Fractional-Flow Models for Foam Enhanced Oil Recovery

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Imperial College, Oct. 7, 2016
“All models are false, but some models are useful.”

- George E P Box
Outline

- Review of fractional-flow theory for waterflood
- Extension to enhanced oil recovery (EOR)
- Applications to foam EOR
  - Effect of foam on gas-water fractional-flow curve
  - Foam Injection with miscible gas flood
  - SAG foam injection
  - Comparison of foam strength in different injection methods
  - Non-Newtonian foam rheology
  - Foam dynamics: within entrance region and at shocks
- Gravity segregation
- Pore-level flow (without capillary-driven flow)
Assumptions of Fractional-Flow Method

- 1D flow through a homogenous permeable medium. No phase changes.
- Incompressible phases (and rock).
- Uniform initial conditions.
- Immediate attainment of local steady-state conditions, which are a function of local phase saturations and compositions only.
- Absence of all dispersive processes: diffusion, dispersion, heat conduction: governing equations are 1st-order p.d.e.’s
- Besides surfactant adsorption, fluids do not react with the rock.
- Newtonian mobilities for all phases.
- For this lecture, only two phases present at any location.
- Absence of gravity forces, fingering and dispersion

Almost all of these assumptions can be relaxed
Conservation equation

\[
\frac{\partial f_w}{\partial x_d} + \frac{\partial S_w}{\partial t_d} = 0
\]

- equation based on material balance:
  \((\text{flux in} - \text{flux out}) = (\text{change in saturation})\)
- \(x_d\) – dimensionless position, defined as
  - \(x/L\) in linear flow
  - \(r^2/Re^2\) in radial flow
- \(t_d\) – dimensionless time, pore volumes injected
- \(f_w\) – fractional flow of water
- \(S_w\) – water saturation
Fractional-flow function (oil-water)

\[ f_w = \frac{k_{rw} / \mu_w}{k_{ro} / \mu_o + k_{rw} / \mu_w} \]

- \( k_{ri} = \) relative permeability of phase i
- \( \mu_i = \) viscosity of phase i
- \( f_w \) is fraction of flow that is water - not same as water saturation (volume fraction in place), but is a function of water saturation (and possibly other variables)

\[ \frac{\partial f_w}{\partial x_d} + \frac{\partial S_w}{\partial t_d} = 0 \]
Solving the equation

conservation equation \[ \frac{\partial f_w}{\partial x_d} + \frac{\partial S_w}{\partial t_d} = \frac{df_w}{dS_w} \frac{\partial S_w}{\partial x_d} + \frac{\partial S_w}{\partial t_d} = 0 \] \quad (I)

total derivative of \( S_w \): \[ dS_w(x_d, t_d) = \frac{\partial S_w}{\partial x_d} dx_d + \frac{\partial S_w}{\partial t_d} dt_d \]
along path defined by \( dx_d / dt_d \):
\[ \frac{dS_w}{dt_d} = \frac{\partial S_w}{\partial x_d} \frac{dx_d}{dt_d} + \frac{\partial S_w}{\partial t_d} \frac{dt_d}{dt_d} \]

define path by \( dx_d / dt_d = df_w / dS_w \)

\( (I) \rightarrow dS_w(x_d, t_d) = 0 \) along this path
Solution by method of characteristics

\[
\frac{\partial f_w}{\partial x_D} + \frac{\partial S_w}{\partial t_D} = 0
\]

- \( S_w \) is constant along paths with slope \( \frac{dx_D}{dt_D} = \frac{df_w}{dS_w} \)
- \( x_D \) – dimensionless position, defined as pore volume between injection point and given position:
  - \( x/L \) for linear flow
  - \( r^2/r_e^2 \) for radial flow
- \( t_D \) – dimensionless time, pore volumes injected
- \( S_w \) is function of \( x_D, t_D \); plot \( S_w(x_D, t_D) \) on \( (x_D, t_D) \) plane
Solution by method of characteristics

- $S_w$ is constant along paths with slope $\frac{dx_D}{dt_D} = \frac{df_w}{dS_w}$
How to eliminate multiple-valued solutions?

- Allow for **discontinuous solutions** $S_w(x_D, t_D)$; discontinuities are “shocks”
- Solve for velocities of shocks by material balance at shocks (more on this shortly)
Material balance gives shock velocity

- Material balance on water: (flux in-out) = accumulation

\[
\left( A uf_{w}^{+} - A uf_{w}^{-} \right) \Delta t = \left( A \varphi S_{w}^{+} - A \varphi S_{w}^{-} \right) \Delta x
\]

\[
\frac{\Delta x}{L} = \frac{\Delta x}{L_D} = \frac{f_{1}^{+} - f_{1}^{-}}{S_{1}^{+} - S_{1}^{-}} = \frac{\Delta f_{w}}{\Delta S_{w}}
\]

Velocity of shock is slope of line on \( f_w(S_w) \) representing shock.
Simplified (Two-Phase) Fractional-Flow Method for EOR Processes
Simplified (Two-Phase) Fractional-Flow Method for EOR Processes

- Something alters the fractional-flow curve – call it EOR AGENT
- Two fractional flow curves – with AGENT, without EOR AGENT

Lake et al., *Fundamentals of EOR*, SPE, 2014
## Examples of EOR Processes and EOR AGENT

<table>
<thead>
<tr>
<th>Process</th>
<th>Agent</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>polymer</td>
<td>polymer</td>
<td>increases viscosity of water</td>
</tr>
<tr>
<td>hot water</td>
<td>heat</td>
<td>reduces viscosity of oil</td>
</tr>
<tr>
<td>surfactant</td>
<td>surfactant</td>
<td>reduces IFT, makes oil and water &quot;miscible&quot;</td>
</tr>
<tr>
<td>wettability alteration</td>
<td>surfactant</td>
<td>changes wettability of rock</td>
</tr>
<tr>
<td>alkaline</td>
<td>alkali</td>
<td>changes wettbility, IFT</td>
</tr>
<tr>
<td>lo-sal</td>
<td>(lower) salinity</td>
<td>changes wettability</td>
</tr>
<tr>
<td>foam</td>
<td>surfactant</td>
<td>reduces mobility of gas</td>
</tr>
<tr>
<td>miscible</td>
<td>solvent</td>
<td>replaces oil; changes viscosity of non-aq phase</td>
</tr>
</tbody>
</table>

Lake et al., *Fundamentals of EOR*, SPE, 2014
Examples of effect of EOR AGENT

- **Base case**: reservoir before introduction of AGENT
- Water more viscous
- Miscible gas replaces oil as nonwetting phase
- Surfactant reduces oil-water IFT to ultra-low values
Simplified (Two-Phase) Fractional-Flow Method for EOR Processes

• Something alters the fractional-flow curve – call it AGENT
• Two fractional flow curves – with AGENT, without AGENT
• Initial state of reservoir I (usually) lacks AGENT; point I is on $f_w(S_w)$ curve without AGENT
• Injected fluid J contains AGENT; point J is on curve with AGENT
• Process must jump between fractional-flow curves

Lake et al., Fundamentals of EOR, SPE, 2014
Simplified Fractional-Flow Method for EOR Processes II

- Perform material balance on AGENT at leading edge of EOR AGENT bank
  - Include AGENT lost to or gained from rock (heat, adsorption of chemical)
  - Also do material balance on, e.g., water
  - Leads to algebraic equation for velocity of jump from AGENT to NO AGENT $f_w(S_w)$ curves
  - Interpret equation geometrically; often it implies that line representing jump between AGENT and NO-AGENT $f_w(S_w)$ curves must pass through some fixed point on $f_w(S_w)$ diagram

Lake et al., *Fundamentals of EOR*, SPE, 2014
Simplified Fractional-Flow Method for EOR Processes II ½: SPOILER ALERT

- Geometric conditions for jump between $f_w(S_w)$ curves
  - Miscible (oil-)solvent flood: line drawn from (1,1) [see additional refinements in Walsh & Lake (1989)]
  - Agent = water-soluble chemical*, no adsorption: line drawn through (0,0)
  - Agent = water-soluble chemical*, w/ adsorption: line drawn through (-D,0); D represents effect of adsorption
  - Agent = hot water: line drawn through (a,b); a, b represent thermal properties of fluids, rock
    * - surfactant, polymer, alkali, (lack of) salinity
Simplified Fractional-Flow Method for EOR Processes III

- Seek path from J to I with monotonically increasing slopes
- Along each separate $f_w(S_w)$ curve, rules for Buckley-Leverett analysis apply: monotonically increasing $df_w/dS_w$ in spreading waves, shocks with velocity $\Delta f_w/\Delta S_w$
- Jump between $f_w(S_w)$ curves must satisfy material-balance conditions (geometric conditions on plot), and also have velocity in sequence with others in displacement

Lake et al., *Fundamentals of EOR*, SPE, 2014
Applications to Foam EOR

- Foam Injection with miscible gas displacement of oil
- SAG foam injection
- Foam injection with surfactant injected in “gas” phase
- Comparison of injection strategies and foam strength in different formations
- Non-Newtonian foam rheology
- Foam dynamics: within entrance region and at shocks
Why foam in gas-injection EOR

- Gas injected for EOR often recovers virtually all remaining oil where it sweeps
- Sweep efficiency is often poor because of
  - Reservoir heterogeneity
  - Viscous instability
  - Gravity override
- Foam can help fight all three causes of poor sweep
- Foam improves sweep; *gas recovers the oil*
How Does Foam Help Sweep?

Problems: reservoir heterogeneity, viscous instability, gravity override

**Heterogeneous formations**: If injected with large gas fraction, foam reduces mobility more in high-k layers; diverts flow to low-k layers*

**Viscous instability**: All foams reduce mobility
Foam increases viscous pressure gradient in competition with gravity; can have high injectivity and low mobility in field; helps fight **override** in a way simply viscosified fluids can’t*

Foam forms as gas passes upward through sharp permeability boundaries; reduces $k_z$ more than $k_x$*

*Only foam does this*
Effect of foam on fractional-flow curve

- Foam greatly reduces gas mobility
- Presence of surfactant in water → foam → shift in $f_w(S_w)$
- Foam collapses at limiting water saturation $S_{w*}$
- Water-soluble surfactant is AGENT.
- Jump between curves passes through $(-D, 0)$; $D$ reflects adsorption
Foam injection with miscible gas

Graph showing the relationship between D, f_w, and S_w.
Foam injection with miscible gas

Ashoori et al., SPE 121579
Foam injection with miscible gas

if gas banks finger through oil, oil bank may advance with velocity of foam bank

Ashoori et al., SPE 121579
Foam injection with miscible gas: Optimal foam quality

Ashoori et al., SPE 121579
SAG injection; gas follows surfactant slug

J at $f_w = 0$; I at $f_w = 1$
SAG injection; gas follows surfactant slug

- J at $f_w = 0$; I at $f_w = 1$
- Shock jumps past strongest foam (and most lab data!) to very low $f_w$
- Leading edge of foam bank is at point of tangency to shock line; can be relatively low mobility, giving good mobility control
- Behind this is spreading wave back to extremely high mobility at well – good injectivity

Gas Injection in SAG: low mobility at front, very high back at well
Gas Injection in SAG: low mobility at front, very high back at well
Better injectivity makes max-$\Delta P$ SAG effective against gravity override

- Fractional-flow analysis predicts extremely high mobility near well, where foam dries out and collapses
- Therefore, get good injectivity and low mobility away from well: perfect for overcoming gravity override
- Verify with simulation

Simulators Underestimate Gas Injectivity in SAG Foam

- In gas injection in SAG process, foam dries out and collapses near injection well; this greatly increases injectivity
- Simulators do not resolve near-well region well
- Fractional-flow simulation allows resolution to arbitrary accuracy (cm or less), depending on number of characteristics
- Show huge errors in simulator injectivity calculations

Foam process with surfactant injected in miscible “gas” (supercritical CO$_2$)

• Only “gas” injected; good injectivity
• Material balance on shock gives graphical condition for shock from foam to gas-water ahead of foam: line must pass through point (a,b), reflecting adsorption and partition coefficient between oil and water
• Foam-bank velocity depends on relative solubility of surfactant in CO$_2$, water (as well as adsorption).
Case $C_{sg} < C_{sw}$ (surfactant partitions into water rather than gas) ($C_{sg}/C_{sw} = 0.5$)

Ashoori et al., SPE 121579
Insights from fractional-flow solution

- Foam bank forms at very low $f_w$; weakened foam, as in SAG
- Foam propagation reduced for surfactant that is much more soluble in water than gas, regardless of extent of solubility in gas
- Always have high-mobility gas banks ahead of foam (unless use preflush)

Ashoori et al., SPE 121579
Non-Newtonian Foam

- Foam often shows non-Newtonian mobility in porous media
- The fractional flow curve depends on superficial velocity; therefore it changes with radial position $r$ around well
- At each new $r$, $f_w(S_w)$ changes slightly
- Characteristics carry forward their values of $f_w$. Characteristics curve; so do shocks
- Can alter injectivity significantly

Rossen et al., *TIPM* 89, 213 (2011)
Non-Newtonian Foam

• Foam often shows non-Newtonian mobility in porous media
• The fractional flow curve depends on superficial velocity; therefore it changes with radial position $r$ around well
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• Characteristics carry forward their values of $f_w$. Characteristics curve; so do shocks
• Can alter injectivity significantly

Rossen et al., *TIPM* 89, 213 (2011)
Fractional-Flow Model for Foam Dynamics

- Foam properties are result of dynamic processes of creation & destruction of bubbles
- “Population Balance” models attempt to account for this
- These processes are at dynamic equilibrium EXCEPT
  - near entrance to porous medium
  - at shock fronts
- At “traveling wave” at shocks, must solve for dynamics of bubble creation and destruction and $\nabla P_c$
  - Determines which shocks are allowed

Ashoori et al., *COLSUA* 377, 217, 228 (2011)
Fractional-Flow Model for Foam Dynamics

- Experimental studies find $\nabla p$ trigger for foam generation
- Kam population-balance model explains $\nabla p$ trigger for foam generation, multiple steady states at same injection condition seen in experiments
- Resulting fractional-flow curve from model has multiple loops; also, $f_w(S_w)$ depends on superficial velocity, and on $r$, as in non-Newtonian foam
- Model predicts foam generation stops as superficial velocity $\downarrow$; explains experiments of Friedmann in 1980s
- Model has man simplifications, assumptions; not predictive

Fractional-Flow Model for Foam Dynamics
Fractional-Flow Model for Foam Dynamics
DIVERSION WITH FOAM IS COMPLEX
In which layer is foam strongest?

<table>
<thead>
<tr>
<th>Layer</th>
<th>$f_{m\text{mob}} \times F_5$ 4 bar/m</th>
<th>$f_{m\text{mob}} \times F_5$ 40 bar/m</th>
<th>$f_{m\text{mob}} \times F_5$ 400 bar/m</th>
<th>$f_{\text{mdry}}$</th>
<th>$k_m(f_{\text{mdry}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bentheimer</td>
<td>47,200</td>
<td>46,200</td>
<td>45,100</td>
<td>0.271</td>
<td>3.43×10^5</td>
</tr>
<tr>
<td>Berea</td>
<td>6.07×10^6</td>
<td>726,000</td>
<td>86,700</td>
<td>0.336</td>
<td>2.34×10^4</td>
</tr>
<tr>
<td>Sister Berea</td>
<td>4.14×10^{11}</td>
<td>1.68×10^7</td>
<td>684</td>
<td>0.396</td>
<td>2.18×10^4</td>
</tr>
<tr>
<td>Bandera Gray</td>
<td>250,000</td>
<td>90,700</td>
<td>33,000</td>
<td>0.549</td>
<td>1.63×10^3</td>
</tr>
</tbody>
</table>
DIVERSION WITH FOAM IS COMPLEX

In which layer is foam strongest? Based on $fmmob$, mobility reduction in low-quality regime:

<table>
<thead>
<tr>
<th>Layer</th>
<th>$fmmob$</th>
<th>$epdry$</th>
<th>$fmdry$</th>
<th>$epcap$</th>
<th>$fmcap$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Bentheimer</td>
<td>47,700</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Berea</td>
<td>869,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Sister Berea</td>
<td>30,700</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Bandera Gray</td>
<td>68,200</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\mu = 0.0\rightarrow 0.2\rightarrow 0.4\rightarrow 0.6\rightarrow 0.8\rightarrow 1.0$

$Sw_{fw}$

Water fractional flow

Foam

No foam

Water saturation

foam

$fmmob$

no foam
DIVERSION WITH FOAM IS COMPLEX

In which layer is foam strongest? Effect of shear-thinning and foam mobility in low-quality regime:

(Shear-thinning behavior extrapolated from narrow range of data)
DIVERSION WITH FOAM IS COMPLEX

In which layer is foam strongest? Based on $k_{rw}(fmdry)$, mobility in high-quality (“coalescence”) foam regime:

<table>
<thead>
<tr>
<th>Layer</th>
<th>$fmmob \times F5$ 4 bar/m</th>
<th>$fmmob \times F5$ 40 bar/m</th>
<th>$fmmob \times F5$ 400 bar/m</th>
<th>$fmdry$</th>
<th>$k_{rw}(fmdry)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bentheimer</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>$3.43 \times 10^5$</td>
</tr>
<tr>
<td>Berea</td>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>$2.34 \times 10^4$</td>
</tr>
<tr>
<td>Sister Berea</td>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>$2.18 \times 10^4$</td>
</tr>
<tr>
<td>Bandera Gray</td>
<td></td>
<td></td>
<td></td>
<td>9</td>
<td>$1.63 \times 10^3$</td>
</tr>
</tbody>
</table>

Water fractional flow

- Foam
- No foam

Water saturation

- fmdry

Graph shows foam and no foam conditions with water fractional flow and water saturation.
DIVERSION WITH FOAM IS COMPLEX

In which layer is foam strongest? Based on mobility at leading edge of shock front in SAG (STARS model):

<table>
<thead>
<tr>
<th>Layer</th>
<th>$S_{w, shock}$</th>
<th>$\left(\frac{df_w}{dS_w}\right)<em>{S</em>{w, shock}}$</th>
<th>$(\lambda_{rt})<em>{S</em>{w, shock}}$</th>
<th>$S_{WR}$</th>
<th>$(\lambda_{rt})<em>{S</em>{WR}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bentheimer</td>
<td>0.266</td>
<td>1.36</td>
<td>4.10</td>
<td>0.25</td>
<td>16.4</td>
</tr>
<tr>
<td>Berea</td>
<td>0.328</td>
<td>1.48</td>
<td>20.0</td>
<td>0.23</td>
<td>369.0</td>
</tr>
<tr>
<td>Sister Berea</td>
<td>0.393</td>
<td>1.64</td>
<td>43.4</td>
<td>0.25</td>
<td>2760</td>
</tr>
<tr>
<td>Bandera Gray</td>
<td>0.507</td>
<td>1.99</td>
<td>8.77</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
DIVERSION WITH FOAM IS COMPLEX

In which layer is foam strongest?

Based on mobility at well in SAG (STARS model):

<table>
<thead>
<tr>
<th>Layer</th>
<th>( S_{w,\text{shock}} )</th>
<th>( \left( \frac{df_w}{dS_w} \right)<em>{S</em>{w,\text{shock}}} )</th>
<th>( (\lambda_{rt})<em>{S</em>{w,\text{shock}}} )</th>
<th>( S_{wr} )</th>
<th>( (\lambda_{rt})<em>{S</em>{wr}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bentheimer</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>16.4</td>
</tr>
<tr>
<td>Berea</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>369.0</td>
</tr>
<tr>
<td>Sister Berea</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2760</td>
</tr>
<tr>
<td>Bandera Gray</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>22.8</td>
</tr>
</tbody>
</table>

![Diagram showing water fractional flow vs. water saturation with foam and no foam]

- Bentheimer: 16.4
- Berea: 369.0
- Sister Berea: 2760
- Bandera Gray: 22.8
DIVERSION WITH FOAM IS COMPLEX

In which layer is foam strongest? Based on mobility at well in SAG (Namdar-Zanganeh model):

<table>
<thead>
<tr>
<th>Layer</th>
<th>$S_{wr}$, shock</th>
<th>$(\lambda_{rt})<em>{S</em>{wr}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bentheimer</td>
<td>0.266, 1.36</td>
<td>29,500</td>
</tr>
<tr>
<td>Berea</td>
<td>0.328, 1.48</td>
<td>49,500</td>
</tr>
<tr>
<td>Sister Berea</td>
<td>0.393, 1.64</td>
<td>23,500</td>
</tr>
<tr>
<td>Bandera Gray</td>
<td>0.511, 2.01</td>
<td>36,500</td>
</tr>
</tbody>
</table>

*Extremely important whether foam collapses completely at $S_{wr}$*
Model for Gravity Override
Gravity Override, Co-Injection

\[ r_g = \sqrt{\frac{Q_t}{k_z \Delta \rho g \pi \lambda_{rt}}} \]
Proving Stone's Model

- Key: change coordinates from \((x, z)\) to \((x, \psi)\)
  - \(\psi\) is stream function

\[
\left( \frac{\partial f}{\partial x} \right)_\psi + k_v (\rho_w - \rho_g) g \left( \frac{\partial F(f)}{\partial \psi} \right)_x = \left( \frac{\partial f}{\partial x} \right)_\psi + k_v (\rho_w - \rho_g) g \frac{dF}{df} \left( \frac{\partial f}{\partial \psi} \right)_x = 0
\]

- Equation is in form of conventional fractional-flow equation, with altered governing functions
- \(f \equiv f_w(S_w)\) is conventional fractional-flow function
- \(F(S_w) \equiv \frac{\lambda_w(S_w) \lambda_g(S_w)}{\lambda_t(S_w)}\)
  - Behavior depends on shape of \(F(f)\) function

Streamlines ($\psi =$ constant) model tracks segregation relative to streamlines; path of streamlines unknown.
Results

• Confirm always get three uniform regions: override, underride, mixed, with sharp fronts between them: comes out of geometric analysis of F(f) function

• Prove Stone's final equation for distance to point of segregation correct, making only standard assumptions of fractional-flow theory

• Cannot confirm Stone and Jenkins' equations for boundaries of underride, override regions in terms of r, z
Pressure Rise at Injection Well

- Then volumetric flux in cylindrical flow is

\[ U_{tr} = \left( \frac{4Q_{\text{inj}}}{\pi r H} \right) \left[ 1 - \left( \frac{r}{r_g} \right)^2 \right] \]

- Pressure rise at injection well given by

\[
p(R_w) - p(R_g) = \frac{k_z (\rho_w - \rho_g) g}{2H k_h} R_g^2 \left[ \ln \left( \frac{R_g}{R_w} \right) - \frac{1}{2} \left( 1 - \left( \frac{R_w}{R_g} \right)^2 \right) \right]
\]

- Distance to segregation point given by

\[
r_g = \sqrt{\frac{Q_t}{k_z \Delta \rho_g \pi \lambda_{rt}}} \]
Same approach works for water injection above gas
Does snap-off occur repeatedly at the same pore throat during foam flow?

- Calculations show that snap-off occurs quickly and repeatedly in foam flow.
- Calculations based on assumed distribution of water in pore corners after foam lamella passes by.
- For simplicity, calculations assumed “convection-dominated flow,” i.e. ignore $\nabla P_c$.
- Equations are purely convective; fit assumptions of method of characteristics.

Impact of Initial Distribution of Water in Pore

Initial Distribution of Water in Pore

a) axial view

- $R_w(x) = 0.53R(x)$

b) cross-section

- Water in pore corners

$R_w^o(z)/R_b$

- $P_c^o(z)/P_c^{sn}$

- $V_zb^o(z)$

Dimensionless radius

Dimensionless capillary pressure

Dimensionless volume

Dimensionless position, $z/L$
Impact of Initial Distribution of Water in Pore

Initial Distribution of Water in Pore

Large mass of water initially in pore body is swept into throat by convection
“All models are false, but some models are useful.”

- George E P Box
What’s *useful* in fractional-flow modeling?

- Exact solutions for benchmarking accuracy and numerical artifacts of simulators
- Identify key parameters in complex models
- Identify key aspects of complex processes
  - Viscous instability in sequence of banks
  - Most important conditions for conducting experiments
- Can lead to improved designs, to be tested and refined with simulation
- Solutions that can be extended to resolution not feasible with finite-difference simulation
Questions?

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