Green-wave analysis in a tandem of traffic-light intersections

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Overview

- Network of intersections
- Stochastic model
- Numerical results
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Network of intersections
Fixed length of each phase.
Network of intersections: fixed control

- Each lane has fixed green and red times.
  - no real-time data
- Fixed common cycle length, $c$, in the network.
  - coordination between intersections
- Control parameters: green times and offsets.
  - offset is time between coordinated phases of two intersections
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Service process is time-dependent.

Discrete-time model
Stochastic model: problems

- Service process is time-dependent.
  - discrete-time model
- High dimension of the system.
  - network decomposition into separate lanes
Stochastic model: problems

- Service process is time-dependent.
  - discrete-time model
- High dimension of the system.
  - network decomposition into separate lanes
- Dependency between lanes.
  - arrival process
Stochastic model: network
Stochastic model: external lane

$P = 0$

$Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y$

$\text{Bernoulli arrivals: i.i.d.}$

$\text{Delayed departure at second } s + P + d_k$, where $s$— beginning of the green time, $P$— distraction variable, $d_k$— deterministic second of the $k$th delayed vehicle.

If the queue becomes empty, all the arrivals proceed without stopping.
Stochastic model: external lane

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- Delayed departure at second $s + P + d_k$, where $s$ — beginning of the green time, $P$ — distraction variable, $d_k$ — deterministic second of the $k$th delayed vehicle.
- If the queue becomes empty, all the arrivals proceed without stopping.
Stochastic model: internal lane

Correlated arrivals.

Acceleration of the delayed departures.
Markovian arrival process

- Underlying Markov chain $L_i, i = 0, \ldots, c - 1$.
- States represent information that determines arrivals, e.g., the number of delayed departures at the upstream lane.
- $\mathbb{P}(Y_i = 1|L_i = l, Y_0, \ldots, Y_{i-1}) = \lambda_i^l$. 
Markovian arrival process

- red second
- green second without departures
- green second with a delayed departure
- green second with a free departure

combined input
The arrivals during different cycles are independent.
Independence assumption

- The arrivals during different cycles are independent.

Under this assumption, we prove that the pgf of the queue length at a lane at the beginning of the cycle has form:

\[
X(z) = \frac{\sum_{j=0}^{n-1} x_j f_j(z)}{z^n - A(z)C(z)},
\]

where \( n \) is the maximum capacity, \( x_j = P(X_0 = j) \), \( A(z) \) — the pgf of arrivals, \( C(z) \) — the pgf of the lost capacity due to randomness of \( P \), \( f_j(z) \) — polynomials.
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What is a **good green wave**?
Green-wave efficiency

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**Definition** The *green-wave efficiency* is the expected number of intersections passed without stopping for an arbitrary vehicle.

- In an ideal green wave, the green-wave efficiency is equal to the expected number of intersections for a vehicle.
- In the worst case scenario, all of the vehicles need to stop, and our measure is equal to 0.
Optimisation: network of intersections

- Cars
- Decelerating car
- Turning car
- Traffic lights
Optimisation: parameters

We consider a tandem of 3 intersections (100 meters apart):

- the arrival rate from west is $\lambda$,
- the arrival rate from east is $0.5\lambda$,
- the arrival rate from north and south is $0.2\lambda$,
- 16% of the major traffic turns south or north,
- 40% (20%) of the minor traffic turns east (west).
Optimisation: objectives and constraints

Optimisation with multiple objectives:
- maximising the green-wave efficiency,
- minimising the average delay
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Optimisation with multiple objectives:
- maximising the green-wave efficiency,
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for
- fixed cycle length of 60 seconds,
- given phase schedule.
Optimisation: approaches

- Genetic algorithm coupled with our model, multiple objectives
- SUMO cycle program generator (SCPG), Webster (proportional) green time allocation
- MAXBAND. bandwidth maximisation
Optimisation results: Pareto optimality

Green-wave analysis for a traffic-light network
Optimisation: phases

Phase 1

Phase 2

Phase 3

Phase 4
Optimisation results: Pareto optimality load 0.7

Green-wave analysis for a traffic-light network

Green times:
- [7, 7, 20, 2], [7, 7, 20, 2], [7, 7, 20, 2]
- [7, 7, 19, 3], [7, 7, 20, 2], [7, 7, 20, 2]
- [8, 7, 18, 3], [7, 7, 20, 2], [7, 7, 20, 2]
- [7, 7, 20, 2], [7, 6, 21, 2], [7, 7, 20, 2] or
- [7, 7, 20, 2], [7, 6, 21, 2], [7, 7, 20, 2]
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- Other
Conclusions

- It is important to take the real behaviour of traffic into account.
- Optimisation for the best green wave may be disadvantageous for the average delay.
- The average delay per vehicle is very sensitive to the changes in the green times.