Cantarella, Mounce, Watling, Ke Han, Himpe, Viti, Schlaich, Haupt, Tampere, Huang, Rinaldi,
Traffic Control and Route Choice
NETWORK WITH ROUTE AND GREEN-TIME CHOICES

Stage 1

Stage 2

S

Signal: \((g_1, g_2)\)

b_2 = bottleneck delay

bottleneck delay b_1

ORIGIN

DESTINATION
Network with Route and Green-Time Choices

Stage 1
Route 1, cost $C_1$

Stage 2
Route 2, cost $C_2$

Signal: $(g_1, g_2)$

$b_2 = \text{bottleneck delay}$

$b_1 = \text{bottleneck delay}$
Control and Route-flow Variables

Controls:
Green-time vector $g$:
$$g_1 + g_2 = l.$$  

Route-flows:
Route-flow vector $X$;
$$X_1 + X_2 = \text{given steady demand } T.$$
$X_2 = \text{route 2 flow}$

Stage 2

Sat flow $s_2 = 2 \ (v/s)$

Signal: $(g_1, g_2)$

Stage 1

Sat flow $s_1 = 1 \ (v/s)$

bottleneck delay $b_1$

bottleneck delay $b_2$

$X_1 = \text{route 1 flow}$

ORIGIN

Destination

SMALL Network
P0: Choose greens so \( \mathbf{b} \) is “normal” to \( S \).

\[ X_1 + X_2 = T \]

\[ s_2 = 2 \]

\[ S = \text{the set of supply-feasible } (X_1, X_2) \]

\( (X_1, X_2) \)

\( (b_1, b_2) \)

\( \text{MUST BE A RIGHT ANGLE} \)

\( s_1 \)

\( s_2 \)
P0: Choose greens so \( \mathbf{b} \) is "normal" to \( S \).

\[
s_1 b_1 = s_2 b_2
\]

\( S \) = the set of supply-feasible \((X_1, X_2)\)

\((X_1, X_2)\) MUST BE A RIGHT ANGLE
STANDARD POLICIES HALVE THE CAPACITY OF THIS NETWORK

Average journey time

Standard

P0

OPT

MAX/2
Demand

MAX
NETWORK WITH ROUTE AND GREEN-TIME CHOICES

Stage 1

Stage 2

Signal: \((g_1, g_2)\)

bottleneck delay \(b_1\)

bottleneck delay \(b_2\) = bottleneck delay

ORIGIN

S

DESTINATION
Equilibrium flow and $P_0$ green-time

EXACT $P_0$ Control Policy:
green-time $g$ satisfies $s_1 b_1 = s_2 b_2$

EXACT route-choice equilibrium:
route-flow $X$ satisfies $C_1 + b_1 = C_2 + b_2$

What if these conditions do not hold?
ROUTE AND GREEN-TIME SWAPS

Route 1

Stage 1

Stage 2

Route 2

ORIGIN

S

DESTINATION
ROUTE AND GREEN-TIME SWAPS

Travel cost along route 1 = $C_1 + b_1$
Travel cost along route 2 = $C_2 + b_2$
ROUTE AND GREEN-TIME SWAPS

Travel cost along route 1 = $C_1 + b_1$
Travel cost along route 2 = $C_2 + b_2$
ROUTE AND GREEN-TIME SWAPS

Pressure on stage 1 = $s_1 b_1$
Pressure on stage 2 = $s_2 b_2$
**ROUTE AND GREEN-TIME SWAPS**

\[ [C_1 + b_1] - [C_2 + b_2] \]

controls black arrow

\[ s_1 b_1 - s_2 b_2 \]

controls green arrow

Route 2

Stage 2

Stage 1

Route 1

ORIGIN

S

DESTINATION

SIGNAL
*** HOPE: Stability ***

\[ V(X, g) = 4 \]
\[ V(X, g) = 3 \]
\[ V(X, g) = 2 \]
\[ V(X, g) = 1 \]

**EQUILIBRIUM:**
\[ V(X, g) = \text{minimum} \]
*** HOPE: Stability ***

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\[ V(X,g) = 2 \]
\[ V(X,g) = 1 \]

**EQUILIBRIUM:**
\[ V(X,g) = \text{minimum} \]
HOPE: WITH $P_0$ MANY DYNAMICAL MODELS HAVE STABILITY!!!

EQUILIBRIUM:
$V(X, g) = \text{minimum}$
Other policies?

STABLE WITH STANDARD POLICIES ???
Other policies?

STABLE WITH STANDARD POLICIES ???

NO!
CONTROL WHICH MAXIMISES THROUGHPUT
Even when demand exceeds capacity
ORIGIN \rightarrow G \rightarrow SIGNAL \rightarrow DESTINATION

\text{MAXIMUM FLOW} = s/2

G
s
s
EQUILIBRIUM G UNDER P₀
EQUILIBRIUM G UNDER P0

OK
EQUILIBRIUM $G$ UNDER $P_0$

OK

NOT OK
EQUILIBRIUM G: modified P0
SUMMARY

P0 modified maximises throughput of ONE network even when demand exceeds capacity
SUMMARY

$P_0$ modified maximises throughput of ONE network even when demand exceeds capacity

Further work: Generalise!!!
QUESTIONS?
Other related work


Peter Kovacs, Tung Le, Rudesindo Nunez-Queija, Hai L. Vu, Neil Walton, Proportional green time scheduling for traffic lights.
YORK RESULTS

These were obtained by Mustapha Ghali using a dynamic equilibrium program called CONTRAM (used to be supported by the Transport and Road Research Laboratory in the UK)
CONTRAM RESULTS FOR THE YORK NETWORK

% CHANGE IN TOTAL JOURNEY TIME

DELAY-BASED
TIME-BASED
DISTANCE-BASED
TOLLS

TOTAL OF ALL CHARGES PAID (1000 x Pounds)
CONTRAM RESULTS FOR THE YORK NETWORK

% CHANGE IN TOTAL JOURNEY TIME

CONTRAM CONTROL POLICY
OUR CONTROL POLICY P0
FIXED SIGNAL SETTINGS

TOTAL OF ALL CHARGES PAID (1000 x Pounds)
CONTRAM RESULTS FOR THE YORK NETWORK

CONTRAM CONTROL POLICY
OUR CONTROL POLICY P0
FIXED SIGNAL SETTINGS

% CHANGE IN TOTAL QUEUEING DELAY

TOTAL OF ALL CHARGES PAID (1000 x Pounds)
Questions?
$P_0: s_1 b_1 = s_2 b_2$

$C_2 = C_1 + \Delta$
$P_0: s_1 b_1 = s_2 b_2$

$P_0$ with prices: $s_1 p_1 = s_2 p_2$

$C_2 = C_1 + \Delta$
$P_0: s_1 b_1 = s_2 b_2$

$P_0$ with prices: $s_1 p_1 = s_2 p_2$

Prices do not block back
\( P_0: s_1 b_1 = s_2 b_2 \)
\( P_0 \) with prices: \( s_1 p_1 = s_2 p_2 \)

**FEASIBLE EQM**

Prices do not block back
$P_0: s_1 b_1 = s_2 b_2 \text{ NO FEASIBLE EQM}$

$P_0 \text{ with prices: } s_1 p_1 = s_2 p_2 \text{ FEASIBLE EQM}$
$P_0: s_1b_1 = s_2b_2 \text{ NO FEASIBLE EQM}$

$P_0 \text{ with prices: } s_1p_1 = s_2p_2 \text{ FEASIBLE EQM}$
\[ P_0: s_1 b_1 = s_2 b_2 \text{ NO FEASIBLE EQM} \]
\[ P_0 \text{ with prices: } s_1 p_1 = s_2 p_2 \text{ FEASIBLE EQM} \]
$P_0: s_1 b_1 = s_2 b_2$

$P_0$ with prices: $s_1 p_1 = s_2 p_2$
Questions?