Produced Water Re-Injection (PWRI)
An Experimental Investigation into Internal Filtration and External Cake Build up

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Rouhollah Farajzadeh
August 2004
In the name of God

To:

My Parents,
who taught me the meaning of life and love
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CHAPTER 1

PRODUCED WATER AND ITS INJECTION

1.1 Introduction

Most oil and gas reservoirs have a natural water layer called formation water beneath the hydrocarbon layer. Also to achieve maximum oil recovery, additional water is usually injected into the reservoir, which may be associated with hydrocarbons in the production. In the case of some gas production, produced water can be condensed water. Thus, the liquid that comes out of reservoir is not just hydrocarbons, but is frequently accompanied by water. The liquid production is in the form of a mixture of free water, an oil/water emulsion and oil. Furthermore, as an oil field matures the amount of produced water increases. This is because, after some time, the formation waters out due to the water injection process. The water-oil ratio varies from reservoir to reservoir. It also varies with time for a particular reservoir. Worldwide 75% of the production is water, but in some places this percentage may increase to 98%. Table 1-1 shows the amount of produced water and oil in North Sea.

The production stream goes to a separation unit called the Free Water Knockout Vessel (FWKV). In this unit free water and loose solids are separated from oil. Produced water from this unit is stored in Water Tanks, while the remaining oil and oil-water emulsion are additionally treated. In Treater Vessels with the combination of heat and some chemicals (Emulsion Breakers or Demulsifiers) oil-water emulsions are broken down and clean oil, water and some solids are produced. Clean oil goes to storage or shipping and produced water is kept for disposal in Water Tanks. Depending on the residence time some of the solids may settle out of the water and residual oil in the water floats to the surface. This oil layer is skimmed off the top and recycled to recover this additional oil. In the fields with smaller
facilities in this stage water will go to be disposed, but in the larger facilities water faces an additional treatment in a Dissolved Air Floatation (DAF) to get cleaner water. After the DAF unit, the water is either sent through filters, which are usually sand and multimedia filters or through hydrocyclones to remove the remaining traces of oil and solids. After final filtration the water is ready to be disposed. Figure 1-1 shows a simple schematic of a separation unit.

![Simple Schematic of a Separation Unit](image)

**Figure 1-1**: Schematic of a simple separation unit. Worldwide 3 barrels out of 4 barrels of production is water and just one barrel is oil.

### 1.2 Composition of produced water

Physical and chemical properties of the produced water mainly depend on geographical location, geological formation and type of hydrocarbons of the field and may differ from one place to another. Since the produced water has been in contact with geological formations for millions of years, its composition is strongly field-dependent.

Produced water consists of a major part, water, and minor amounts of organic and inorganic constituents from the source geologic formation and the associated hydrocarbons. The composition of produced water may change through the production lifetime of the reservoir, because more water is injected to maintain the pressure of the reservoir. Produced water may also contain small amounts of chemicals that have been added in the treatment of water. These treatment chemicals could be listed as: Hydrate Inhibitors, Dehydrators, Scale Inhibitors, Corrosion Inhibitors, Bactericides, Emulsion Breakers, Coagulants, Flocculants, Defoamers, Paraffin inhibitors and solvents. In terms of salinity, most produced waters are more saline than sea water.

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of installations</th>
<th>Water quantity (millions of tonnes)</th>
<th>Oil levels (ppm)</th>
<th>Oil quantity (tonnes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996</td>
<td>59</td>
<td>210</td>
<td>27</td>
<td>5,706</td>
</tr>
<tr>
<td>1997</td>
<td>64</td>
<td>234</td>
<td>25</td>
<td>5,764</td>
</tr>
<tr>
<td>1998</td>
<td>67</td>
<td>253</td>
<td>22</td>
<td>5,690</td>
</tr>
</tbody>
</table>

**Canadian Association of Petroleum Product, Technical Report, August 2001**

### 1.3 Produced water disposal methods

Produced water is the largest single wastewater stream in oil and gas production. With the increasing amount of produced water, handling of produced water has become one of the main issues in petroleum industry. Required facilities and equipment for treatment of the produced water are expensive. Therefore companies are trying to develop new technologies to minimize the production of water and consequently reduce the costs of water treatment.
methods and look for ways that with existing facilities can handle larger volumes of water. Produced water from onshore wells is recycled and reused.

There are different ways of disposing produced water. Some of the common methods are:

- Evaporation pits:
- Surface discharge/overboard disposal
- Deep aquifer injection
- Agricultural use (Irrigation of fruit trees or forage land…)
- Industrial use (Dust control, Vehicle washwater, power generation…)
- Desert flooding / livestock water pits
- Shallow water aquifer recharge
- Produced Water Re-Injection (PWRI)

It should be noted that the choice of produced water disposal methods is dependent on several factors, such as site location, regulatory acceptance, technical feasibility, cost and availability of infrastructure and equipment.

After treatment, produced water still contains traces of oil, chemicals and solids and because of this discharging into the environment is strictly controlled by the legislation. Besides some international rules, each country has its own tough and strict regulations which forbid companies from discharging contaminated produced water into the environment. Also according to rules the amount of discharged water per day should be controlled and limited. Table 1-2 compares existing standards of four countries. Because of the severe environmental effects, discharge of water for most onshore wells is prohibited. Table A-1 (in Appendix A) shows an example of effects of discharging of produced water in the sea. Tables A-2 also shows different aspects of disadvantages of discharge of produced water. Another problem comes from the fact that many oil fields are far from cities and costs of transportation facilities are added to previous costs.

**Table 1-2: Produced Water Treatment Standards Comparison, OLF, The Norwegian Oil Industry Association. (1991).**

<table>
<thead>
<tr>
<th>Country</th>
<th>“Best available Technology”</th>
<th>Effluent Limits</th>
<th>Monitoring Requirements</th>
<th>Exception Thresholds</th>
<th>Routine Reporting</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>Gas Floatation</td>
<td>29 mg/L monthly avg. 42 mg/L daily max.</td>
<td>Total O&amp;G Gravimetric</td>
<td>Any exception</td>
<td>Annual</td>
</tr>
<tr>
<td>UK</td>
<td>Gas Floatation Hydrocyclones</td>
<td>40 ppm monthly avg. 30 ppm annual avg.</td>
<td>Dispersed O&amp;G 1/day composite O&amp;G 1/yr comprehensive</td>
<td>&gt; 100 ppm</td>
<td>Monthly O&amp;G Annual Comprehensive</td>
</tr>
<tr>
<td>Norway</td>
<td>Gas Floatation Hydrocyclones</td>
<td>40 ppm monthly avg.</td>
<td>Dispersed O&amp;G 1/day composite O&amp;G 1/yr comprehensive</td>
<td>&gt; 40 ppm monthly avg.</td>
<td>Quarterly O&amp;G Annual Comprehensive</td>
</tr>
<tr>
<td>Canada</td>
<td>Not stated</td>
<td>40 ppm 30 day avg. 80 ppm 2 day avg.</td>
<td>Dispersed O&amp;G 2x/day</td>
<td>Any exception</td>
<td>Monthly</td>
</tr>
</tbody>
</table>
The only potential alternative to surface discharge of produced water is underground injection. In underground injection of produced water some factors like availability of space or load capacity for added equipment required for injection, including additional tanks, water treatment equipment and injection pumps should be considered. The subsurface formation must be capable of receiving the injected water at rates equal to or greater than the production rate. This may require several wells. The subsurface formation must also have long term capacity to accept the produced water volumes that will be generated.

The pressure of a reservoir decreases with time leading to significant reduction in the production. Thus in order to produce more oil pressure should be maintained. There are different ways to enhance oil recovery; one of them is water injection. Therefore, one of the most economical and environmentally friendly methods of discarding the produced water is its re-injection into a suitable subsurface formation. Figure 1-2 shows the schematic of the produced water re-injection process. By re-injecting the produced water into the actual oil reservoir the other problem of pressure maintenance is also solved simultaneously. Thus, the produced water that would have been a waste product has been transformed to a money resource. In summary the benefits of produced water re-injection (PWRI) are:

- Disposal of produced water
- Pressure maintenance and displacing the crude oil in the reservoir
- Environmentally friendly
- Economic advantages
- Meets new regulations

1.4 Problems

Produced water handling has become a major effort of all waterflood operations. With increasing environmental regulations, more and more produced water is being re-injected. But like other operations, PWRI suffers some problems. The water that comes from the separators is definitely not pure water. It is comprised of water containing residual hydrocarbons, heavy metals, radionuclides, numerous inorganic species, suspended solids and chemicals used in treatment and hydrocarbon extraction. Presence of these contaminants will result in injectivity decline and formation damage.

1.4.1 Injectivity Decline due to PWRI

The ease with which a fluid can be pumped into a well is measured as the ratio of the volume entering the formation during a period of time to the pressure differential between the well bore and the reservoir. The value of the ratio is called “Injectivity Index”.

\[
\frac{q}{\Delta P_{\text{skin}}} = \frac{\mu}{2\pi k h} S_i 
\]

Here \( S_i \) is the total skin factor, \( q \) the injection flow rate and \( k \) the permeability of the reservoir. The expression can be combined with Darcy’s law to give:

\[
\frac{q}{(P_{\text{inj}} - P_e)} = \frac{\mu}{2\pi k h} \left( \ln \frac{r_e}{r_w} + S_i \right) \]

(1.2)
This can be re-arranged to give:

\[
\frac{q}{(P_{oij} - P_c)} = \frac{2\pi kh}{\mu \left( \ln \frac{r_e}{r_w} + S_i \right)}
\]  

Figure 1-2: Schematic of Produced Water Re-Injection (PWRI)

The total pressure drop due to different variables is compared to the pressure drop calculated for an ideal open hole completion, and the result used to calculate the skin factor. The skin presents the combined effect of permeability damage, geometrical effects, and non-Darcy flow.

A rapid injectivity decline is usually observed in the injection processes because of the oil droplets and particles in the injected water. Suspended particles and oil droplets in produced water may deposit in porous media. The deposition of particles in porous media will cause a severe damage to the formation with a significant reduction in permeability. According to (1.3) any decrease in permeability would decrease the Injectivity index and consequently the production, which is not favorable at all.

Damage due to flow of dilute oil-in-water emulsions in porous media depends on several factors. The permeability of the core, the droplet concentration, the concentration of emulsifier present, the flow rate, and the properties of the crude oil all affect the permeability impairment. High pressure gradients, high flow rates, low permeabilities, high oil concentrations and large droplet sizes contribute to a more rapid decline in permeability.

Formation injectivity decline may lead to fracture initiation and growth if the injection rates are to be maintained. Fracture propagation and fracture face plugging are coupled phenomena. [Prasad Saripalli et al]. Fracture propagation can be characterised by the combined influence of injection pressure, changes in pore pressure and thermal stresses due to injection of cold water and an additional pressure drop due to particulate plugging.
1.4.2 Formation Damage due to PWRI

Subsurface injection of produced water generally leads to damage of the formation into which the water is injected. Formation Damage can be defined as a reduction in permeability of the formation. Formation damage leads to loss of production and/or injectivity and consequently loss of money. Several parameters play role in the process of formation damage. The rate and extent of the damage strongly depends on the properties of the porous medium in which the produced water is injected and the characteristics of the injected water itself. In porous media size and distribution of pores and throats and connectivity of pores are important parameters. Concentration and size distribution of particles and also injection rate of the produced water are the parameters which determine the damage to the formation. The problem of formation damage due to PWRI can be decomposed into separate distinct problems. These include internal filtration, external filter cake build-up and the associated permeability reduction (formation damage).

Depending on the details of the case under investigation (e.g. contaminant size with respect to porous medium pore throat size, surface charge, injection rate, etc.) one of the following scenarios may be applicable:

- Pure internal filtration of the contaminants
- Pure external filter cake build-up
- Initial internal filtration followed by external filter cake build-up after a certain transition time $t^*$
- Initial internal filtration followed by simultaneous internal filtration and external cake build-up after a certain transition time.

It is important to know the performance of injection wells as a function of the amount of the suspended particles in the water. Some important decisions should be made in order to quantify the damage. This, being a major issue, attracted researches and many investigations have been conducted. Barkman and Davidson [Barkman et al. (1972)] were among the first ones who investigated the injectivity decline for PWRI. They defined the injectivity half time, which is the time required for the initial injection rate to decrease to 50% of this rate. The formation damage is caused by four mechanisms: wellbore narrowing, invasion, perforation plugging, and wellbore fill-up. For each mechanism they introduce an equation to predict the half-life of an injection well based on the water quality ratio. Water quality ratio (WQR) is the ratio of the concentration of solids to the permeability of the filter cake formed by those solids. The WQR can be measured in the laboratory with constant pressure drop test using cores or membrane filters. As cake filtration proceeds, the filtration volume $V$ will grow linearly with $\sqrt{t}$. The slope of the $V-\sqrt{t}$ plot may be used to calculate the WQR. Barkman and Davidson could apply their equations just for constant pressure conditions, whereas real injectors are rate-controlled. Hofsaess and Kleintz (2003) introduce a revised equation for the WQR after Barkman and Davidson. They introduced the half-volume concept and presenting the injectivity decline in terms of accumulated injection volume, they overcome the problem of using constant pressure. The latest proposed models [M.M. Sharma (1994), D.Marchesin, P.Bedrikovetsky (2002)] about the injectivity decline are mainly based on classical deep bed filtration in which they try to find a filtration coefficient and damage coefficient which conform with the obtained data from the laboratory experiments. This theory will be explained more in chapter 2.
1.5 The Objectives of This Research

Due to the importance of the environmental issues and the huge amount of produced water, the subsurface injection of produced water has earned a lot of attention. Although it is known that the re-injection of the produced water would damage the formation and result in the reduction of the production, the extent and mechanism of the damage is still unclear. The objective of this study is to carry out laboratory experiments in order to investigate and predict the damage. This research is done both on internal filtration and external cake build up. For the internal filtration results the classical deep bed filtration theory will be applied. For the external cake build up a new model which is developed by the PWRI group is presented.
CHAPTER 2
INTERNAL FILTRATION

2.1 Introduction

The problem of formation damage is split into two separate parts which includes internal filtration and external cake build-up. In this chapter we will discuss the theories and proposed models of internal damage resulting from produced water re-injection.

In this chapter, first flow of the suspension in a porous medium is explained. Then the retention sites and filtration mechanism are introduced. The filtration theory and its mathematical modelling are also given in this chapter. In the following section, the filtration coefficient and effect of the different parameters will be explained. Finally, the permeability reduction models will be discussed. The permeability reduction model tries to model the effect of particle deposition on flow properties such as permeability. Consequently, different measurement techniques are discussed and description is given to how they are used to obtain the unknown parameters in the filtration theory.

2.2 Suspension flow in porous media

Porous media (rocks, soils, catalyst pellets, packed beds...) and particulate systems (suspensions, colloids, emulsions, bacteria...) are present in most engineering practices. Consequently a lot of investigations have been done in order to understand the mechanisms of the transport of the particulate matters through a porous medium. However, due to the complexity of the transport phenomena this issue is not fully understood.
Throughout this review colloidal materials suspended in water are referred to as particles. A porous medium is a fixed bed of granular materials with pores through which fluid can flow. Apparently, when a suspension flows through a porous medium, particles are brought in contact with retention sites. In this case there are two possibilities for particles: either they are deposited or they are carried away by the stream. Separation of particles can change the fluid properties as well as the permeability of the medium. The reduction in permeability value can be simply determined by applying Darcy’s law for different conditions.

2.2.1 Retention sites

Each sand grain can collect a number of particles in the porous medium. Therefore the deposition surface is called the collector. There are various sites that can retain a particle when transported through a porous medium.

2.2.1.1 Surface sites

In this case, a particle stops when it reaches a grain surface, and is deposited on the surface of the grain. This happens when the particle is too large to penetrate into the medium.

2.2.1.2 Crevice sites

Sorting and ordering of the grains may result in forming convex surfaces. Thus, some particles will get stuck at the convex surfaces.

2.2.1.3 Constriction sites

When the diameter of a particle is larger than that of a pore, particle will stop and get deposited.

2.2.1.4 Cavern sites

A particle may find some sheltered area in its path and stay there. These sheltered areas are small pockets formed by several grains. In these shelters the particles are protected from the flow stream.

Figure 2-1: Different retention sites: (a) surface site, (b) crevice site, (c) constriction site, (d) cavern site

2.3 Filtration Theory

When fluid containing particles reaches a porous medium, the liquid and the solid phases in the suspension can be separated, either by depositing in the pore or accumulating in front of the surface. This is similar to what happens in the filtration process. Then, the retention
process of particles when flowing through a porous medium is called Deep Bed Filtration. With deep filtration the particles to be captured are entrapped in the interstices among particles held in a deep bed. The deep filtration occurs because of several mechanisms: the contacting of particles with the retention site, the fixing of particle sites, and the breaking away of previously retained particles [Hertzig et al. (1970)].

2.3.1 Filtration mechanisms

Particles are deposited due to different mechanisms that are listed as:

2.3.1.1 Interception

Interception happens when a particle following a streamline hits the surface of a grain and attaches to it. Particles with a density equal to the fluid density follow the streamline in the porous media, especially at low velocities. Eventually particles will be brought into contact with grain surfaces, because of the finite size of the particles. To calculate the probability of collision the stream model of Happel is used.

\[ \eta_i = \frac{3}{2} A_s \frac{d_p^2}{d_g^2} \]  

Here, \( \eta_i \) is the collision probability, \( d_p \) diameter of the particle and \( d_g \) the diameter of the grain. \( A_s \) is the porosity dependent parameter which is calculated by:

\[ A_s = \frac{2(1 - \gamma^5)}{2 - 3\gamma + 3\gamma^2 - 2\gamma^6} \]  

with \( \gamma = (1 - \phi) \frac{1}{3} \).

When the particle is retained by a previously deposited particle we refer to it as bridging. The simplest bridging process that exists is blocking or sieving of the mobile particles. Case B in Figure 2-2 shows the bridging of two particles in a pore throat. This effect will occur when two particles arrive at the same moment to pass through. As mentioned earlier, another possibility is when one particle is already attached to the grain and the second particle wants to pass. Case C in Figure 2-2 shows the bridging of three particles.

2.3.1.2 Impaction (Inertial impact)

When the density of particle is larger than that of the fluid, inertia forces will deviate the particle from the stream line and it may attach to the surface of a grain.

The collision probability due to impaction (\( \eta_{im} \)) can be written as:

\[ \eta_{im} = \frac{(\rho_p - \rho_f)d_p^2v_p}{18\mu d_m} \]  

Here, \( \rho_p \) and \( \rho_f \) are the densities of the particles and fluid, respectively, \( v_p \) is the velocity of the particle and \( \mu \) is the fluid viscosity. In fact impaction is important when the inertia of the approaching particulates is greater than the hydrodynamic forces. This is the reason why impaction is more significant for air filtration and not for water filtration.
2.3.1.3 Sedimentation

When the density of particles is different than the density of the fluid, the velocity of the particles will be different from the velocity of fluid and as result particles will deviate from streamlines. Therefore, particles will be subjected to gravity and get into the contact with grains.

The collision probability due to sedimentation ($\eta_s$) is equal to:

$$\eta_s = \frac{d_p^3 (\rho_p - \rho_f) g}{18 \mu U}$$  \hspace{1cm} (2.4)

Where $\rho_p$ and $\rho_f$ are the densities of the particles and the fluid, respectively, $g$ is the acceleration due to gravity, $\mu$ fluid viscosity and $U$ the fluid velocity.

2.3.1.4 Diffusion

Small particles will be subject to random Brownian motion that increases the number of collisions between particles and grains. The collision probability due to diffusion ($\eta_d$) is equal to:

$$\eta_d = 0.9 \left( \frac{kT}{\mu d_p d_U} \right)^{2/3}$$  \hspace{1cm} (2.5)

Where, $k$ is the Boltzman constant and $T$ is the absolute temperature. Diffusion is more important for small particles ($d_p < 1 \mu m$) and is usually neglected for larger particles.

2.3.1.5 Straining

When a particle enters a throat that is too small for it to pass through, it gets stuck there. This phenomenon is called straining or size exclusion. The most critical factor determining straining within a porous medium is the ratio of the media (throat) diameter to the particle diameter. When this ratio is small it means that the throat size is too small for a particle to pass through. When this ratio is too small (less than 10) cake will be built up on the surface of the media. Hertzig et al. [1937] consider straining as a purely geometrical process. Case A in Figure 2-2 shows the straining of a particle in a porous medium.

![Figure 2-2: Relation between bridging effects and particle–pore distribution. (de Zwart et al., 2004)](image-url)
2. Internal Filtration

2.3.1.6 Surface forces

Van der Waals attraction forces and double layer repulsion forces are also significant for the capture of the particles. (For more details see Chapter 4)

The above mentioned mechanisms will play role in the deposition process of the particle in contact with the grain surface. The dominance of the force is depending on filtration medium properties, particle properties and flow. Details of surface forces (van der Waals attraction forces and double layer repulsive forces) will determine whether a particle will stick to the grain or not. In other words, if the sum of the hydrodynamic and electrostatic forces is attractive, a particle will be retained and if the sum is repulsive particles will not adhere to the grain.

Rajagopalan proposed an equation to calculate the collision probabilities in which the collision probability due to electrostatic forces is calculated by:

\[
\eta_E = A \left( \frac{4A}{9\pi \mu} \right)^{1/8} \left( \frac{d_p^{13/8}}{(\varphi v_p)^{1/8} d_g^{15/8}} \right) \tag{2.6}
\]

Where, \( A \) is the Hamaker constant \( \left( 10^{\text{-}20} \text{ J} \right) \).

The total collision probability due to diffusion, interception, impaction, surface forces and sedimentation is calculated by:

\[
\eta = \eta_d + \eta_i + \eta_{im} + \eta_s + \eta_E \tag{2.7}
\]

The total collision probability can be used to estimate the filtration coefficient, which will be explained later.

<table>
<thead>
<tr>
<th>Table 2-1: Input Data for Figure 2-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity</td>
</tr>
<tr>
<td>Diameter of Grains</td>
</tr>
<tr>
<td>Density of Particles</td>
</tr>
<tr>
<td>Density of Fluid</td>
</tr>
<tr>
<td>Viscosity of Fluid</td>
</tr>
<tr>
<td>Velocity</td>
</tr>
</tbody>
</table>

Depending on the size and shape of the particles one of the above mentioned mechanisms will prevail. For example for very small particles surface and diffusion forces become most important while for larger particles these mechanism become less important. Figure 2-3 shows the effect of size of the particles and media on the total collision probability and the dominant mechanisms in different sizes of particles. To obtain this graph values in Table 2-1 were used, which are characteristic of the core samples, the fluids and the velocities we used in our experiments. Figure 2-3 indicates that in our experiments (\( 1 < d_p < 10 \mu m \)), the more dominant mechanism is interception. Also it can be seen that for the range of velocities we use, impaction is negligible according to the model we use. Since the effect of diffusion is an order of magnitude smaller than that of interception we neglect the effect of diffusion in our modelling. We have to notice that the model we have used to plot this graph does not take
into account the mechanisms like straining, bridging and surface forces. As a result, this graph may change with other models.

![Graph showing effect of particle size on collision probability](image)

**Figure 2-3: Effect of particle size on collision probability**

### 2.4 Mathematical model for one-Dimensional flow of water with suspended particles

Deposition of the particles in porous media is described mathematically by the mass balance equation (Equation (2.8), for more details see Appendix B) and the kinetic equations describing the rate of deposition.

Generally the mass balance equation can be simplified since diffusion of particles is not important nor are suspended particles with respect to the retained ones [Hertzig (1970)]

Injectivity decline and formation damage occur in most waterflood projects to some degree. Damage is due to deposition of the suspended particles in the displacing fluid. In most of the cases suspended particles would block the throats, so that the permeability will decline.

Subsequently we are interested in the flow of water containing suspended particles. It is important to predict the damage and also the performance of the injectors as a function of injection water quality.

Starting with a simple model, we assume that:

- The displacing fluid (water) and the deposited particles are incompressible.
- The density of the solid particles is equal in both dispersed and deposited states.
2. Internal Filtration

- The velocity of the fluid along the core is constant. Also we assume a constant velocity with time. Therefore, the conservation of the total flux is:
  \[ \text{div}\,U = 0 \]

- The volume of the entrapped particles is negligible compared to the effective porosity. \((\sigma \ll \varphi)\)

- The kinetics of the particles is linear.

- The dependency of the viscosity on concentration is negligible.

- Diffusive and dispersive effects are negligible.

For such a problem, the mass conservation equation for the retained and deposited particles in linear flow of an incompressible fluid can be written as:

\[
\frac{\partial}{\partial t} (\varphi c + \sigma) + U \frac{\partial c}{\partial x} = 0
\]  
\[ (2.8) \]

Here \(\varphi\) is the effective porosity, \(c(x,t)\) and \(\sigma(x,t)\) are the volumetric concentrations of the suspended and deposited particles respectively. \(c\) can have values between 0 and 1, whereas the quantity \(\sigma\) has values between 0 and \(\varphi\). \((0 \leq c \leq 1, 0 \leq \sigma \leq \varphi)\). (2.8) basically states that net increase of the particles in the system is equal to the upstream entering particles minus the downstream exiting particles minus the deposited particles.

In this model we assume that permeability reduction is only due to the deposition of particles and that permeability is a decreasing function of the deposited concentration.

Darcy’s law relates superficial velocity to pressure:

\[
U = -\frac{k_0 k(\sigma)}{\mu} \frac{\partial P}{\partial x}
\]  
\[ (2.9) \]

Here \(k_0\) is the absolute permeability, \(k(\sigma)\) is the relative permeability, when expressed as a function of \(\sigma\), it is also called “Formation Damage Function”. It can be normalized so that: \(k(0) = 1\)

The rate of deposition is proportional to the concentration of suspended particles and fluid velocity:

\[
\frac{\partial \sigma}{\partial t} = \lambda(\sigma)|U|c
\]  
\[ (2.10) \]

The constant of proportionality \(\lambda(\sigma)\) is the “Filtration Coefficient” function. (2.10) is only valid if \(\sigma \ll \varphi\) otherwise \(\varphi\) would change with deposition and problem would be more complicated.

The filtration coefficient \(\lambda\) is a function of deposited particles \(\sigma\) and fluid velocity. In our model we assume that the velocity is constant, thus automatically we neglect the effect of velocity and we write \(\lambda(\sigma)\) instead of \(\lambda(\sigma, U)\). However, it is important to mention that for radial flow near the borehole the velocities are very large and the velocity effect becomes important.

We introduce dimensionless length, time, scaled concentration \(C\) and the filtration coefficient \(\lambda\) as:
Produced Water Re-Injection

\[
\begin{aligned}
X &= \frac{1}{L} x \\
T &= \frac{U}{\phi L} \\
C &= \frac{c}{c_0} \\
S &= \frac{\sigma}{\phi c_0} \\
\Lambda(S) &= L\lambda(\sigma)
\end{aligned}
\]  

Then equations (2.8) and (2.10) become:

\[
\frac{\partial C}{\partial T} + \frac{\partial C}{\partial X} = -\frac{\partial S}{\partial T} 
\]

(2.12)

\[
\frac{\partial S}{\partial T} = \Lambda(S)C 
\]

(2.13)

Combining (2.12) and (2.13) yields:

\[
\frac{\partial C}{\partial T} + \frac{\partial C}{\partial X} + \Lambda(S)C = 0
\]

(2.14)

As can be seen to solve (2.14) we need to know the equation for filtration coefficient, \(\lambda\). For example assuming a constant value for \(\lambda\), the concentration profile can be obtained by:

\[
C = e^{-\lambda_0 x}
\]

(2.15)

[For more details see appendix C]. Marchesin et al. [2002] have solved (2.14) with taking different functions for \(\Lambda(S)\).

### 2.4.1 Relation between Concentration of Suspended and Deposited Particles

Putting (2.15) into (2.13) and solving the obtained equation will give the solution for deposition which is:

\[
\sigma = (T - X)\Lambda_0 \exp(-\Lambda_0 X)
\]

(2.16)

This solution is valid when \(T \geq X\).

Hertzig et al. [1970] proved the following relation between \(\sigma\) and \(c\) and their initial values:

\[
\frac{c(X,T)}{c(0,T)} = \frac{\sigma(X,T)}{\sigma(0,T)}
\]

(2.17)

(2.17) applies when \(c\) and \(\sigma\) are taken at the same depth \(X\), and \(c, c_0, \sigma\) and \(\sigma_0\) are taken at the same \(T\). Hertzig et al. [1970] derived the following equation for the clogging front velocity:

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2. Internal Filtration

\[
V_F = -\frac{\partial \sigma}{\partial T} = U \frac{c}{\sigma}
\]  
(2.18)

The existence of such a constant clogging front can be only seen in the plot of \( \sigma \) against \( x \). Since \( \partial c / \partial x \) depends on both \( c \) and \( \sigma \), this feature can be seen from the plot of \( c \) against \( x \) only if the concentration at the inlet of the core is assumed constant.

2.5 Filtration Coefficient

The basic filtration equation (2.10) was proposed by Iwasaki [1937]. This equation basically indicates that the rate of deposition is a function of the concentration and velocity of the suspension with the filtration coefficient as proportionality constant.

Equation (2.10) is sometimes called the “Kinetic Equation”. The filtration coefficient \( \lambda \) is a function of several parameters and has the dimension of \( m^{-1} \). According to literature filtration coefficient is mainly dependent on the number of previously deposited particles, concentration of the suspension, velocity, size of the particles and size of the grains.

\[
\lambda = \lambda(\sigma, c, u, d_p, d_g, \ldots)
\]  
(2.19)

For the dilute suspensions the dependency on concentration is usually neglected in order to get a linear relationship between \( \sigma \) and \( c \).

Hertzig et al. [1970], proposed different filtration coefficients for the different mentioned mechanisms. Generally, \( \lambda \), can be related to superficial velocity and size of particles and grains as:

\[
\lambda \propto \begin{cases}
    u^{-\alpha} \\
    d_p^\beta \\
    d_g^{-\gamma}
\end{cases}
\]  
(2.20)

2.5.1 Effect of Velocity

As mentioned before for large particles, size exclusion or straining is the main mechanism of deposition. When straining is the case or at least for the viscous flow regime, the filtration coefficient is not a function of velocity, leading to \( \alpha = 0 \). When \( \alpha = 1 \), the retention rate, \( \frac{\partial \sigma}{\partial t} \) is independent of velocity. Fitzpatrick and Spielman [1973] did various experiments with different particle sizes. They found out that \( \alpha \) increases with increase in size of particles and its value is mostly between zero and two. Nevertheless, larger values for \( \alpha \) have been reported (Ison et al. [1969]). In conclusion the dominant filtration mechanism determines the effect of velocity in the value of filtration coefficient. Moreover, Darcy’s velocity definition \( \left( v = \frac{u}{\varphi} \right) \) underestimates the interstitial velocity, because it does not take into account the tortuosity and dead end porosity. Thus, it has been a big discussion whether Darcy’s velocity should be used or not (Rochon et al [1996]). When the effect of porosity on \( \lambda \) is considered separately, Darcy’s velocity can be used (Wennberg et al. [1997]).
2.5.2 Effect of Particle and Grain Size

For small particles ($d_p < 1\mu m$) diffusion might be the main mechanism of the particle transport through the porous medium. This will result in negative values for $\beta$ in (2.20). For slightly larger particles $\beta$ is positive. This means that the larger the size of the injected particle, the larger the value of the filtration coefficient. Negative and positive values mean the existence of a minimum value for $\beta$. This minimum can be determined experimentally (Fitzpatrick and Spielman [1973]). Also experiments show a value of between 0-2. The average value is about 0.9.

2.5.3 Effect of Porosity

Higher porosity values will result in lower values for the filtration coefficient, because in high porous rocks the available space for particles to be captured is higher. Fitzpatrick and Spielman [1973] did experiments at two different porosity values, 0.38 and 0.41. They found out that for latter case the values of the filtration coefficient were 30% lower than for the other case under all conditions.

2.5.4 Effect of Ion Concentration

Experiments show that at electrolyte concentrations below $10^{-3} \text{ M}$, the filtration coefficient starts to drop rapidly, because with increasing concentration the zeta potential $\xi$ increases. A higher zeta potential means higher double layer repulsion forces. Therefore, higher concentrations make a barrier for the depositing of the particles.

2.6 Determination of Filtration Coefficient

The filtration coefficient is a key parameter in the modeling of the water injection and, in general, in formation damage problems. In order to solve the governing partial differential equation (2.14) we need to find a value or a function for $\lambda$. In this section different methods for finding $\lambda$ are explained.

2.6.1 Collision probabilities

Using (2.7) one can determine the total collision probability due to different filtration mechanisms and calculate the filtration coefficient by:

$$\lambda = \frac{3}{2} \left(1 - \varphi\right) c \xi \eta$$  \hspace{1cm} (2.21)

Where $\alpha_c$ is clean-bed collision efficiency. $\eta$ can be calculated using any of the available models and $\alpha_c$ is determined from experimental data. Putting (2.21) into (2.15) yields:

$$\frac{c}{c_0} = \exp\left(-\frac{3}{2} \left(1 - \varphi\right) \alpha_c \eta L\right)$$  \hspace{1cm} (2.22)

Although this method is acceptable to fit the experimental data (Jiamhong et al. [2001]) it does not consider some mechanisms like strain. Nevertheless, it can be used to guess the initial value of the filtration coefficient.
2.7 Permeability Reduction Models

Due to deposition the porous medium will change during filtration. This effect is incorporated in the filtration function $F(\sigma)$ defined as: $\lambda = \lambda_0 F(\sigma)$, which defines the dependency of the filtration coefficient on the initial coefficient and deposition of the particles. Also for the filtration function a variety of empirical relations are found, based on different stages during filtration (filter ripening, re-entrainment or blocking). In Hertzig et al. [1970] a list of expressions of $F(\sigma)$ is given. The most general form is proposed by Ives [1970]:

$$\lambda = \lambda_0 \left( 1 + \frac{\beta \sigma}{\phi_0} \right)^y \left( 1 - \frac{\sigma}{\phi_0} \right)^z \left( 1 - \frac{\sigma}{\sigma_M} \right)^x$$

(2.23)

where $\beta, x, y$ and $z$ are empirical parameters and $\sigma_M$ is the maximum value of $\sigma$ for which $\lambda$ drops to zero.

The extent of damage depends on the amount of deposited particles. Therefore, the reduction in permeability is linked to $\sigma$. Deposition of particles in the pores and throats of the medium will decrease the permeability. Since in the experiments one of the measured parameters is the pressure drop along the core, it is necessary to have the relationship between the permeability and the deposition concentration. To that end, the Kozeny-Carman equation is used:

$$i_0 = -\frac{d\Phi}{dx} = \frac{6 \mu U}{g} \frac{(1-\varphi)^2}{\varphi^3} \left( \frac{6}{d_g} \right)^2$$

(2.24)

$\Phi$ is the hydraulic head and $i_0$ is the clean bed gradient. Combining Darcy’s law and (2.24) gives:

$$K = \frac{g d_g^2}{216 \mu \left( 1 - \varphi \right)^2}$$

(2.25)

$K$ is the hydraulic conductivity which is related to medium permeability by:

$$K = \frac{k g}{\mu}$$

(2.26)

Combining (2.25) and (2.26) leads to:

$$k = \frac{d_g^2 \varphi^3}{216 \left( 1 - \varphi \right)^2}$$

(2.27)

(2.27) indicates that the permeability is only a function of the medium parameters. Nevertheless, deposition of the particles in the porous media increases the medium surface area to volume ratio and decreases the porosity. In literature there are some proposed models for increased head loss caused by deposited particles. Many of them are in the form of:

$$R_\sigma = \frac{i}{i_0} = \frac{1}{(1 - a \sigma)^n}$$

(2.28)
where \( R_\sigma \) is the resistance due to deposition. \( \alpha \) and \( n \) are positive parameters (McDowell-Boyer et al. [1986]. Using a power series (2.28) becomes:

\[
R_\sigma = \left( 1 + n \alpha \sigma + \frac{n(n+1)}{2} \alpha^2 \sigma^2 + \ldots \right)
\] (2.29)

By introducing \( \beta_n \) as formation damage factor a permeability reduction model can be written as:

\[
k_\sigma = \frac{1}{R_\sigma} = \frac{1}{1 + \beta_1 \sigma + \beta_2 \sigma^2 + \ldots}
\] (2.30)

### 2.8 Utilising the pressure drop measurements

Impedance \( j \) is the inverse of the injectivity index.

\[
j = \frac{\Delta P}{U}
\] (2.31)

The non-dimensional form of impedance is:

\[
J = \left[ \frac{\Delta P}{U} \right] \left[ \frac{\Delta P_0}{U_0} \right]^{-1}
\] (2.32)

where \( \frac{\Delta P_0}{U_0} \) is the initial impedance of the system prior to injection of produced water.

The 3-point pressure method (Bedrikovetsky et al. [2001]) utilises Darcy’s law (2.33) and a linear resistance relationship for the resistance (2.34).

\[
U = -k_0 k(\sigma) \frac{\partial P}{\partial x}
\] (2.33)

\[
R_\sigma = \frac{1}{k_\sigma} = 1 + \beta \sigma
\] (2.34)

Using these equations Bedrikovetsky et al. [2001] developed a model to calculate the filtration coefficient using pressure data at 3 points and the following equations:

\[
J = 1 + m T \quad \text{Where } m = \beta \left( 1 - \exp(A_0) \right)
\] (2.35)

\[
J = 1 + m T_{w_t} \quad \text{Where } m_t = \beta \left( 1 - \exp(A_0 \omega_t) \right)
\] (2.36)

where \( T_{w_t} = \frac{T}{\omega_t}, \omega_t = \frac{x_t}{L} \) and \( T \) is the number of pore volumes injected.

As can be seen from (2.35) and (2.36) if we plot \( J \) vs. \( T \), its slope will present the value of \( m \) and \( m_t \). With these values and having the equation of \( R_\sigma \), filtration coefficient could be calculated.

Although the linear relationship (2.34) is widely accepted, Al-Abduwani et al. [2004a] introduced quadratic relationship to calculate the filtration coefficient.
2.9 . Utilising the deposition profile

The deposition profile can be utilised in quantifying the filtration coefficient in two different ways:

2.9.1 Utilising a single deposition profile at a given time

In order to obtain the deposition profile Al-Abduwani et al. [2003] used a method called post-mortem analysis. This analysis gives an average value per segment $< S_i >$. Therefore, the cumulative deposition $M_i$ in a given segment $i$ can be obtained by:

$$M_i = < S_i > (X_i - X_{i-1})$$  \hspace{1cm} (2.37)

or:

$$M_i = \int_{X_{i-1}}^{X_i} S(X) dX$$  \hspace{1cm} (2.38)

Thus using $S = T\Lambda_0 \exp(-\Lambda_0 X)$ we can write:

$$\frac{M_i}{T} + y^{X_i} = y^{X_{i-1}}$$  \hspace{1cm} (2.39)

where $y = \exp(-\Lambda_0)$. (2.39) can be solved for each segment separately or with putting $X_i = 0$ and $X_i = 1$, it can be used to calculate the cumulative deposition over the entire core.

2.9.2 Utilising the deposition profile along the core at different parts

To use this method we have to know the concentration and deposition profile at the same position at different times. Such information can be obtained if we assume that the average concentration in the last segment is what we measure in the effluent. The deposition profile can be obtained by non-invasive techniques such as CT-scanner monitoring (Al-Abduwani et al. [2004b]). The invasive way is to run multiple experiments with the same parameters and end them in different times. Then (2.13) becomes:

$$\frac{S_2 - S_1}{T_2 - T_1} = \Lambda_0 C_{1.2}$$  \hspace{1cm} (2.40)

from which the filtration coefficient can be calculated. This technique is a differential technique and suffers instability and magnificence of measurement errors.

2.10 Concept of Transition Time

Experimental results show that when we start to inject, particles mostly deposit at inlet and near inlet face, consequently pores will become filled with particles and after that external cake starts to build up. The time that this happens is called transition time. Transition occurs when porosity reaches critical porosity. Pang [1996] gave the following formula to calculate
the transition time:

\[
t^* = \frac{1}{C_d a_g} \int_0^{n^*} \frac{d_n}{\eta(n)}
\]

(2.41)

where, \( n \) is the number of particles attached to one grain, \( n^* \) the number of particles attached to one grain when \( \eta = 1 \), \( a_g \) is the cross-sectional area of a grain, \( C_n \) the number concentration of particle per unit liquid volume. To calculate \( \eta(n) \) Pang determined the function from Stokesian dynamics simulation and found out that: \( \eta = \eta_0 + b'n \) and therefore:

\[
\int_0^{n^*} \frac{d_n}{\eta(n)} = \frac{1}{b'} \ln \frac{1}{\eta_0}
\]

(2.42)

Pang and Sharma [1997] modified this concept. They choose to operate in terms of filtration coefficient rather than the collection efficiency. The concept of transition time is just an approximation and can never be exactly true.
3.1 Introduction

Various forces act on a particle in a colloidal system. Particles are in a continuous motion under the influence of these forces until they get deposited or go out of the system. These forces are:

- Drag forces
- Lift forces
- Net Gravity force
- Brownian force
- Screened electrostatic and steric repulsions
- Van der Waals attractions
- Friction Force

To understand the problem better and to be able to model the problem, origin and nature of these forces should be known and in order to simplify the problem some assumptions should be made. Figure 3-1 shows a particle on the cake surface with various forces acting on it. It should be noted that in this chapter we will consider both static (dead end) and dynamic (crossflow) filtration.

3.2 Drag Force

Drag is the component of the force acting on a body parallel to the opposite direction to motion. In colloidal systems drag forces arise due to the resistance of surrounding fluid to the motion of particles. The resistance is caused by the viscous shear of the fluid flowing over the particles and by the pressure difference between the upstream and downstream sides of the
moving particle. The viscosity of the suspending fluid is a measure of the resistance of the fluid to particle motion. The drag force acts in a direction opposite to the particle velocity, thus opposing the particle motion. For a spherical particle with diameter $d_p$, moving through a viscous incompressible and Newtonian fluid with speed of $V$, the drag force is dependent on the fluid properties, i.e. viscosity and density.

$$F_p = f(d_p, V, \mu, \rho)$$

In the crossflow filtration the velocity has two components: one is parallel to the surface and the other is perpendicular to the cake surface. Consequently, there are two drag forces in the crossflow filtration: Tangential drag force that is caused by the crossflow velocity ($u_{cf}$) and Normal drag force that is caused by permeate velocity $u_p$, also known as filtrate velocity.

![Diagram of forces acting on a particle in cake surface in crossflow filtration](image)

**Figure 3-1: Schematic of the forces acting on a particle in the cake surface in a crossflow filtration process, GRC poster**

### 3.2.1 Flow over a sphere: Friction and Pressure Drag

In this case both the friction (tangential) drag and the pressure (normal) drag contribute to the total drag [Fox (1994)]. In colloidal suspensions in water, the Reynolds number, which is the ratio of inertial force to viscous force, is small.

$$Re = \frac{\text{inertial force}}{\text{viscous force}} = \frac{\rho V^2}{d} \frac{d}{\mu V} = \frac{\rho V d}{\mu}$$

In our case with a maximum velocity of $V = 0.006$ m/sec, a maximum particle size of $d_p = 10\mu m$ and assuming that the suspension has the same properties of water, since it is dilute, maximum value of the Reynolds number is: $$Re = \frac{(1000)(0.006)(10^{-5})}{10^{-5}} = 0.06$$
3. Forces Acting on a Particle in a Colloidal System

For a Reynolds number of this magnitude the flow is accurately predicted by Stokes flow. Stokes flow indicates that at very low Reynolds numbers, \( \text{Re} \leq 1 \), there is no flow separation from a sphere and the drag is predominantly friction drag. Stokes has shown analytically that for very low Reynolds numbers for which inertia forces may be neglected, the drag force on a sphere of diameter \( d_p \), moving at speed \( u_p \), through a fluid of viscosity \( \mu \), is given by:

\[
F_D = 3\pi \mu d_p u_p \tag{3.1}
\]

This equation indicates that the drag force has a higher magnitude for a fluid with higher viscosity and for larger particles. As particle velocity increases, the resistance to motion increases linearly resulting in higher magnitude of drag force. (3.1) can be used to estimate the drag forces caused by permeate or filtrate velocity in crossflow filtration process. With (3.1) drag coefficient can be calculated as:

\[
C_D = \frac{F_D}{\frac{1}{2}\rho V^2} = \frac{24}{\text{Re}} \tag{3.2}
\]

### 3.2.2 Flow over a flat plate parallel to the flow: Friction Drag

In this case pressure gradient is zero, and then the total drag is equal to the friction drag:

\[
F_D = \int \tau dA
\]

Here \( F_D \) is the drag force caused by crossflow velocity, \( \tau \) the shear force, \( A \) is the total surface area in contact with the fluid (the wetted area). Then the following formula can be written for drag coefficient:

\[
C_D = \frac{F_D}{\frac{1}{2}\rho V^2} = \frac{\int \tau dA}{\frac{1}{2}\rho V^2}
\]

The drag coefficient for a flat plate parallel to the flow depends on the shear stress distribution along the plate. Shear stress will result in higher drag force. According to theory and results of experiments done by Rubin et al. [1977] the drag force in a linear wall-bounded shear flow is given by:

\[
F_D = 2.11 F_{\text{Stokes}} = 6.33 \pi \mu \rho d_p u_{ef} \tag{3.3}
\]

### 3.3 Lift Force

In a colloidal system lift force is caused by the shear flow. Lift on a particle plays a central role in several applications e.g. in the oil industry we can consider the removal of drill cuttings in horizontal drill holes and sand transport in fractured reservoirs. Lift force is a consequence of the velocity gradient. When the fluid reaches particle it is believed that velocity on top of the particle is higher than in bottom part. As a result, according to
Bernoulli’s equation \( \frac{dp}{\rho} + \frac{1}{2}V^2 + gh = cte \), pressure in top part is less than pressure in bottom part. Particle will move up because of this pressure difference.

According to the experiments and theoretical studies from Rubin et al. the lift force can be calculated as:

\[
F_{L} = 0.761 \frac{\tau_{\nu}^{1.5} d_{p}^{3} \rho_{p}^{0.5}}{\mu}
\]  

(3.4)

Lift force induced by shear flow is always perpendicular to drag force caused by that velocity. It is in the direction of permeate flow. So, its magnitude with respect to normal drag force will play an important role in deposition of the particles. It means if lift force acting on a particle is higher than drag force caused by permeate velocity, it will not stay on the layer or if it is already on the cake layer it will be removed and re-entrained to the fluid. In other words the particle will deposit only if drag forces are higher than lift forces. This is the case in higher filtration rates. Also (3.4) shows that lift force is dependent on the size of particles and the larger the size of the particles the higher the lift forces are.

### 3.4 Net Gravity Force

Buoyancy force results from the density difference between particles and the fluid in the suspension. Depending on the magnitude of particle density, buoyancy can act vertically upwards or vertically downwards. If the density of particle is higher than the density of fluid the force will act in vertically upward direction and if the density of particle is less than the density of fluid the buoyancy will act in vertically downward direction. This can also be concluded from the general formula of buoyancy force for fully immersed particle:

\[
F_{B} = \frac{1}{6} \pi d_{p}^{3} g(\rho_{p} - \rho_{w})
\]  

(3.5)

Here, \( F_{B} \) is the Buoyancy force, \( d_{p} \) mean diameter of the particles and \( \rho_{p} \) and \( \rho_{w} \) the densities of particles and fluid, respectively. (3.5) shows that buoyancy force is only dependent on the size of particles and density difference between the particles and the fluid and not velocity. In our case density of hematite \( (\rho_{hem} = 5.3 \text{gr/cm}^3) \) is higher than water and the buoyancy force is vertically upwards. Sometimes buoyancy force is called Net Gravity Force \( (F_{G}) \).

### 3.5 Interparticle forces

The interparticle forces can be estimated by using the DLVO theory. This theory is a very good approximation for low concentrated solutions. According to this theory the major long-range interaction forces acting on two approaching particles are electrostatic force, \( F_{e} \), and van der Waals force, \( F_{vdW} \). (Figure 3-2)
3.5.1 Electrostatic Interactions

The electrostatic repulsions between the particles create an energy barrier that prevents the coalescence of the particles. However, the electrostatic interaction between charged colloidal particles involves not only the interactions between the colloidal particles, but also the interactions between colloidal particles and the sea of ions surrounding the particles. As every particle has a finite surface charge there should be an excess of ions of opposite charge in the solution to maintain overall electric neutrality of the system (DLVO theory 1942). Oppositely charged ions in the system are attracted towards the ions present on the surface of particles. But the thermal motion of the ions counteracts this attraction. Therefore, the ions take an equilibrium position to balance the electrostatic interaction energy with the thermal energy. This results in a diffuse double layer of ions surrounding the particle: one layer formed from the charge on the surface of the particles and other layer form from excess of oppositely charged ions present in the solution.

One of the governing equations in the electrical double-layer region is Poisson’s equation, which can be derived from Coulomb’s law of electrostatics [Jackson (1975)]:

$$\frac{d^2 \psi}{dz^2} = -4\pi \frac{\rho_z}{\varepsilon}$$

For the case where the thickness of an electric double layer is much smaller than the radius of the particle the repulsive electrostatic force between two particles with the same radius of a can be given by below equation proposed by Hunter [1987]:

$$F_e = -2\pi \varepsilon_0 \varepsilon_a k \psi^2 \frac{\exp(-\kappa R)}{\ln[1-\exp(-\kappa R)]}$$ (3.6)
Where $\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{J}^{-1} \text{m}^{-1}$ is the absolute permittivity of the free space, $\varepsilon_r$ the dielectric constant of the fluid between particle (for water: 78.5), $\psi$ Stern potential, and $\kappa$ the reciprocal of the thickness double layer. The measurable value of zeta potential $\xi$ can be used for most conditions instead of $\psi$, and the $\kappa$ can be estimated by:

$$\kappa = \left[ \frac{e^2 \sum n_i^0 z_i^2}{\varepsilon_0 \varepsilon_r k_B T} \right]^{1/2} \quad (3.7)$$

Where $e$ is the electrical charge ($1.6 \times 10^{-19} \text{C}$), $n_i^0$ and $z_i$ are the number of ions per unit volume in the bulk solution and valence of type I, respectively, $k_B$ the Boltzmann constant ($= 1.4 \times 10^{-23} \text{J} \text{K}^{-1}$) and $T$ is the absolute temperature in Kelvin.

Tent and Nijenhuis [1992] obtained the shortest distance $R$ between two neighboring particles by proposing a hexagonal packing structure of the deposited particles:

$$R = \left[ \frac{\pi}{K_1 (1 - \varepsilon_0)} \right]^{-0.33} d_p \quad (3.8)$$

McDonogh et al. [1994] indicated that the only interparticle forces between neighboring particles of filter cake should be considered. The proposed the following equation to evaluate $K_1$ with assuming the hexagonal packing structure of the deposited particles:

$$K_1 = 3 \cos \theta \quad (3.9)$$

$\theta$ varies from 0 to 90 and for hexagonal packing $\theta$ is 54.7°.

### 3.5.2 Van der Waals Forces

Van der Waals attractive forces (or London dispersion forces) are weak forces that exist between uncharged molecules as a result of polarity. These forces become significant when sizes of the particles are very small and the distances are in nano level. It is important to remember that van der Waals' forces are forces that exist between molecules of the same substance. They are quite different from the forces that make up the molecule. For example, a water molecule is made up of hydrogen and oxygen, which are bonded together by the sharing of electrons. These electrostatic forces that keep a molecule intact are existent in covalent and ionic bonding but they are not van der Waals' forces.

The origin of the van der Waals attractions may be related to the fluctuations in the charge distribution of atoms. It is important to note that electrons are constantly moving within a bond. There are moments that electrons are crowded in one side. This gives a temporary polarity to the atom. This induces polarity to the adjacent atom by repelling the electron cloud. However, this temporary charge disappears as quickly as it appeared because the electrons are moving so fast. According to the theory of London [1930], the attractive forces between atoms are additive in nature. Because of this nature, the attractive interaction between two colloidal particles containing many atoms is appreciable.
3. Forces Acting on a Particle in a Colloidal System

The Van der Waals attractive forces between atoms are very small. Hamaker [1937] derived an expression for the attractive interaction between colloidal particles. For two particles of radii \(a_1\) and \(a_2\) separated by distance \(r\), the Van de Waals potential energy is given by:

\[
U_{\text{v/w}}(r) = -\frac{A}{6} \left( \frac{2a_1a_2}{r^2} + \frac{2a_1a_2}{r^2} + \ln \left( \frac{r^2 - 4a_1a_2}{r^2} \right) \right)
\] (3.10)

For two particles of the same size and when the distance between particle surfaces much less than the radius of particle, equation (3.10) becomes:

\[
U_{\text{v/w}}(r) = -\frac{Aa}{12} \frac{1}{r - 2a}
\] (3.11)

A is the Hamaker constant which has values generally in order of Jouls.

To calculate the van der Waals force we put: \(F_{\text{v/w}} = \frac{\partial U_{\text{v/w}}(r)}{\partial r}\). Then for the particles with identical diameter of \(d_p\), van der Waals that we assumed to be the main source of attraction between colloidal molecules can be calculated with the following equation:

\[
F_{\text{v/w}}(r) = -\frac{Ad_p}{24} \frac{1}{(r - d_p)^2}
\] (3.12)

Van der Waals attraction forces can be calculated by the formula proposed by Hunter, regarding the condition that the thickness of the double layer is much smaller than the diameter of the particles:

\[
F_{\text{v/w}} = -\frac{Ad_p^6}{6R^2(R + 2d_p)^3(R + d_p)^3}
\] (3.13)

3.6 Diffusion
3.6.1 Brownian Diffusion

In crossflow filtration permeate flux drags particles towards the filter medium, but from the other side the crossflow induces particles backtransport into the bulk. Belfort et al. [1994] have proposed that besides inertial lift, diffusion of the particles helps to the backtransport of the particles. Trettin and Doshi [1980] solved the governing convective-diffusion equation for crossflow filtration assuming that diffusion is the only mechanism of backtransport of the particles and proposed the following equation for the length averaged permeate flux, \(J\) for dilute suspensions:

\[
J = 0.81 \left( \frac{\tau_w D_{0,b}}{\mu_0 L} \right)^{1/3} (\Phi_w / \Phi_b)^{1/3}
\] (3.14)

Where \(\tau_w\) is the wall shear stress, \(\mu_0\) the permeate viscosity, \(L\) the filter length, \(\Phi_w\) the particle volume fraction at the filter surface, and \(\Phi_b\) the particle volume fraction in the bulk. \(D_{0,b}\) is the Brownian diffusion coefficient for dilute suspensions of spheres, which is given by Stokes-Einestein equation:
Produced Water Re-Injection

\[ D_{0,Bo} = k_B T / 6 \pi \mu_0 a \]  

(3.15)

Here \( a \) is the particle radius.

### 3.6.2 Shear induced diffusion

Shear induced hydrodynamic diffusion of particles occurs because individual particles undergo random displacements from the streamlines in a shear flow as they interact with and tumble over other particles [Belfort (1994)]. The shear induced diffusion coefficient can be calculated from the formula suggested by Eckstein et al. [1977]:

\[ D_s = 0.3\tau_w a^2 \]  

(3.16)

Equation indicates that the shear induced hydrodynamic diffusivity is proportional to the square of the particle size by the shear rate. It is evident from the comparison of (3.15) and (3.16) that unlike shear induced hydrodynamic diffusivity, Brownian diffusivity is independent of the shear rate. As a result Brownian diffusivity is important for low shear rates and sub-micron particle, whereas in the larger particles and higher shear rates shear induced hydrodynamic diffusivity is dominant. The resulting particle migration by shear induced diffusion can be calculated by using an effective diffusion coefficient (Leighton and Acrivos, 1986-7):

\[ D_{so} = 0.33\tau_w a^2 \Phi^2 (1 + 0.5 \exp(8.8\Phi)) / \mu \]  

(3.17)

Numerical calculations of shear-induced diffusion have been performed by Davis and Sherwood [1990]. For low particle concentrations and \( \Phi_{max} = 0.58 \) they found that:

\[ J = 0.06 \frac{\tau_w}{\mu_0} (a^4 / \Phi L)^{1/3} \]  

(3.18)

### 3.7 Friction Force

Coulomb conducted hundreds of experiments with different material in broad range of conditions. He found out that the magnitude of frictional force is proportional to the sum of normal forces.

\[ F_f = f \cdot F_N \]

The proportionality, \( f \), is called Coulomb’s frictional coefficient. Coulomb found out that frictional force is approximately independent of contact area and velocity magnitude. In a colloidal system frictional force can be written as:

\[ F_f = f \left( F_D + F_e + F_G \right) \]  

(3.19)

where, \( F_f \) is the frictional force, \( F_D \) the drag force of permeate velocity, \( F_e \) electrostatic forces and \( F_G \) represents the net gravity force.
3.8 Force Analysis

Analysis of forces for the hematite particles which are used in our experiments have been done. Table 3-1 shows the values that we used to calculate forces using the equations in this chapter.

**Table 3-1: Input data for Figure 3-3**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Crossflow velocity</td>
<td>$3.38 \times 10^{-2}$ [m/sec]</td>
</tr>
<tr>
<td>Minimum Crossflow velocity</td>
<td>$4.82 \times 10^{-4}$ [m/sec]</td>
</tr>
<tr>
<td>Maximum Permeate velocity</td>
<td>$5.8 \times 10^{-3}$ [m/sec]</td>
</tr>
<tr>
<td>Minimum Permeate velocity</td>
<td>$2.41 \times 10^{-4}$ [m/sec]</td>
</tr>
<tr>
<td>Density of Fluid</td>
<td>1000 [kg/m$^3$]</td>
</tr>
<tr>
<td>Viscosity of fluid</td>
<td>0.001 [Pa.sec]</td>
</tr>
<tr>
<td>Maximum particle diameter</td>
<td>10 [micron]</td>
</tr>
<tr>
<td>Minimum particle diameter</td>
<td>0.1 [micron]</td>
</tr>
</tbody>
</table>

Figure 3-3 shows hydrodynamic forces acting on a hematite particle used in experimental solution.

Figure 3-3 shows hydrodynamic forces acting on a hematite particle. This figure implies that the magnitude of the forces increase with increasing size of the particles. Obviously, any increase in the velocities will result in increase in drag forces. Figure 3-3 shows that the lift...
force compared to the drag force is negligible and we can ignore the lift force in our force balance. Net gravity force becomes important with larger particles and can not be ignored.
4.1 Introduction

In 1907 Bechhold [1907] found out that at the filtration process with flow parallel to the filter medium (crossflow) the volume of filtrate can be increased before a compact cake layer completely blocks the pores of the filter. The phenomena may occur during activities in which a suspension flows tangentially to some porous media, e.g. fluid loss from a drilling mud or flow of injected fluids in fractures resulting in formation of the external cake.

Crossflow filtration refers to a pressure driven separation process in which the permeate flow is perpendicular to the feed flow. In other words in crossflow filtration the main direction of suspension flow is perpendicular to the flow direction of the recovered (or separated) liquid. The term crossflow just describes the direction of the fluid and does not describe the type of medium where particles are deposited. Different parameters affect the crossflow filtration e.g. crossflow velocity, transmembrane pressure, membrane resistance, layer resistance, size distribution of the suspended particles, particle form, agglomeration behavior and surface effects of the particles. However, the main and central issue for all investigation about crossflow filtration is: How does one estimate the fraction of particles transported to membrane surface associated with the permeating liquid become deposited. It is obvious that the particles that come to the surface of membrane just a fraction of them become deposited.

In petroleum industry transport of fluid containing particles in the subsurface is similar to crossflow filtration. For example in the fractures, transport of injected fluid will result in formation of cake on the surface of the rock. Another example is the build up of mud cake during the drilling of well. Therefore, external cake build-up is another important phenomena that should be investigated in order to predict the extent of the damage. Dynamic or crossflow
filtration is important because the erosive effects associated with flow limit the build-up of the external filter cake that was predicted by static filtration.

In this chapter, using Navier-Stokes equations and force balance a novel model is developed. Based on these equations we made a simulator which can give the profile of external cake.

It should be noted that Prof. Bedrikovetsky was the motivator of undertaking the external filter cake study. The basis of the following work was formulated by Profs. Bedrikovetsky and Currie, and the presented work was developed by Al-Abduwani and the author (Farjzadeh).

### 4.2 Experimental Setup

![Figure 4-1: Geometry of experimental Setup (F. Al-Abduwani – internal report 2004)](image)

In this chapter, an equation will be derived for the experimental setup shown in Figure 4-1. In this schematic figure $h$ is the thickness of the cake, $L$ is the length of setup in axial direction, $u_p$ is the permeate velocity and $u_{cf}$ (or $u_{cf}$) is the crossflow velocity.

### 4.3 Momentum Equations

In order to derive the motion equations we assume the geometry of the setup justifies an infinite plate approximation. Moreover, the following assumptions are made:

- Changes of velocities in $y$ direction are so small that we can neglect them.
- To get simpler equations we neglect the fluctuations of velocities in axial direction. However, in the later stages we will violate this assumption in the other derivations.
- The velocity profile in any section of the setup is fully developed.
- Shear forces are just in $x$ direction.
- Effects of gravity forces are neglected.
- Porosity of cake is constant.
- Filter cake has a constant permeability $k_c$.
- The membrane (or porous medium) has a constant permeability $k$.
- The injected suspension is dilute and Newton’s law is applicable.
With these assumptions the boundary conditions of the problem are:

\[
\frac{du}{d\psi} \bigg|_{\psi=0} = 0 \tag{4.1}
\]

\[u_{ef} = 0 \text{ at } \psi = h = \frac{H-h}{2} \tag{4.2}\]

Thus, solving of the momentum equation follows:

\[u_{ef} = \frac{\dot{h}^2}{2\mu} \frac{dp}{dx} \left[ 1 - \left( \frac{\psi}{h} \right)^2 \right] \tag{4.3}\]

Average crossflow velocity can be calculated with:

\[\bar{u}_{ef} = \frac{\dot{h}^2}{3\mu} \frac{dp}{dx} \tag{4.4}\]

Shear stress in the transverse direction \(\psi\) is given by:

\[\tau_{\psi}(\psi) = \frac{3\mu\bar{u}_{ef}}{\dot{h}^2} \psi \tag{4.5}\]

(4.5) is used to calculate shear stress at the surface of the cake as:

\[\tau_{\psi}(\psi) = \frac{3\mu\bar{u}_{ef}}{h} \tag{4.6}\]

To calculate the velocity in \(y\) direction Darcy’s law is applied:

\[u_p = \frac{k_i \partial p}{\mu \partial y} \tag{4.7}\]

Applying Darcy’s law to the two separate media, the membrane and the cake itself, yields:

\[u_p = \frac{\Psi}{h + \Theta} \tag{4.8}\]

with

\[\Psi = \frac{k_c (p_a - p_i)}{\mu} \tag{4.9}\]

and

\[\Theta = \frac{k_c l}{k} \tag{4.10}\]

It should be noticed that \(\psi\) is not constant along the channel, because according to (4.4) pressure is not constant along the channel.

4.4 Forces acting on a particle on the surface of cake
4.4.1 Tangential Drag Force:

The tangential drag force can be calculated by one of three ways:

4.4.1.1 Poiseuille’s Law For Infinite Plate:

\[ F_d = 24\pi a^2 \frac{\mu u_{ef}}{(H-h)} \]  
(4.11)

4.4.1.2 Poiseuille’s Law for Equivalent Tube:

\[ F_d = 16\pi a^2 \frac{\mu u_{ef}}{W(H-h)} \]  
(4.12)

4.4.1.3 Stokes law:

\[ F_d = 18\pi a^2 \frac{\mu u_{ef}}{(H-h)} \]  
(4.13)

4.4.1.4 General Form of the Tangential Drag Force:

\[ F_d = \alpha \pi a^2 \frac{\mu u_{ef}}{(H-h)} \]  
(4.14)

\( \alpha \) will be determined depending on the method we use to calculate the tangential drag force.

4.4.2 Normal Drag Force:

The normal drag force acts on a particle due to permeate velocity and can be calculated using Stoke’s law:

\[ F_n = 6\pi \mu a u_p \]  
(4.15)

4.4.3 Electrostatic Forces

According to DLVO theory there are two main electrostatic forces acting on a particle: double layer forces and Van der Waals forces.

4.4.4 Frictional Force

\[ f_f = f \cdot F_N \]  
(4.16)

4.5 Solving for the Force Balance Equality

During a crossflow filtration process particle will be deposited if friction force overcomes the tangential drag force. Therefore, while the tangential force overcomes the friction force, cake reaches its critical or equilibrium thickness and forces satisfy the inequality:

\[ F_D \geq f \left( F_e + F_c + F_G \right) \]  
(4.17)
Using the above mentioned equations:

\[ \alpha \pi a^2 \frac{\mu u_{cf}}{H-h} = f \left( F_e + 6\alpha \mu a u_p + F_G \right) \]  
(4.18)

Three cases could be considered:

- Both Electrostatic and Net Gravity forces are negligible
- Net Gravity Force is not negligible but Electrostatic Forces are negligible
- Electrostatic and net Gravity Forces are not negligible

In this report all the equations are derived for the first case where we ignore the effect of electrostatic and gravity forces. In this case (4.18) reduces to:

\[ \alpha \pi a^2 \frac{\mu u_{cf}}{H-h} = f \left( 6\alpha \mu a u_p \right) \]  
(4.19)

\[ \frac{\bar{u}_{cf}}{u_p} = \frac{6\gamma (H-h)}{a} \]  
(4.20)

Where,

\[ \gamma = \frac{f}{\alpha} \]  
(4.21)

### 4.6 Mass Balance Equations

To derive the equations we use Figure 4-2.

#### 4.6.1 Steady State Case

In steady state case cake reaches to equilibrium and macroscopically no net deposition occurs.
4.6.1.1 Mass Balance Equation for Fluid Phase

\[ u_p = -\frac{\partial}{\partial x} \left[ \frac{\pi_{df} (H - h)}{\alpha} \right] \]  \hspace{1cm} (4.22)

4.6.1.2 Mass Balance Equation for Particles

\[ \frac{dc}{c} = \frac{a}{(H-h)^2} dx \]  \hspace{1cm} (4.23)

4.6.1.3 Equation for the Pressure Change along the Channel

Replacing (4.20) into (4.4) follows:

\[ \frac{dp}{dx} = -\frac{72 \gamma \mu \psi}{a(H-h)(h+\Theta)} \]  \hspace{1cm} (4.24)

(4.24) is valid for both transient and steady state cases.

4.6.1.4 Equation for the Thickness of Cake

\[ \frac{dh}{dx} = \left[ \frac{h+\Theta}{h+2\Theta + H} \right] \left[ \frac{u_p}{u_{df}} - \frac{72 \gamma k_e \psi}{a u_p} \right] \]  \hspace{1cm} (4.25)

To calculate the thickness of cake in a given flow boundary we can use the following equation:

\[ h = H + \frac{Qa \pm \sqrt{Qa(Qa + 24\gamma W\psi (H+\Theta))}}{12\gamma W\psi} \]  \hspace{1cm} (4.26)

4.6.2 Transient Case with Erosion

In filtration the rate of cake height growth can be obtained from mass balance for particles. Mass balance for the particle mass flow in Figure 4-3 can be written as:

\[ \dot{V}_{p,\text{in}} - \dot{V}_{p,\text{back}} = \frac{dV_p}{dt} \]  \hspace{1cm} (4.27)

(4.27) indicates that the change of control volume equals to the volume of deposited particles minus the volume of eroded ones.
4.6.2.1 Deposition Rate

For the rate of deposition particles on the surface on control volume shown in Figure 4-3 the following equation can be obtained by writing a simple mass balance equation:

\[
(1 - \varphi_c) \frac{dh^*}{dt} = u_p \cdot c 
\]  
(4.28)

4.6.2.2 Back Transport or Erosion Rate

The deposition of particles on the surface is assumed to be an irreversible process. Thus, once a particle attaches to the surface will stay there and will not be detached. This is because of the energy barrier caused by electrostatic forces. However in our model, for simplicity of the case, we ignore the effect of surface forces. As a result, when tangential drag force overcomes frictional force particle will not be deposited and may go back to the suspension or out of the channel. The back transport of the particles will result in the decrease in thickness of the cake. Erosion or back transport of the particles will happen when:

\[
\frac{\eta_{cf}}{u_p} > \frac{6 \gamma}{a} (H - h)  
\]  
(4.29)

The main parameter which causes the back transport of the particles into the system is crossflow velocity. However, another parameters such as thickness of the cake and permeate velocity may have effect on erosion of the particles. Here, we report three different equations for back transport of the particles:

4.6.2.2.1 When the main parameter is just crossflow velocity:

\[
u_{back} \cdot c = \omega \eta_{cf} 
\]  
(4.30)

Therefore for the equation for thickness of cake will be:

\[
(1 - \varphi_c) \frac{dh}{dt} = u_p \cdot c - \omega \eta_{cf} 
\]  
(4.31)
\( \omega \) is the proportionality coefficient and may be called “Erosion Coefficient” and assumed to be constant along the channel. Erosion coefficient can be calculated by applying (4.31) for steady state case when time derivative of height is zero and averaging it along the core:

\[
\omega = \text{MEAN} \left[ \frac{u_p}{U_{ef}} c \right]_{SS} \quad (4.32)
\]

### 4.6.2.2 When both crossflow velocity and height of cake are important parameters

\[
U_{back}c = \omega \frac{U_{ef}^n}{(H-h)^n} \quad (4.33)
\]

then,

\[
(1-\varphi_c) \frac{dh}{dt} = u_p c - \omega \frac{\overline{U}_{ef}^m}{(H-h)^n} \quad (4.34)
\]

with:

\[
\omega = \text{MEAN} \left[ \frac{a(H-h)^{n-1} c}{6\gamma U_{ef}^{m-1}} \right]_{SS} \quad (4.35)
\]

When \( m=n=1 \), we have:

\[
(1-\varphi_c) \frac{dh}{dt} = u_p c - \omega \frac{\overline{U}_{ef}}{H-h} \quad (4.36)
\]

with:

\[
\omega = \text{MEAN} \left[ \frac{a c}{6\gamma} \right]_{SS} \quad (4.37)
\]

Although it is initially assumed that erosion is independent of initial concentration of the particles in the suspensions, the final equations for \( \omega \) shows the dependency of erosion on concentration.

### 4.6.2.3 Mass Balance Equation for Fluid

Writing mass balance equation for fluid in the suspension and also in the cake will lead to the following equation:

\[
-\frac{\partial}{\partial x} \left[ \overline{U}_{ef} (H-h) \right] = u_p + \frac{\partial}{\partial t} \left[ H-h(1-\varphi_c) \right] \quad (4.38)
\]

### 4.6.2.4 Mass Balance Equation for Particles

In this case we will get an equation for concentration. Mass balance is written for particles in the suspension as well as the cake.
\[(H - h)\frac{\partial C}{\partial t} = -\left[\frac{\partial}{\partial x}\left[C\pi\varphi (H - h)\right] + (1 - \varphi_e - C)\frac{\partial h}{\partial t}\right]\]  \hspace{1cm} (4.39)

All of the above equations are solved numerically and put in the simulator to build the profiles of cake thickness and velocities. This will be explained in details in chapter 7.
Produced Water Re-Injection
5.1 Introduction

Solid particles contained in produced water will be deposited along the hole and fractures in the reservoir. Water will propagate in elliptical shape (pressure profile) and in some cases will help of propagation of unwanted fractures. Figure 5-1 shows a normal water injection well and front of water contained particles along fracture and bore hole. Injected water will propagate with elliptical shape but the problem is that we are not sure about the invasion profile. In order to investigate the mechanism of damage several experiments have been conducted in Deitz Laboratory of TU Delft with special designed set up that will be introduced in this chapter.

Figure 5-1: The invasion profile of particles is still unclear (Al-Abduwani, presentation 2003)
Zooming on area highlighted with A in Figure 5-1 it can be seen that flow has only one opportunity to go through. This situation can happen at the bottom of wells or tips of the fractures. The simulated model of this case is shown in Figure 5-2. Deposition and filter cake built up process in this case is called “Static Filtration” or “Dead End Filtration”. Static filtration is a filtration process in which the slurry being filtered remains static. Filter cake continues to grow thicker as filtration continues. Under static conditions, no cake erosion occurs.

![Figure 5-2: Schematic of Static Filtration](image)

Zooming on area highlighted with B in Figure 5-1, one can observe that there are two possibilities for fluid to pass through, either into the formation or along the fracture. This is mostly to happen in the real situations. The simulated model of case B is shown in Figure 5-3. Deposition and filter cake built up process in this case is called “Dynamic Filtration”, a filtration process in which the slurry being filtered is being circulated over the filter cake, so that the cake is simultaneously eroded and deposited. The erosion rate depends on the shear rate of the fluid at the face of the cake. If the shear rate remains constant, cake thickness and filtration rate reach steady state, usually in a matter of hours. When the conditions change, a new steady state will be established.

![Figure 5-3: Schematic of Dynamic Filtration](image)

### 5.2 Description of the set up

Based on the explained concepts, a set up was designed to study the mechanism of deposition, damage and filtration in both static and dynamic circumstances and in different conditions. With this set up the effects of different parameters such as velocity, concentration and etc. are investigated. This set up consists of three parts: Injection Unit (IU), Dynamic Filtration Unit (DFU) and Effluent Concentration Detection Unit (ECDU). Figure 5-4 shows the schematic of set up.
5. The Set up and Experiments

5.2.1 Injection Unit

The injection Unit consists of two separate tanks, one filled with distilled water and the other with the injection solution. In this project the solution is a hematite suspension, latex or brine. Fluids are transferred through pipes by using a screw pump. Screw pumps are rotary, positive displacement pumps that can have one or more screws to transfer high or low viscosity fluids along an axis. The main reason to use this kind of pump in this set up is the high constant velocities that this pump can handle.

5.2.2 Dynamic Filtration Unit (DFU)

The DFU is the unit in which the sample is replaced. Figure 5-5 shows a schematic of the DFU. The brown rectangle is the core. Y is connected to the pump and the suspension is injected through this pipe into the DFU. The designed dynamic filtration unit can be used both for static and dynamic filtration experiments. When valve C is closed the liquid will just go into the core. In this case the unit acts as a static filtration unit. However, when valve C is open, the liquid will be divided into two parts, one part will penetrate into the core and another part will flow out either through U or W. In order to measure the pressure drop caused by the deposition of the particles, small holes are drilled in the core sample. These holes are connected to pressure gauges. There are six pressure gauges in our system and therefore, six pressure difference values are obtained. These data are used to calculate the permeability and extent of damage in the porous media, which will be explained more thoroughly in Chapter 6.

5.2.3 Effluent Concentration Detection Unit (ECDU)

Measuring the concentration along the core is not easy in conventional experiments. Therefore, the determination of the deposited particles is not possible. However, to obtain the deposition profile along the core concentration of the particle in the effluent is measured.

Figure 5-4: Schematic of the set up A and B are tanks filled with distilled water and injection solution, C the screw pump, D core sample in DFU, E back pressure gauge, F laser unit (ECDU) and G the flow regulator (F. Al-Abduwani – Internal report, 2003)
To measure the effluent concentration, the Sympatec HELOS laser diffraction (LD) system is used. Figure 5-6 shows the schematic of the device. Using this setup we are able to measure both size distribution and concentration of the particles in the suspension which comes out of the core.

The LD sensor, called the HELOS central unit, is the heart of measuring device. It consists of a laser light source including a focusing lens, a measuring zone (where the laser light and the particles interact) and an optical system that converts the diffraction of the laser light in the spectrum into an image that can be detected by the photo detector. A multi-component photo-detector with an integrated auto focus unit, followed by an electronic device, converts the intensity of the light into electrical signals. These are processed by computational means using the software provided. The computer program converts the data into the particle size distribution (PSD). The LD allows the particle size analysis of dispersed matter in gas or liquid regardless of whether the particles are solid, gaseous (bubbles) or liquid (droplets). The analysis is based on the difference of the optical properties of the matter with regard to its environment.

The measuring range of the HELO S sensor can be adapted to the requirements by a change in the lens/objective setup. These are seven measuring ranges available. Particles from 0.1 µm up to 3500 µm can be measured. Every measuring range includes 32 particle size classes.

---

Figure 5-5: Schematic of Dynamic Filtration Unit (Al-Abduwani, Internal report 2003)

Figure 5-6: Schematic of the ECDU
It should be noted that the DFU is connected to the ECDU by a nylon tube. Figure G-3 indicates that the dispersion deposition in the mentioned pipe is negligible meaning that what comes out of the DFU is what we measure in the ECDU.

### 5.3 The Core (Porous Medium)

The porous media which is used in our experiments is Bentheim sandstone. Properties of Bentheim sandstone are summarized in Table 5-1. These properties were obtained independently by the Dietzlab and by Pan Terra Geoconsultants B.V at the behest of PWRI group for confirmation.

<table>
<thead>
<tr>
<th>Components</th>
<th>Quartz 60-85 %</th>
<th>Feldspar 2-5 %</th>
<th>Rock Fragments 1-2 %</th>
<th>Heavy minerals &lt;1 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity</td>
<td>22.1 – 22.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Permeability</td>
<td>1200 – 1400 mD</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grain Density</td>
<td>2.65 [gr/ml]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5-7 shows the pore size distribution of the Bentheim sandstone. According to the report the radius of pore throats are mainly in the range of 10-30 µm. However, with increasing pressure the size of throats decreases.

5.4 Preparation of the core sample

For the pervious setup (used by the previous MSc student in year 2002) the required sandstone samples had a cylindrical shape with the diameter of 1 inch (25.4cm) and length of 134 mm. For the new setup the samples are cut in cuboids. In order to maintain the same superficial injection rates as previous experiments and also to be able to use the same pump,
cross sectional area of both samples must be approximately the same. Therefore to calculate the dimensions of the new sample we can simply write:

$$\pi r^2 = a^2$$

Here, $r$ is the radius of the previous cylindrical sample and $a$ is one side of the squared cross section of the new cuboid sample. If we substitute $r$, we will get: $a = .89" = 24mm$

The range of superficial velocity in the filtration experiment is 10-20 m/hr. Since the sides of designed square cross-section are 24 mm, an injection rate of $\sim 6-11.2$ L/hr would be required to maintain the same superficial injection velocity range. Figure 5-8 shows the cross-sectional view of the required sample.

The cuboid sandstone has the following dimensions: $W=H=24 \text{ mm}, \ L=134 \text{ mm}$ (Figure 5-9). This cuboid shape sample is inserted into a cylindrical plastic tube that has the same length as the sample. The inner diameter of this cylinder is 34 mm and it has 3 mm thickness. The diagonal of the cuboid’s cross-section should be tightly fit into the plastic tube. The gap between the tube and the sandstone sample is then filled up with viscous glue that hardens forming a strong bond between the two components. The two ends of the cores are then sawn off, yielding a final length of 127 mm (5 inches). The glue would penetrate the sample forming a layer with thickness of $\sim 1\text{ mm}$. Thus this should be considered in all calculations of velocities.
For the dynamic filtration experiments which will be explained in Chapter 6, short cylindrical core samples were used ($L=2\, cm$, $D=2.4\, cm$).

### 5.5 Particle Properties

Two kinds of particles were used in the experiments:

#### 5.5.1 Hematite Particles

Hematite particles ($Fe_2O_3$) which are used in our experiments are spherical colloids with the diameter of 2µm. However particles in the range of 1-5µm are also present. The hematite particles are strongly positively charged and therefore, will be attached to the negatively charged quartz grains. There are four reasons that we use this material in our experiments:

- Iron is one of the metals that are present in the produced water in the form of hematite. The hematite comes predominantly from corroded metal pipes.
- Hematite particles are easy to detect. They can be visualised quite easily because of their intense colour. Also hematite particles are solvable in hydrochloric acid (HCl) which is useful in chemical analysis.
- They can be found in desired sizes.
- They can be easily detected by CT-scanner.

#### 5.5.2 Latex Particles

A latex microsphere is a polymer particle in the colloidal size range. The particles are spherical and are formed from amorphous polymer. Polystyrene is the polymer of choice for many applications. In our experiments we used mono-dispersed 5µm latex particles in three colors, blue, red and green. The first idea to use different colors of latex particles was to see the different stages of the damage, since different colors are injected in different times. The particles were supposed to have the same chemical properties, but several experiments showed that the blue latex is an unstable emulsion and the red and green one are stable emulsion. This resulted in different slopes for pressure graphs [see Appendix G].

### 5.6 Preparation of the Injected Suspension

In order to prepare the suspension the required amount of the hematite (or latex) particles is weighed by a very accurate mass balance. About 1 litre of distilled and de-ionised water is added to the hematite (or latex). To obtain a uniform suspension, the container is put in sonic bath for 10-15 minutes. After we make sure that the suspension is uniform, it is added to the big injection Tank. Since the hematite particles have a settling time, use of mixer to stir the suspension during the experiments is recommended.

### 5.7 Experiments

Before starting the experiments we have to make sure that the core is placed correctly in the DFU. All the holes in the core should be aligned with the holes of steel or PEEK (Poly Ethylene Ether Keton) core holder. This will prevent leaking of the liquid from connections and will increase the accuracy of the pressure measurements. Also all connection should be checked before starting the experiments. Needless to say the safety rules of the laboratory
should be adhered to during the experiments. The summary of the experimental procedure is listed below:

1. Saturate the core sample.
2. Make sure that valves are in the right position.
3. Set the ECDU and take the reference measurement while injecting the distilled water.
4. Check the pressure response of the system.
5. Purge the air out of all pipes.
6. Synchronise all the times.
7. Set the back pressure to 2 bars.
8. Wait until the flow rates are stable.
9. Take a sample of the prepared suspension.
10. Switch to the injection tank. For the safety of the pump the injection valve is opened before closing of the water tank valve.
11. Write down the starting time.
12. Check the system frequently and never leave the setup.
13. To terminate the experiment close the valve of regulator in order to save the external filter cake. Reduce the flow rate gradually to zero.
14. Switch off the pump.
15. Flush the pipes to remove the settled particles, especially the flow regulator. Be careful not to set the flow rate to the flow rates larger than the maximum capacity of the flow regulator.
16. Clean up the system and laboratory.
6.1 Introduction

In order to investigate the internal filtration and the damage caused by produced water the data from both static and dynamic filtration were used. 1 static and 7 dynamic filtration experiments were conducted using the CT scanner to obtain the deposition profile. During this project we found out that the X-ray tomography is a good and reliable method to obtain the deposition profile. Also another valuable data such as dynamic inlet concentration and two filtration coefficients can be directly extracted using X-ray Computed tomography. In this chapter a novel method of deposition profile extraction will be briefly introduced. Next the experiments will be explained. Finally we will analyse the obtained data and discuss the results. Please note that the experiments presented was jointly conducted with Al-Abduwani.

6.2 Deposition Profile Obtaining Methods

6.2.1 Chemical Analysis

In this method, which is part of the post mortem analysis, after conducting the experiment the core is split into several parts. After breaking the sample, each part is labelled and crushed into powder. In the next stage hydrochloric acid is added. The iron atoms of hematite are dissolved in the acid. The following reaction takes place between acid and hematite:

\[ \text{Fe}_2\text{O}_3 + 6\text{HCl} \rightarrow 2\text{FeCl}_3 + 3\text{H}_2\text{O} \]

With chemical analysis the amount of present iron in each segment is determined. Taking an average value, one point is obtained for each segment and Figure 6-1 is plotted.

Although the error is acceptable, this method is time consuming and may take several days to complete the analysis. Besides the number of points is another important issue. The resolution of the profile is extremely important for the mathematical modelling. This method
is limited to a number of data points equivalent to the number of the segments the core is broken into.

It should be noted that in the dynamic filtration experiments, which will be explained later, to obtain the effluent concentration, the chemical analysis method was used. Similar to the explained method, acid was added to the suspension samples to dissolve the iron atoms. To obtain the amount of iron in the suspension AAS (Atomic Adsorption Spectroscopy) method was used in the chemistry laboratory of Applied Earth Science department of TU Delft.

![Deposition profile obtained from chemical analysis of the sample](image)

6.2.2 X-ray Tomography

Using X-ray has proved to be a better way of determining the amount of deposited particles. In this method the resolution is much higher. For example for the dynamic filtration experiments each slice had 1 mm thickness which gives more than 240 points for a 5 inch length core. It is also possible to get the deposition profile at multiple times while conducting the experiment. Due to availability of CT-Scanner at TU Delft (Siemens SAMATOM) the possibility of detection of hematite particles in the core was studied. The results are given in this section.

6.2.2.1 An introduction to X-Ray CT

X-Ray CT was developed in early seventies for medical purposes. It was introduced to industrial applications in late eighties. Industrial computed X-ray tomography (ICT) is a non-destructive testing (NDT) procedure and differs from normal X-Ray Radiography only in the way X-Rays penetrates the object. It gives a cross-sectional image of the object. This image is digital and its values corresponds to a physical property of tested object. Figure 6-2 shows the schematic of how an image is obtained by CT Tomography. It is believed that CT-Scanning is the best available tool to study flow of fluid in porous media without no or minimum external aid.

6.2.2.2 General Equation

The general equation defining the attenuation of the CT-signal is:
\[ \mu = \rho \left[ \sigma(E) + b \frac{Z_e^{3.8}}{E^{3.2}} \right] \]  

(6.1)

where,
\( \rho \) = the electron or bulk density
\( \sigma(E) \) = the Klein-Nishina coefficient
\( b \) = a constant \((= 9.8.10^{24})\)
\( Z \) = the effective atomic number of the chemical species
\( E \) = the X-ray photon energy in (KeV)

The measurement of the signal is normalized with respect to the attenuation of water at the given conditions of the measurement (Hounsfield Units):

\[ HU = 1000 \left( \frac{\mu}{\mu_w} - 1 \right) \]  

(6.2)

The effective atomic number is given by:

\[ Z_e = \left( \sum_{i} g_i Z_i^{3.8} \right)^{1/3.8} \]  

(6.3)

where \( g_i \) is the fraction of electrons of constitution \( i \) and \( Z_i \) is the atomic number of \( i \)-the atom species. For example for water which consists of two atoms of hydrogen (2 electrons) and oxygen (8 electrons) effective atomic number is:

\[ Z_{ew} = \left( \frac{2}{10} 1^{3.8} + \frac{8}{10} 8^{3.8} \right) = 7.5439 \]

Our calculation showed that hematite can be detected by CT scanner (Appendix H)
For a dilute suspension in porous media $\mu$ reads:

$$\mu = \varphi \mu_w + (1 - \varphi) \mu_s + \left[ \varphi c + (1 - \varphi) \sigma \right] \mu_p$$

(6.4)

where $\mu_w$, $\mu_s$ and $\mu_p$ are the attenuation coefficients of water, sandstone and the particle in the suspension. From (6.4) the local concentration is given by:

$$C_i = \left[ \varphi c + (1 - \varphi) \sigma \right]$$

(6.5)

or,

$$C_i \approx \frac{(\mu - \mu_s)}{\mu_p}$$

(6.6)

$\mu_s$ is the attenuation coefficient for the wet core just before injection of the hematite particles.

### 6.3 Experiments

#### 6.3.1 Static Experiment and Data Analysis

The simulated synthetic produced water was a suspension of hematite particles ($d_p = 2\mu m$) in water. The suspension was found out to have a static settling time of about 5 hours and a $pH = 6.8 \pm 0.2$. In this pH according to Kuhnen et al. [2000] hematite particles are positively charged while the surface of quartzitic sandstone is negatively charged. This means that the filtration mechanism will be controlled by electrostatic attraction and also interception. Also the repulsion force between two hematite particles should result in monolayer type of deposition. This justifies the use of classical deep bed filtration model. The water that we used to make the solution was double distilled de-ionized with $pH = 5.1 \pm 0.1$. The static filtration experiment was conducted on cylindrical Bentheim sandstone ($D = 4.4 cm, L = 18 cm$). The dry core is saturated with double distilled and de-ionised water. It should be mentioned that the sample was scanned before and after saturation. The X-ray system we used is a third generation SAMATOM AR made by Siemens. For the whole object examination, the X-ray source-detector system rotates continuously, utilizing the traversing slice method. The volumetric concentration of hematite in water in this experiment was $1.2 \times 10^{-3}$. The suspension was injected continuously into the core with the velocity of $u = 2.11 \times 10^{-3} m/sec$. In order to get the deposition profile the core sample which was replaced in a PEEK core-holder, was scanned using X-ray at different times. The core was scanned slice by slice and the thickness of slices was 2 millimetres. This means for our sample we got 91 images.

Obtained data from the CT scanner was visualized using OSIRIS program and a Matlab subroutine written by the PWRI group of TU Delft. Figure 6-3 shows a partial set of images (voxel size $0.35 \times 0.35 \times 2.0 mm$) obtained at several times and positions along the sample. The bright colour represents the hematite particles in the cross section. It is observed from the figure that the deposition of particles in the bottom left corner is higher than that of the other side of the core. This is due to misalignment of the core in the CT scanner, which has the effect of presenting the deposition as non-uniform. In fact the deposition front is uniform once the images are corrected for the tilt. However, these images do show the applicability of X-ray method to observe the presence (deposition) of particles in the pores of the sand. The bright colours in the graph are representatives of hematite particles due to the definition in
Matlab program. As can be seen, the deposition of hematite particles increases with time and decreases along the core.

![CT scan images obtained during an online experiment](image)

**Figure 6-3:** Example of CT scan images obtained during an online experiment, GRC conference Poster

Moreover, reading the CT numbers we are able to construct the deposition profile of the hematite particles along the core. This could be done using (6.6). Figure 6-4 shows the deposition profile of the hematite particles along the core. It should be noted that the $y$-axis does not show the exact amount of hematite particles, but it is a measure of present hematite in porous media. In order to obtain the exact amount, calibration of CT scanner to hematite is necessary. The numbers in $y$-axis are normalised to wet core.

$$N_y = \frac{(\mu - \mu_w)}{\mu_w}$$

(6.7)
The sensitivity of the CT apparatus for the hematite particles can be estimated using:

\[ \Delta C = 10^{-3} \frac{\mu \Delta H}{\mu_p} \quad (6.8) \]

Table 6-1: The minimum sensitivity of the CT scanner is \( \Delta H = 3 \). As a result:

\[ \Delta C = 10^{-3} \frac{0.1570}{12.150} (3) = 3.87 \times 10^{-5} \]

<table>
<thead>
<tr>
<th>Species</th>
<th>Purity (%)</th>
<th>Bulk Density (g/cm³)</th>
<th>Effective Z (-)</th>
<th>Cross Sections (cm²/g)</th>
<th>Linear Attenuation (cm⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂O</td>
<td>99.2</td>
<td>1.0</td>
<td>7.54</td>
<td>0.157</td>
<td>0.157</td>
</tr>
<tr>
<td>SiO₂</td>
<td>99.2</td>
<td>2.7</td>
<td>12.65</td>
<td>0.16</td>
<td>0.4320</td>
</tr>
<tr>
<td>α-Fe₂O₃</td>
<td>99.2</td>
<td>5.4</td>
<td>23.56</td>
<td>2.250</td>
<td>12.150</td>
</tr>
</tbody>
</table>
Also analytically and with literature values we can get the concentration profile using (6.6) and data in Table 6-1. Figure 6-5 shows the concentration of hematite along the core in different time intervals. To obtain this graph we used equation (6.6).

Figure 6-5: Concentration profile along the core

It is clear from Figure 6-4 and Figure 6-5 that the deposition is mainly in the first few centimeters of the sample, which will be followed by external cake build-up later.

Internal filtration will be followed by filter cake build-up on the front surface of the core. The accumulation of particles on the surface, where fluid tries to enter the porous media will increase the inlet concentration, which can be clearly observed from Figure 6-5.

Assuming a constant $\lambda$ for the filtration theory equation and deposition of $\sigma(x,t)$, initial hematite concentration of $c_0$, the overall concentration $C(x,t)$ can be calculated by:

$$C(x,t) = c_0 \phi [1+(1-\phi)\lambda_0 (Vt-x)] \exp(-\lambda_0 x)$$

(6.9)

and

$$C(x,t) = \phi c(x,t) + (1-\phi) \sigma(x,t)$$

(6.10)

However, in the reality the inlet concentration is not constant. Therefore, the solution for filtration theory reads:

$$C(x,t) = C_0(x,t) \exp(-\lambda x)$$

(6.11)

Comparison between (6.9) and (6.11) follows:
Produced Water Re-Injection

\[ C_0(x, t) = c_0 \varphi \left[ 1 + (1 - \varphi) \lambda_0 V t \right] \quad (6.12) \]

Therefore if we take the concentration values at \( x=0 \) and plot it versus \( V t \), from the slope of curve we can calculate the filtration coefficient and also estimate the hematite concentration of injected suspension. The fitting of points gives:

\[ C(0, t) = 2.4 \times 10^{-5} V t + 7.4 \times 10^{-5} \quad (6.13) \]

which means,

\[ c_0 = \frac{7.4 \times 10^{-5}}{0.22} = 3.36 \times 10^{-4} \]

and

\[ \lambda_\infty = 0.41 \text{ m}^{-1} \]

The concentration is an order of magnitude larger than the injected suspension concentration. The reason is explained below.

From (6.11) we can say that:

\[ \frac{C(x, t)}{C(0, t)} \propto \exp(-\lambda_0 x) \quad (6.14) \]

Figure 6-7 shows the plot of reduced concentration against distance. The fitting graph shows that for obtained data from X-ray tomography a master curve can be found. Using an exponential fitting we find that: \( \lambda_0 \approx 68.53 \). This number is in good agreement with (6.9) and shows the high validity and reliability of X-ray tomography data. However, the difference between two obtained values for filtration coefficient indicates that \( \lambda \) is not constant in the whole process of filtration. The filtration coefficient is high in the beginning of the experiment and suddenly drops to a low value in later stages. This discrepancy could be explained by the different mechanisms (Net Forces) of the filtration process in different stages of the experiment. When the surface forces are dominant mechanism in deposition of hematite particles, in the beginning of the experiment the sand grains and hematite particles have opposite charges. This means particles have high tendency to attach to the grain. However while injection continues and \( \sigma \) increases and part of the pore surface is covered with particles the net force reduces and as a result deposition decreases. When \( \sigma \) reaches an equilibrium this process is limited and in this case filtration coefficient has reached its lowest value. In fact, the much larger value of \( c_0 \) compare to the injected one shows that the linear behavior given by (6.12) is a good approximation of the true behavior of the system only for large \( \sigma \) and consequently for large values of \( t \).
6. Internal Damage Results and Analysis

Figure 6-6: Initial concentration with $V_t$, Experimental data and linear fitting

$y = 2.4 \times 10^{-5} V_t + 7.4 \times 10^{-5}$

Figure 6-7: Reduced concentration profile which is proportional to the reduced deposition
6.3.2 Dynamic Filtration Experiments

7 dynamic filtration experiments were conducted in the CT scanner room under different conditions. The aim of these experiments was to study and quantify the damage caused by the flow of the produced water in the fractures and the borehole. In the dynamic filtration experiments both internal and external cakes could be studied, because of the two different velocities. In this section we will just discuss the internal filtration results.

The simulated synthetic produced water was again a suspension of hematite particles in double distilled and de-ionised water. This water has $PH=5.1 \pm 0.1$. All dynamic or crossflow filtration experiments were conducted on cylindrical Bentheim sandstone ($D=2.4 \pm 0.1 cm$, $L=2.1 \pm 0.1 cm$). The reason to choose the shorter core was to accelerate the pressure build up rate which leads to the earlier external cake build up. The core was inserted in a plastic tube and the gap between the rock and tube was filled with glue. Glue would penetrate into the core and reduce the cross sectional area of the core. To calculate the velocity the diameter of the core is assumed $D=21mm$, because of the glue penetration. The volumetric concentrations of suspensions and experimental velocities are presented in Table 6-2.

The suspension was injected continuously into the core with the velocities of Table 6-2. To get the deposition profile the core samples which were replaced in the PEEK (poly ether ether ketone) core holder were scanned using X-ray at different time intervals. Since the core samples were shorter in comparison with the one used in the static filtration experiment, we were able to choose 1 mm thick slices, which improved the resolution of the deposition profile. The imaging setting was the same for all these experiments and is presented in Table 6-3.

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Material</th>
<th>Concentration [mg/lit]</th>
<th>Permeate flow rate [lit/hr]</th>
<th>Permeate velocity [m/sec]</th>
<th>Crossflow flow rate [lit/hr]</th>
<th>Crossflow velocity [m/sec]</th>
<th>Flow rate ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Hematite</td>
<td>40</td>
<td>5.50</td>
<td>4.41E-3</td>
<td>5.50</td>
<td>4.41E-3</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Hematite</td>
<td>40</td>
<td>5.50</td>
<td>4.41E-3</td>
<td>2.75</td>
<td>4.41E-3</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>Hematite</td>
<td>30</td>
<td>7.33</td>
<td>5.87E-3</td>
<td>2.75</td>
<td>2.20E-3</td>
<td>0.375</td>
</tr>
<tr>
<td>4</td>
<td>Hematite</td>
<td>30</td>
<td>7.33</td>
<td>5.87E-3</td>
<td>3.67</td>
<td>2.94E-3</td>
<td>0.5</td>
</tr>
<tr>
<td>5</td>
<td>Hematite</td>
<td>60</td>
<td>3.67</td>
<td>2.94E-3</td>
<td>3.67</td>
<td>2.94E-3</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>Hematite</td>
<td>60</td>
<td>3.33</td>
<td>2.67E-3</td>
<td>5.0</td>
<td>4.00E-3</td>
<td>1.5</td>
</tr>
<tr>
<td>7</td>
<td>Hematite</td>
<td>60</td>
<td>3.67</td>
<td>2.94E-3</td>
<td>1.83</td>
<td>1.4E-3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube current</td>
<td>250 mA</td>
</tr>
<tr>
<td>Tube Voltage</td>
<td>80 kV</td>
</tr>
<tr>
<td>Slice Thickness</td>
<td>1 mm</td>
</tr>
<tr>
<td>Scan Time for each core</td>
<td>8.25 sec</td>
</tr>
<tr>
<td>Image Reconstruction Kernel</td>
<td>B45 S medium</td>
</tr>
<tr>
<td>Scan Mode</td>
<td>Spiral</td>
</tr>
<tr>
<td>Couch Feeding</td>
<td>5mm</td>
</tr>
</tbody>
</table>
The obtained data were analysed similarly to the static filtration experiment. The principles are the same and the above equations are still valid for these experiments. The same graphs were plotted (see Appendix I) and two filtration coefficients were extracted from the data. Moreover, it was found out that there is a relation between the filtration coefficient and the injected time. For each time interval using the CT data, reduced concentrations were calculated. As time passes, the available space for particles to be deposited decreases. This will result in the reduction of the filtration coefficient with time. With data fitting curves we found out that for all experiments the following relation exists between $\lambda$ and $t$:

$$\lambda = \lambda_{SS} e^{\frac{b}{t}}$$  \hspace{1cm} (6.15)

or,

$$\ln \lambda = b \left( \frac{1}{t} \right) + \ln \lambda_{SS}$$  \hspace{1cm} (6.16)

When $t \to \infty$, from (6.15) we can say that:

$$\lambda = \lambda_{SS}$$  \hspace{1cm} (6.17)

The filtration coefficient has a high value in the beginning of the experiment due to the virginity of the rock. As experiment continues, the particles will deposit in the retention sites and decrease the available space for the deposition. The filtration coefficient will decrease with time. However, in the first stages of the experiment the reduction in $\lambda$ is sharp but as the experiment continues, this reduction becomes smaller and smaller until it reaches a constant value which will not change. This ultimate filtration coefficient is called “steady state filtration coefficient” and can be obtained from the experimental data using (6.16). A plot of $\ln \lambda$ against $1/t$ will give the value of $\lambda_{SS}$ which is close to what we get for $\lambda_0$, because the dynamic filtration experiments are stopped when the system has reached the steady state condition. Figure 6-8 shows an example of this graph plotted using data from experiment number 2 ($\lambda_0 = 82.90$, $\lambda_{ss} = 79.71 \text{ m}^{-1}$).

![Figure 6-8: Filtration coefficient vs. inverse of time, experimental data and linear fitting (Experiment 2)](image_url)
For all experiments the initial concentration and three values of filtration coefficient were calculated with the same method explained for the static filtration experiment. Table 6-3 shows the obtained results for all experiments.

**Table 6-4: Extracted data from dynamic experiments**

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>(c_0)</th>
<th>(\lambda_{\infty} [m^{-1}])</th>
<th>(\lambda_0 [m^{-1}])</th>
<th>(\lambda_{ss} [m^{-1}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.0 \text{\times} 10^{-4}</td>
<td>2.91</td>
<td>86.75</td>
<td>86.56</td>
</tr>
<tr>
<td>2</td>
<td>4.0 \text{\times} 10^{-4}</td>
<td>1.51</td>
<td>82.90</td>
<td>79.710</td>
</tr>
<tr>
<td>3</td>
<td>3.91 \text{\times} 10^{-4}</td>
<td>2.08</td>
<td>63.47</td>
<td>58.40</td>
</tr>
<tr>
<td>4</td>
<td>5.91 \text{\times} 10^{-4}</td>
<td>1.18</td>
<td>58.46</td>
<td>54.795</td>
</tr>
<tr>
<td>5</td>
<td>Na</td>
<td>Na</td>
<td>Na</td>
<td>Na</td>
</tr>
<tr>
<td>6</td>
<td>6.36 \text{\times} 10^{-4}</td>
<td>2.74</td>
<td>78.20</td>
<td>76.420</td>
</tr>
<tr>
<td>7</td>
<td>4.27 \text{\times} 10^{-4}</td>
<td>2.04</td>
<td>75.20</td>
<td>75.045</td>
</tr>
</tbody>
</table>

As can be seen from the results, the experiments with the same permeate velocity have the same filtration coefficient (\(\lambda_0\)). This means in the dynamic filtration experiments the crossflow velocity does not affect the internal filtration and the permeate velocity brings the particles into contact with the sand grains. This allows researchers to study the internal filtration and external cake build up together. It is consistent with the results that the internal filtration and external cake build up are two independent problems. In fact this data confirms the existence of the transition time from internal filtration into external filtration. Furthermore, the value of \(\lambda_{\infty}\) reaches one number (\(\approx 2\)) for all experiments. It can be said that \(\lambda_{\infty}\) is a characteristic of the sand sample when no further internal filtration occurs. The values for \(C_0\) are still much higher than the injected concentration. This is because of the accumulation of the particles at the inlet of the core sample and external cake build-up meaning that this value of the concentration corresponds to the later stages of the experiment when the external cake has already built up. This phenomenon is similar to the concept of the “concentration polarization” [Elimelech et al. (1999) and Song et al. (1995)] for nano sized particles.

Also from the pressure data we can extract the normalized permeability graphs. As expected these graphs confirm the damage caused by the produced water re-injection. In the initial stages of the injection the damage is more severe though after some time the permeability of the formation reaches a constant value. Figure 6-9 shows the normalized permeability graph for experiment 3. For the other experiments the graphs are shown in Appendix I.

Since in the dynamic filtration experiments the ECDU were not available in the CT scanner room the effluent breakthrough curves were plotted by taking samples and analyzing them with the above explained chemical analysis method. Figure 6-10 shows the breakthrough curve for experiment 5. For the other experiments the breakthrough curves are shown in Appendix I.
6. Internal Damage Results and Analysis

Figure 6-9: Normalised Permeability for Experiment 3

Figure 6-10: Breakthrough curve for Experiment 5
7.1 Introduction

The derived equations for external cake (Chapter 4) were solved numerically using a Matlab subroutine written by members of the PWRI Group (Al-Abduwani & Farajzadeh). These equations predict the velocity and thickness profiles for a channel in which the suspension flows at any given condition. Using this program and obtaining proportionality coefficient \( \gamma \) from the experiments one is able to predict the external cake build up as well as the crossflow and permeate velocity profiles. This Matlab subroutine also takes into account the changes in the concentration of the particles along the hypothetical channel, although the change does not lead to diffusion effects to the particle transport mechanisms. To obtain the graphs in this chapter and Appendix J the properties of the cake, the porous medium and setup listed in Table 7-1 were used. The discretisation of the equations is presented in Appendix J. In this section the results for steady state case will be discussed. Figure J-2 shows the scheme of the simulator for the steady state case. Discretisation of the equations and the codes of the Matlab program are also shown in Appendix J. In this chapter we will discuss the simulator results. Also we will analyze the effect of different parameters and sensitivity of the profile to the changes in parameters.
7.2 Simulation Results for Steady State case

The permeate flow carries particles toward the porous membrane and they deposit on its surface. However, contrary to the static filtration the cake growth is not uniform along the channel (green line in Figure 7-1). This is a direct consequence of the shear forces caused by the crossflow velocity which leads to the erosion of the cake. This is the reason why the cake thickness increases along the channel. Since the shear forces decrease, any increase in the cake thickness will result in higher resistance to the permeate velocity to enter the porous medium and therefore, with increasing thickness the permeate velocity decreases along the channel. The growth of the external cake reduces the cross section for crossflow to pass along the channel. Thus, the shear rate at the cake surface increases. These two effects, the permeate flow decrease and the shear rate increase, cause a steady state cake to eventually be reached when the rate of particle convection to the cake surface is balanced by shear back-transport of the particles. Crossflow velocity also decreases with distance from the injection inlet of the channel because of the leak off of the liquid in the channel (permeate flow). The bottom-left graph shows that the volumetric concentration of the particles increases moving toward the end of the channel. The reason is that because of the increase in thickness of the cake, the volume of the cells in the simulated cells decreases for a constant number of particles. The effect of diffusion is neglected due to the high velocities we used. In fact we assume that the only mechanism in transporting of the particles is advection.

<table>
<thead>
<tr>
<th>Table 7-1: Base case conditions used to run the simulator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cake Permeability ( (k_c) )</td>
</tr>
<tr>
<td>Porous Medium Perm ( (k_m) )</td>
</tr>
<tr>
<td>Porous Medium thickness ( (L_m) )</td>
</tr>
<tr>
<td>Back Pressure ( (P_{bp}) )</td>
</tr>
<tr>
<td>Cake Porosity ( (\phi_c) )</td>
</tr>
<tr>
<td>Gamma ( \gamma = \frac{\phi}{\alpha} )</td>
</tr>
<tr>
<td>Setup Length ( (L) )</td>
</tr>
<tr>
<td>Channel Height ( (H) )</td>
</tr>
<tr>
<td>Channel Width ( (W) )</td>
</tr>
<tr>
<td>Injection rate ( (Q_0) )</td>
</tr>
<tr>
<td>Inlet concentration ( (c_0) )</td>
</tr>
<tr>
<td>Particle Radius ( (a) )</td>
</tr>
</tbody>
</table>

Figure 7-2 shows that the pressure decreases slightly along the channel for base case. However because of the laminar flow the change is not significant and the pressure could be assumed constant. The same story is true for \( \psi \), since it is a direct function of the pressure.
7. External Cake Results and Analysis

Besides the base case, we ran the simulator for different cases to see the effect of each parameter on the thickness of cake individually. We chose different parameters (particle size, back pressure, porous medium and cake permeability and the proportionality coefficient gamma) by the factors of 0.1, 0.5 and 2 of the base case. For example Figure 7-3 shows the sensitivity of the cake thickness to the size of the particles. This graph shows that as particle radius decreases the thickness of the cake increases. This is quite interesting because it is contrary to what one should see in the static filtration. In static filtration the particles with bigger size will form a higher porosity and thicker cake. In the dynamic or crossflow filtration velocity has two components which insert drag forces on the entering particles. Our model takes into account the drag forces and frictional force. Besides the velocity, according to Stokes law for laminar flow, drag force depends on the radius of the particles. As particle size increases the tangential drag force on the particle increases. This means that the drag force is bigger for the bigger particles. For an open end channel when the drag force overcomes the frictional force the particle will go further or out of the channel but small particles will stay on the surface of the channel and will form the cake. Figures for other parameters are shown in Appendix J and their effect on the cake thickness will be explained in the next section.
Figure 7-2: Pressure slightly decreases along the channel

Figure 7-3: Sensitivity of the steady state cake thickness to particle size
7.3 Sensitivity Analysis

Figure 7-3 to Figure 7-6 show the sensitivity of the cake thickness to different parameters at different parts of the cake. We do sensitivity analysis for different parts of the cake because the cake profile is not uniform and to get a feel of the non-linear behavior. All figures indicate that the permeability of the porous medium has no effect on the thickness of the cake because of the assumption of constant permeability for the porous medium. In fact we run the simulator for the external cake without considering the internal filtration. In our analysis the porous medium is just a membrane. Moreover, since the cake has much lower permeability compared to the porous medium the main resistance to the fluid is in the cake itself. The three figures clearly show that the cake thickness has linear relationship with the cake thickness. The thickness of the cake will be changed by the factor that the base cake permeability is multiplied to. It should be noticed that in our modeling the cake permeability is also assumed constant which may not be true in reality.

The cake thickness has also linear relationship with the back pressure. If we double the back pressure, the cake thickness will be doubled and if we reduce the pressure to the half of its initial value, the cake thickness will be half as well. The back pressure is related to the permeate velocity with Darcy’s law. Therefore when the back pressure is higher the permeate velocity is higher which increases the probability of contact of particle with the cake surface. This increases the chance of attachment to the neighborhood particles and will result in higher cake thickness growth.

![Sensitivity Analysis Graph](image)

**Figure 7-4: Sensitivity analysis for the first cell**

The story of the particle size and Gamma is more complicated. The changes in their values show different effects in different parts of the cake. We chose three sites on the hypothetical channel, the first cell where the fluid enters to the channel, the middle cell and the last. The proportionality coefficient gamma and the particle radius are the
parameters that play a significant role in the force balance equations as well as the external cake model equations in Chapter 4. The combinations of these two effects cause the unpredictable effects.

When the magnitude of Gamma is doubled, either the frictional force is doubled or the drag force is halved. In either case, the particle has more chance to be attached to the other particles on the cake surface. But the model doesn’t show the linear relationship between Gamma and the cake thickness. In the first cell when Gamma is doubled the cake thickness increases by the factor of 1.5. But also when Gamma is halved the thickness doubles and when it is reduced to 10% of its initial value the cake becomes much thicker. This nonlinear behavior of the Gamma shows the importance of the Gamma to be measured experimentally for each setup. Without having an idea about the value of the Gamma it is difficult to predict the cake profile along the channel.

![Figure 7-5: Sensitivity analysis for the middle cell](image)

![Figure 7-6: Sensitivity analysis for the last cell](image)
8.1 Conclusions

The following conclusions can be drawn from the results of the experiments of this work.

- Presence of the solid particles is a potential source of formation damage.
- X-ray computed computer tomography is suitable for extraction of the deposition profile of the particles along the core sample. With this method the actual longitudinal profiles can be obtained in a matter of hours which is much shorter than the other conventional methods. Also the deposition profile can be obtained at different times of the experiments using CT scanner.
- The classical filtration models could predict the extent of the internal damage. The profiles extracted from the experimental data are in a good agreement with the existing filtration theory.
- There is a relation between the filtration coefficient and the injection time. Using X-ray computed tomography the filtration coefficient in different times of the experiment can be obtained. The value of the filtration coefficient in the initial stages of the experiment is quite high due to the availability of more space for particle to be retained. This value drops to a small value at the last stages of the
experiment. This value for our core sample was $\lambda_\infty = 2 \text{ m}^{-1}$, which can be a property of the Bentheim sandstone.

- In the dynamic or the crossflow experiments the magnitude of the crossflow velocity does not affect the internal filtration. For the experiments with the same permeate velocities and different crossflow velocities the same deposition profile were extracted from the experimental data.

- The results of the experiments confirm the existence of the transition time proposed by Pang and Sharma [1997].

- A novel model for the external cake build up was developed during this project. Using Matlab, a simulator was written which predicts the steady state profiles of the external filter cake.

- The crossflow velocity is the main parameter which affects the erosion of the cake. This can be seen from the calculated $C_0$ for all experiments which has higher valued for higher crossflow velocity.

### 8.2 Recommendations

- In order to study the different stages of the damage and also check the validity of the transition time experiments with colorful particles are recommended. The possibility of being detected by CT scanner should be checked for selected particles. For colorful latex particles which are not detectable by CT scanner the problem can be solved by coating hematite particles with different colors of the latex, since the hematite particles are detectable using CT scanner.

- In this project the simulated produced water was a suspension of hematite particles. Since the real produced water has another composition for the future experiments the simulated produced water could be selected closer to the reality (for example by adding oil droplet, bacteria...)

- The Bentheim sandstone is homogenous sand with good permeability and porosity which is not the case in most of the reservoirs which suffer for injectivity decline due to produced water re-injection. The experiments can be conducted using other sands.

- To verify the crossflow simulations, experiments should be done on a 1m long setup.

- In the external cake model some simplifying assumptions were made. We ignored the effect of the electrostatic, net gravity and lift forces in our equations. Also the equations were derived for mono-sized particles. For the future work these simplifications can be changed.
8. Conclusions and Recommendations

- The transient simulator should be completed to see the pressure build up profiles. This could help to predict the approximate time of the formation fracturing.

- The viscosity of the fluid decreases with increasing temperature. Also temperature changes some properties of the porous medium. Therefore, other experiments can be done in the temperatures above the room temperature. Temperature should also be incorporated in the presented model.

- In order to omit the noises caused by the aluminum plates used in the core holder, a new design is recommended for the core holder.
Produced Water Re-Injection
REFERENCES


[37] Pang, S. (1996) “colloid retention and flow in porous media”, phd dissertation, the university of Texas at Austin


NOMENCLATURE

\( S_t \): skin factor
\( \mu \): viscosity
\( P_{inf} \): injection pressure
\( q \): flow rate
\( k \): permeability
\( \eta \): total collision probability
\( \eta_I \): collision probability due to interception
\( \eta_D \): collision probability due to diffusion
\( \eta_{lm} \): collision probability due to impaction
\( \eta_s \): collision probability due to sedimentation
\( \eta_E \): collision probability due to surface forces
\( d_p \): particle diameter
\( d_g \): grain diameter
\( \phi \): effective porosity
\( \rho_p \): particle density
\( \rho_f \): fluid density
\( U, u \): Darcy’s velocity
\( g \): gravity acceleration
\( k_B \): Boltzman constant (=10^{23})
\( T \): absolute temperature
\( A \): Hamaker constant
\( v_p \): particle velocity
\( c(x,t) \): volumetric concentrations of the suspended particles
\( \sigma(x,t) \): volumetric concentrations of the deposited particles
\( k_0 \): absolute permeability
\( k(\sigma) \): Formation Damage coefficient (relative permeability)
\( \lambda \): Filtration coefficient
\( L \): length of the porous media
\( t \): time
\( c_0 \): initial concentration of suspended particles
\( X = \frac{x}{L} \): scaled length
\( T = \frac{Ut}{\phi L} \): scaled time
\( C = \frac{c}{c_0} \): scaled concentration of the suspended particles

\( S = \frac{\sigma}{\varphi c_0} \): scaled concentration of the deposited particles

\( \Lambda \): scaled filtration coefficient

\( \lambda_0 \): initial filtration coefficient

\( \lambda_\infty \): final filtration coefficient

\( \Lambda_0 \): scaled initial filtration coefficient

\( V_F \): clogging front velocity

\( \alpha_c \): clean-bed collision efficiency

\( \Phi \): hydraulic head

\( i_0 \): clean bed gradient

\( K \): hydraulic conductivity

\( R_\sigma \): resistance due to deposition

\( I \): injectivity index

\( j \): inverse of injectivity index

\( J \): non-dimensional impedance

\( t^* \): transition time

\( n \): number of particles attached to one grain

\( a_g \): cross-sectional area of a grain

\( F_D \): drag force

\( C_D \): drag coefficient

\( \tau \): shear force

\( F_L \): lift force

\( F_B \): buoyancy force

\( F_G \): net gravity force

\( F_e \): electrostatic force

\( F_f \): frictional force

\( F_{vdW} \): van der Waals force

\( F_N \): sum of Normal forces

\( \psi \): Stern potential

\( \varepsilon_0 \): absolute permittivity of the free space

\( \varepsilon \): dielectric constant of the fluid

\( \kappa \): reciprocal of the thickness double layer

\( \xi \): zeta potential

\( \tau_w \): wall shear stress

\( \mu_0 \): permeate viscosity

\( \Phi_w \): particle volume fraction at the filter surface
\( \Phi_b \): particle volume fraction in the bulk

\( D_{\text{b,Bo}} \): Brownian diffusion coefficient

\( f \): Coulomb frictional coefficient

\( \gamma \): proportionality coefficient
## ENVIRONMENTAL EFFECTS AND COMPOSITION OF PRODUCED WATER

### Table A-1: Examples of Effects of Produced Water

<table>
<thead>
<tr>
<th>Concentration</th>
<th>Test duration</th>
<th>Test organisms</th>
<th>Effects</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dilution to 25 µg/l</td>
<td>Hours and days</td>
<td>Larvae of Atlantic cod (North Sea)</td>
<td>Reduced survival</td>
<td>Davies et al., 1981</td>
</tr>
<tr>
<td>0.15% with 5-15 µg/l of oil</td>
<td>100 days</td>
<td>&quot;Natural zooplankton&quot; (North Sea)</td>
<td>Reduced survival at population level</td>
<td>&quot;</td>
</tr>
<tr>
<td>5%</td>
<td>48 hours</td>
<td>Larvae/embryos of oyster <em>Crassostrea gigas</em> (North Sea)</td>
<td>50% mortality</td>
<td>Somerville et al., 1987</td>
</tr>
<tr>
<td>10%</td>
<td>24 hours</td>
<td>Copepod <em>Calanus Finmarchus</em> (North Sea)</td>
<td>50% mortality</td>
<td>&quot;</td>
</tr>
<tr>
<td>5%</td>
<td>10 days</td>
<td>Hydroid <em>Campularia Flexuosa</em> (North Sea)</td>
<td>50% decrease in population growth</td>
<td>&quot;</td>
</tr>
<tr>
<td>5-6%</td>
<td>15 minutes</td>
<td>Luminescent bacteria (North Sea)</td>
<td>50% decrease in photoluminescence intensity</td>
<td>&quot;</td>
</tr>
<tr>
<td>5-6%</td>
<td>48 hours</td>
<td><em>Daphnia Daphnia magna</em></td>
<td>Immobilization of 50% of test organisms</td>
<td>&quot;</td>
</tr>
<tr>
<td>10%</td>
<td>24 hours</td>
<td>Trout <em>Salmo gairdneri</em></td>
<td>50% mortality</td>
<td>&quot;</td>
</tr>
<tr>
<td>0.1-0.3%</td>
<td>60-130 days</td>
<td>Natural communities of plankton in mesocosms (North Sea)</td>
<td>Reduced abundance of copepod larvae, disturbances of trophic structure, biochemical changes in fish larvae</td>
<td>Gamble et al., 1987</td>
</tr>
<tr>
<td>3-30%</td>
<td>96 hours</td>
<td>(Gulf of Mexico) Invertebrates and fish</td>
<td>50% mortality</td>
<td>Middleditch, 1981</td>
</tr>
<tr>
<td>1-10%</td>
<td>Chronic tests</td>
<td>(California Shelf) Macrophytes <em>Macroystis pyrifera</em></td>
<td>Reduced ability of zoospores to settle on the bottom</td>
<td>Lewis &amp; Reed, 1994</td>
</tr>
<tr>
<td>20-70mg/l</td>
<td>1-2 days</td>
<td>Fish eggs, fry and larvae (Caspian Sea)</td>
<td>Death of juveniles at 60-70mg/l; larvae at 35-40mg/l; eggs at 20-25mg/l.</td>
<td>Alekperov, 1989</td>
</tr>
<tr>
<td>2-30mg/l</td>
<td>&quot;Several days&quot;</td>
<td>Invertebrates (Caspian Sea)</td>
<td>Death of crustaceans at 30mg/l; reduced survival at 2mg/l.</td>
<td>&quot;</td>
</tr>
<tr>
<td>Technology</td>
<td>Processes</td>
<td>Advantages</td>
<td>Disadvantages</td>
<td>Costs</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>------------------------------------------------</td>
<td>-----------------------------------------------------------------------------------------------</td>
<td>---------------------------------------------------------------------------------------------------------</td>
<td>----------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Carbon adsorption</td>
<td>Modular granular activated carbon systems.</td>
<td>Removes hydrocarbons and acid, base and neutral compounds; low energy requirements; higher throughput than other treatments (except biological); treats a broad range of contaminants; very efficient at removing high Mwt. Organics.</td>
<td>Fouling of carbon granules is a problem; produces waste stream of carbon and backwash; requires some pre-treatment of produced water stream.</td>
<td>&quot;Middle range&quot; of costs.</td>
</tr>
<tr>
<td>Air stripping</td>
<td>Packed tower with air bubbling through the produced water stream.</td>
<td>Can remove 95% of volatile compounds as well as benzene, toluene, naphthalene, phenanthrene, anthracene, pyrene and phenols; H2S and ammonia can be stripped but pH must be adjusted; higher temperature improves removal of semi-volatiles; small size, low weight and low energy requirements; simple to operate; well-known technology.</td>
<td>Can be fouled by oil; risk of iron and calcium scales forming; generates an off-gas waste stream that may require treatment; requires some pre-treatment of produced water stream.</td>
<td>Low capital and operating costs; overall treatment cost US$0.02 to $0.10/1,000 gallons, plus $0.50 to $1.50/1,000 gal if off-gas control by activated carbon is required.</td>
</tr>
<tr>
<td>Filtration</td>
<td>Very fine membranes.</td>
<td>Effective removal of particles and dispersed and emulsified oil; small size, low weight and low energy requirements; high throughput rates.</td>
<td>Does not remove volatiles or dissolved compounds. Does not affect salinity; oil, sulfides or bacteria may foul membrane, which requires daily cleaning; waste streams may contain radioactive material; requires some pre-treatment of produced water stream.</td>
<td>Low capital and operating costs (similar to air stripping).</td>
</tr>
<tr>
<td>Ultra-violet light</td>
<td>Irradiation by UV lamps</td>
<td>Destroys dissolved organics and both volatile and non-volatile organic compounds, including organic biocides; does not generate additional waste stream; handles upset or high-loading conditions.</td>
<td>Will not treat ammonia, dispersed oil droplets, heavy metals or salinity; relatively high energy requirements; UV lamps may become fouled; residues may be toxic if peroxide used; requires some pre-treatment of produced water stream.</td>
<td>Similar capital costs to chemical oxidation with ozone but operating costs lower because no waste stream.</td>
</tr>
<tr>
<td>Chemical oxidation</td>
<td>Ozone and/or hydrogen</td>
<td>Removes H2S and particulates; treats</td>
<td>High energy inputs for ozone system; oil may</td>
<td>&quot;Middle range&quot; of costs.</td>
</tr>
</tbody>
</table>
APPENDIX A

<table>
<thead>
<tr>
<th>Biological treatment</th>
<th>Peroxide oxidation</th>
<th>Hydrocarbons, acid, base and neutral organics, volatiles and non-volatiles; low energy requirements if peroxide system used; straightforward to operate.</th>
<th>Foul catalyst; may produce sludge and toxic residues; requires some pre-treatment of produced water stream.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aerobic system with fixed film biotower or suspended growth (e.g. deep shaft)</td>
<td>Treats biodegradable hydrocarbons and organic compounds, H2S, some metals and, in some conditions, ammonia; &quot;fairly low&quot; energy requirements; handles variable loadings, if acclimated.</td>
<td>Large, heavy plant required for long residence times; build-up of oil and/or iron may hinder biological activity; aeration may cause calcium scale to form; may produce gas and sludge requiring treatment; requires some pre-treatment of produced water stream.</td>
<td></td>
</tr>
</tbody>
</table>

"No costs estimated

Table A-3: Aromatic Hydrocarbons in North Sea Produced Water, Pre-1994

<table>
<thead>
<tr>
<th>Compound</th>
<th>Concentration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benzene</td>
<td>4 - 5 ppm</td>
</tr>
<tr>
<td>Toluene</td>
<td>0.01 - 2 ppm</td>
</tr>
<tr>
<td>Xylenes</td>
<td>1 - 7 ppm</td>
</tr>
<tr>
<td>Naphthalene</td>
<td>66 ppb</td>
</tr>
<tr>
<td>Biphenyl</td>
<td>4 ppb</td>
</tr>
<tr>
<td>Dibenzothiophene</td>
<td>0.5 ppb</td>
</tr>
<tr>
<td>Fluorene</td>
<td>&lt;2ppb</td>
</tr>
<tr>
<td>Phenanthrene</td>
<td>&lt;2ppb</td>
</tr>
<tr>
<td>Acenaphthene</td>
<td>&lt;2ppb</td>
</tr>
<tr>
<td>Acenaphthylene</td>
<td>&lt;2ppb</td>
</tr>
<tr>
<td>Fluoranthene</td>
<td>&lt;2ppb</td>
</tr>
<tr>
<td>Anthracene</td>
<td>&lt;2ppb</td>
</tr>
<tr>
<td>Pyrene</td>
<td>&lt;2ppb</td>
</tr>
<tr>
<td>Benzo(a)pyrene</td>
<td>&lt;2ppb</td>
</tr>
<tr>
<td>Perylene</td>
<td>&lt;2ppb</td>
</tr>
</tbody>
</table>
Table A-4: Comparison of Heavy Metal Concentration in Produced Waters (µg/L)

<table>
<thead>
<tr>
<th></th>
<th>Copan(3)</th>
<th>Venture(3)</th>
<th>Gulf of Mexico(1)</th>
<th>Gulf of Mexico(2)</th>
<th>North Sea(2)</th>
<th>North Sea(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Gulf of Mexico</td>
<td>Platform study</td>
<td>Oil Platforms</td>
<td>Gas Platforms</td>
<td></td>
</tr>
<tr>
<td>Arsenic</td>
<td>-</td>
<td>90</td>
<td>&lt;0.11-320</td>
<td>-</td>
<td>0.004-12</td>
<td>&lt;1-100</td>
</tr>
<tr>
<td>Barium</td>
<td>-</td>
<td>13500</td>
<td>1.0-650000</td>
<td>-</td>
<td>4-220</td>
<td></td>
</tr>
<tr>
<td>Cadmium</td>
<td>2-6</td>
<td>&lt;10</td>
<td>0.06-98</td>
<td>-</td>
<td>4-220</td>
<td></td>
</tr>
<tr>
<td>Chromium</td>
<td>80</td>
<td>10</td>
<td>&lt;0.01-390</td>
<td>-</td>
<td>4-220</td>
<td></td>
</tr>
<tr>
<td>Copper</td>
<td>8-2400</td>
<td>137</td>
<td>&lt;0.05-210</td>
<td>-</td>
<td>10.0-55</td>
<td></td>
</tr>
<tr>
<td>Lead</td>
<td>8-45</td>
<td>&lt;10</td>
<td>&lt;0.08-5700</td>
<td>-</td>
<td>&lt;1-400</td>
<td></td>
</tr>
<tr>
<td>Mercury</td>
<td>-</td>
<td>&lt;10</td>
<td>0.06-0.19</td>
<td>-</td>
<td>0.000001-75</td>
<td>&lt;0.2-33</td>
</tr>
<tr>
<td>Nickel</td>
<td>20-420</td>
<td>40</td>
<td>0.1-1674</td>
<td>-</td>
<td>20-95</td>
<td>10-170</td>
</tr>
<tr>
<td>Zinc</td>
<td>90-260000</td>
<td>10</td>
<td>7.3-10200</td>
<td>-</td>
<td>5-350000</td>
<td>20-150000</td>
</tr>
<tr>
<td>Total Radium</td>
<td>-</td>
<td>-</td>
<td>0-2802</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Total Radium 226</td>
<td>-</td>
<td>-</td>
<td>0-1565</td>
<td>262 (avg.)</td>
<td>45 (avg.)</td>
<td>-</td>
</tr>
<tr>
<td>Total Radium 228</td>
<td>-</td>
<td>-</td>
<td>0-1590</td>
<td>278 (avg.)</td>
<td>105 (avg.)</td>
<td>-</td>
</tr>
</tbody>
</table>

* Estimated from Sulfate ion concentration and Barium Sulfate solubility product constant


(3) Data provided by SOEI (2000)

B.1 Mass Balance Equation for C.V.

Assuming that the Figure B-1 shows the control volume of a porous medium in which a suspension containing component \( n \) with the volumetric concentration of \( c_n \) (volume of particle / volume of solution) and velocity of \( \vec{u} \) flows, the mass conservation equation will be:

\[
\frac{\partial \rho_n c_n}{\partial t} + \frac{\partial \rho_n c_n u_x}{\partial x} + \frac{\partial \rho_n c_n u_y}{\partial y} + \frac{\partial \rho_n c_n u_z}{\partial z} = -r_n - \rho_n \Delta y \Delta z \Delta t + \frac{\partial c_n}{\partial t} \rho_n \Delta t \Delta y \Delta z
\]

(B.1)

Dividing both sides by \( \rho_n \Delta x \Delta y \Delta z \Delta t \) (assuming that particles are incompressible) yields:

\[
\frac{\partial c_n}{\partial t} + \frac{\partial (c_n u_n,)}{\partial x} + \frac{\partial (c_n u_n,)}{\partial y} + \frac{\partial (c_n u_n,)}{\partial z} = r_n
\]

(B.2)

\[
\nabla \cdot (c_n \vec{u}_n) = r_n
\]

(B.3)

B.2 Convection- Diffusion Equation
In a binary system of just one solute and one solvent, the diffusion flux of the solute is defined as:

\[ \mathbf{J} = \nabla u_n - c \bar{u} \]  \hspace{1cm} (B.4)

Then, (B.3) becomes:

\[ \frac{\partial c_n}{\partial t} + \text{div}(c_n \bar{u}) + \text{div} \mathbf{J} = r_n \]  \hspace{1cm} (B.5)

Using Fick’s law we have:

\[ \frac{\partial c_n}{\partial t} + \text{div}(c_n \bar{u} - D \text{grad}(c_n)) = r_n \]  \hspace{1cm} (B.6)

Finally according to the filtration theory for the rate of deposition in the porous media we can write:

\[ r_n = -\frac{\partial \sigma}{\partial t} = -\lambda \bar{u} c_n \]  \hspace{1cm} (B.7)

Substituting (B.7) into (B.6) yields:

\[ \frac{\partial c_n}{\partial t} + \text{div}(c_n \bar{u} - D \text{grad}(c_n)) + \frac{\partial \sigma}{\partial t} = 0 \]  \hspace{1cm} (B.8)

Writing (B.8) in the x direction considering the porosity of the porous media:

\[ \frac{\partial (c_n + \sigma)}{\partial t} + \frac{\partial (c_n u_n)}{\partial x} - D \frac{\partial^2 c}{\partial x^2} = 0 \]  \hspace{1cm} (B.9)
Appendix C

SOLUTION OF THE CONVECTION EQUATION WITH CONSTANT FILTRATION COEFFICIENT USING METHOD OF CHARACTERISTIC

C.1 Solution

To solve the equation we take $\lambda(\sigma)$ as a constant, so that we could write $\lambda(\sigma) = \lambda_0$. Thus:

$$\frac{\partial S}{\partial T} = \Lambda_0 C$$  \hspace{1cm} (C.1)

$$\frac{\partial \sigma}{\partial T} = \lambda_0 c \hspace{1cm} (8-1)$$

If we substitute (C.1) in (2.7) we find:

$$\frac{\partial C}{\partial T} + \frac{\partial C}{\partial X} + \Lambda_0 C = 0$$  \hspace{1cm} (C.2)

And initially we have: $C(0, T) = 1$.

To solve this equation the method of characteristic is applied. We introduce $Y$ and $Z$ as follows:

$$Y = X - T$$
$$Z = X + T$$  \hspace{1cm} (C.3)

$$\frac{\partial c}{\partial T} = \frac{\partial c}{\partial Y} \frac{\partial Y}{\partial T} + \frac{\partial c}{\partial Z} \frac{\partial Z}{\partial T} = (-1) \frac{\partial c}{\partial Y} + (1) \frac{\partial c}{\partial Z}$$
$$\frac{\partial c}{\partial X} = \frac{\partial c}{\partial Y} \frac{\partial Y}{\partial X} + \frac{\partial c}{\partial Z} \frac{\partial Z}{\partial X} = (1) \frac{\partial c}{\partial Y} + (1) \frac{\partial c}{\partial Z}$$

Substituting these equations in (C.2), we obtain:

$$2 \frac{\partial c}{\partial Z} + \Lambda_0 C = 0$$  \hspace{1cm} (C.4)
The solution for (C.4) is:

\[ \ln C = -\frac{1}{2} \Lambda_0 Z + g(Y) \]  

(C.5)

and if we replace Y and Z in (C.5), using (C.3) the final solution is:

\[ \ln C = -\frac{1}{2} \Lambda_0 (X + T) + g(X - T) \]  

(C.6)

with \( C(0,T)=1 \)

to determine \( g(X - T) \) we use the initial condition:

\[ \ln(1) = -\frac{1}{2} \Lambda_0(T) + g(-T) \]

\[ -\frac{1}{2} \Lambda_0(-T) = g(-T) \]

Then

\[ g(X - T) = -\frac{1}{2} \Lambda_0(X - T) \]  

(C.7)

And if we put (C.7) in (C.6) we will get:

\[ C = e^{-\Lambda_0 X} \]  

(C.8)

Figure 1 clearly shows that the solution is in the area \( X \geq T \).

C.2 Second method

\[ \frac{\partial C}{\partial T} + \frac{\partial C}{\partial X} + \Lambda_0 C = 0 \]

\[ \frac{\partial C}{\partial T} + \frac{\partial C}{\partial X} = -\Lambda_0 C \quad \text{and we have:} \quad C(0,T) = 1 \]

\[ \frac{dT}{ds} = 1 \quad \Rightarrow \quad T = s + c_1(r) \quad \text{with} \quad T(0,0) = r \Rightarrow c_1(r) = r \quad \Rightarrow \quad T = s + r \]  

(C.9)

\[ \frac{dX}{ds} = 1 \quad \Rightarrow \quad X = s + c_2(r) \quad \text{with} \quad X(0,0) = 0 \Rightarrow c_2(r) = 0 \quad \Rightarrow \quad X = s \]  

(C.10)

\[ \frac{dz}{ds} = -\Lambda_0 z \quad \Rightarrow \ln z = -\Lambda_0 s + c_3(r) \quad \text{with} \quad z(0,0) = 1 \Rightarrow c_3(r) = 0 \Rightarrow \ln z = -\Lambda_0 s \]

Then we can simply write:

\[ c(X,T) = z(r(X,T), s(X,T)) = e^{-\Lambda_0 s} \quad \text{and since we have} \quad X = s \quad \text{then we would have:} \]

\[ C = e^{-\Lambda_0 s} \]
C.3 Construction of the solution using Maple

Maple has strong ability in solving partial differential equations under PDEtools. In this section, for better understanding of the geometry of the solution, we try to construct our solution to the problem.

Using (C.9) and (C.10), we can draw some of characteristic lines:

```maple
c:=(x,t)->exp(-2*x);
c→\(e^{-2x}\)
```

First, we see that it really is a solution for the partial differential equation.

```maple
plot3d(c(x,t),x=0..1,t=0..30,axes=normal,orientation=[-45,65]);
```
It is interesting to see that Maple can do all the graphics. Then we start:

\[ c := 'c'; \]

\[ c := c \]

We define the PDE:

\[ pde := \frac{\partial}{\partial t} c(x,t) + \frac{\partial}{\partial x} c(x,t) + 2c(x,t) = 0; \]

\[ pde := \left( \frac{\partial}{\partial t} c(x,t) \right) + \left( \frac{\partial}{\partial x} c(x,t) \right) + 2c(x,t) = 0 \]

For the boundary condition we have:

\[ bond := [0, t, 1]; \]

\[ bond := [0, t, 1] \]

To make the graph of characteristics we write:

\[ \text{PDEplot}(pde, c(x,t), bond, x=0..1, axes=normal, scene=[x,t,c], t=0..1, orientation=[-65,65], basechar=only); \]
To see the graph of the solution:

```latex
\texttt{PDEplot(pde,c(x,t),bond,x=0..1,axes=normal,scene=[x,t,c],
        t=0..1,orientation=[-65,65]);}
```

And finally to see both graph of the solution and characteristics lines, we write:

```latex
\texttt{PDEplot(pde,c(x,t),bond,x=0..1,axes=normal,scene=[x,t,c],
        t=0..1,orientation=[-65,65],basechar=true,style=wireframe);}
```
Projection of the solution curve on the x-t surface would be the characteristics lines:
D.1 Force balance

\[ p \cdot \Delta \zeta \Delta \psi - \left\{ p \cdot \Delta \zeta \Delta \psi + \frac{d}{dx} \left( p \cdot \Delta \zeta \Delta \psi \right) \Delta x \right\} \]
\[ + \tau_{\zeta} \cdot \Delta x \Delta \psi - \left\{ \tau_{\zeta} \cdot \Delta x \Delta \psi + \frac{d}{d\zeta} \left( \tau_{\zeta} \cdot \Delta x \Delta \psi \right) \Delta \zeta \right\} \]
\[ + \tau_{\psi} \cdot \Delta x \Delta \zeta - \left\{ \tau_{\psi} \cdot \Delta x \Delta \zeta + \frac{d}{d\psi} \left( \tau_{\psi} \cdot \Delta x \Delta \zeta \right) \Delta \psi \right\} = 0 \]

\[ \frac{-dp}{dx} - \frac{d\tau_{\psi}}{d\psi} - \frac{d\tau_{\zeta}}{d\zeta} = 0 \quad (D.1) \]

Newton’s law of viscosity states that:

\[ \tau_{n} = -\mu \frac{du}{dn} \quad (D.2) \]

Therefore:

\[ \frac{1}{\mu} \frac{dp}{dx} = \frac{\partial^2 u}{\partial \psi^2} + \frac{\partial^2 u}{\partial \zeta^2} \quad (D.3) \]
Given the following boundary conditions:

\[
\frac{du}{d\psi}\bigg|_{\psi=0} = 0 \quad \text{(D.4)}
\]

\[
\frac{du}{d\zeta}\bigg|_{\zeta=0} = 0 \quad \text{(D.5)}
\]

\[
u = 0 \quad \text{at} \quad \psi = \hat{h} = \frac{H - h}{2} \quad \text{(D.6)}
\]

\[
u = 0 \quad \text{at} \quad \zeta = \frac{W}{2} \quad \text{(D.7)}
\]

I don’t know if an analytical solution exists for the above mentioned problem. However, the problem can be simplified in two different ways to yield an analytical solution:

1. Approximate the problem to that of two infinite plates
2. Assume that the problem can be approximated to that of a circular pipe and obtain the effective hydraulic radius from the area of the rectangular cross-section.

### D.2 Infinite Plate Approximation

The infinite plate approximation reduces (D.3) to:

\[
\frac{1}{\mu} \frac{dp}{dx} = \frac{\partial^2 \nu}{\partial \psi^2} \quad \text{(D.8)}
\]

where boundary conditions (D.4) and (D.6) still hold.

Integrating twice with respect to \( \psi \) and imposing the boundary conditions:

\[
\frac{\partial u}{\partial \psi} = \frac{1}{\mu} \frac{dp}{dx} \psi + C_1 \quad \text{and} \quad C_1 = 0
\]

\[
u = \frac{1}{\mu} \frac{dp}{dx} \frac{\psi^2}{2} + C_2 \quad \text{and} \quad C_2 = -\frac{1}{2\mu} \frac{\partial p}{\partial x} h^2
\]

Thus:

\[
u(\psi) = \frac{1}{2\mu} \frac{dp}{dx} \left[\psi^2 - h^2\right] \quad \text{(D.9)}
\]

or

\[
u(\psi) = -\frac{h^2}{2\mu} \frac{dp}{dx} \left[1 - \left(\frac{\psi}{h}\right)^2\right] \quad \text{(D.10)}
\]

Utilising the mass balance presents a relationship between the pressure gradient and the mean velocity \( \bar{\nu} \) as illustrated below:
\[ \rho \bar{u} W \cdot 2h = \rho W \cdot 2 \int_0^h u(\psi) \, d\psi \]  \hspace{1cm} \text{(D.11)}

which yields:

\[ 2\bar{u} h = \frac{1}{\mu} \int_0^h \left( \psi^2 - h^2 \right) d\psi \]

\[ 2\bar{u} h = -\frac{1}{\mu} \int_0^h \left( h^2 \psi - \frac{\psi^3}{3} \right) d\psi \]

and thus,

\[ \frac{dp}{dx} = -\frac{3 \mu \bar{u}}{h^2} \]  \hspace{1cm} \text{(D.12)}

Combining equations (D.10) and (D.12) yields:

\[ u(\psi) = \frac{3}{2} \frac{\bar{u}}{h^2} \left( h^2 - \psi^2 \right) \]  \hspace{1cm} \text{(D.13)}

or

\[ u(\psi) = \frac{3}{2} \frac{\bar{u}}{h} \left( 1 - \left( \frac{\psi}{h} \right)^2 \right) \]

Given that:

\[ \tau_\psi (\psi) = -\frac{dp}{dx} \cdot \psi \]

the shear stress at a given position \( \psi \) is:

\[ \tau_\psi (\psi) = \frac{3 \mu \bar{u}}{h^2} \cdot \psi \]  \hspace{1cm} \text{(D.14)}

And the shear stress at the wall:

\[ \tau_{\psi, w} = \tau_{\psi, h} = \frac{3 \mu \bar{u}}{h} \]  \hspace{1cm} \text{(D.15)}

or

\[ \tau_{\psi, w} = \tau_{\psi, h} = \frac{6 \mu \bar{u}}{H - h} \]

### D.3 Equivalent Circular Tube Approximation

Equating the cross-sectional area to that of an equivalent circle yields the following:

\[ \pi R_h^2 = W \cdot (H - h) \]
Using the equations derived for radial flow, we have:

\[ \tau = \frac{4 \mu \bar{u}}{R_h^2} \cdot r_h \]  

(D.17)

which is in terms of the measured dimensions:

\[ \tau = \frac{2 \pi \mu \bar{u}}{W_h} \cdot r_h \]  

(D.18)

The shear stress at the wall is given by:

\[ \tau_w = \frac{4 \mu \bar{u}}{R_h} \]  

(D.19)

which is in terms of the measured dimensions:

\[ \tau_w = 4 \mu \bar{u} \cdot \sqrt{\frac{\pi}{2W_h}} \]  

(D.20)

### D.4 Comparison between the two approaches

The shear stress at the cake surface \( \tau_{cs} \) was obtained using two methods:

1. Infinite plate assumption yields:

\[ \tau_{cs} = \left. \tau \right|_{\psi = h} = \frac{3 \mu \bar{u}}{h} \]

2. Effective circular tube assumption yields:

\[ \tau_{cs} = 4 \mu \bar{u} \cdot \sqrt{\frac{\pi}{2W_h}} \]

It can be seen that the results obtained using the two different approaches yield;

\[ \tau_{cs} = \tau_{cs} \left( h^{-1} \right) \text{ and } \tau_{cs} = \tau_{cs} \left( h^{-1/2} \right) \]

respectively.

The results obtained using the two infinite plate approximation will be used.
Appendix E

DERIVATION OF EQUATIONS FOR EXTERNAL CAKE

Figure E-1: Filter cake build up in the experimental cell (Al-Abduwani, internal report 2004)

Nomenclature

\( x \)  axial coordinate (along the length of the cell)

\( y \)  Cartesian coordinate in the direction perpendicular to the surface plane of the porous medium (along the height/depth of the cell)

\( z \)  Cartesian coordinate (along the width of the cell)

\( \xi \)  centreline streamline coordinate (not used in this derivation)

\( \psi \)  coordinate from the centre of the effective flow channel in the \( y \) direction

\( \zeta \)  coordinate from the centre of the effective flow channel in the \( z \) direction

\( h \)  thickness of cake layer at the given position

\( H \)  distance between the two plates (the porous medium and the top plate)

\( W \)  width of the flow channel

\( h \)  half-height of the effective flow channel

\( Q \)  volumetric carrier fluid flux

\( u \)  superficial flow velocity

\( \bar{u} \)  average superficial velocity

\( k \)  permeability

\( p \)  pressure

\( \phi \)  porosity

\( \mu \)  viscosity

Subscripts

\( cf \)  crossflow

\( p \)  permeate (filtrate)

\( c \)  cake

\( cs \)  cake surface
Produced Water Re-Injection

\[ \text{pm} \quad \text{porous medium} \]

\[ \text{w} \quad \text{pressure in the wellbore at a given cross-section, which also the pressure acting on the cake surface for the same cross-section.} \]

\[ \text{i} \quad \text{pressure at the interface between the porous medium and the filter cake} \]

\[ \text{l} \quad \text{location } l \text{, which is the radius of drainage of the given well} \]

**E.1 Force Balance**

As was mentioned in chapter (?), various forces act on a particle in colloidal system. These forces are:

- Normal (Permeate) Drag Force \( F_n \)
- Tangential (Crossflow) Drag Force \( F_d \)
- Lift Forces \( F_L \)
- Electrostatic Forces \( F_e \) and Particle interaction
- Diffusion
- Friction Force \( F_f \)
- Net Gravity Force \( F_G \)

**E.1.1 Tangential Drag Force**

The tangential drag force can be calculated by one of three ways:

**Poiseuille’s Law For Infinite Plate**

\[ F_d = \tau w A_s \]  \hspace{1cm} (E.1)

\[ F_d = 4\pi a^2 \tau w \bigg|_{\psi=h-a} \]  \hspace{1cm} (E.2)

Using Navier-Stokes derivation in Appendix (?):

\[ \tau = \frac{3 \mu \nu_{eff}}{h^2} \psi \]

\( \psi = h - a \), thus:

\[ F_d = 4\pi a^2 \frac{3 \mu \nu_{eff}}{h^2} (h - a) \]

\[ F_d = 12\pi a^2 \frac{\mu \nu_{eff}}{h} \left( 1 - \frac{a}{h} \right) \]  \hspace{1cm} (E.3)

Radius of particle is very small compare to half height of the channel; therefore the tangential drag force can be approximated as:

\[ F_d \approx 12\pi a^2 \frac{\mu \nu_{eff}}{h} \]  \hspace{1cm} (E.4)
Poiseuille’s Law for Equivalent Tube

\[ F_d = 4\pi a^2 \cdot \tau \bigg|_{\alpha=\alpha_0} \]

\[ \tau = \frac{4 \mu \bar{u}_{ef}}{R_h^2 - r_h^2} \]

\[ R_h = \sqrt{\frac{W(H-h)}{\pi}} \quad \text{and} \quad r_a = \sqrt{\frac{W(H-h-a)}{\pi}} \]

Thus,

\[ \left. \tau \right|_{\alpha} = 4 \mu \bar{u}_{ef} \cdot \frac{\pi}{W(H-h)} \cdot \sqrt{\frac{W(H-h-a)}{\pi}} \]

\[ \left. \tau \right|_{\alpha} = 4 \mu \bar{u}_{ef} \cdot \frac{\pi}{W(H-h)} \cdot \left[ 1 - \frac{a}{\sqrt{H-h}} \right] \]

\[ \left. \tau \right|_{\alpha} = 4 \mu \bar{u}_{ef} \cdot \frac{\pi}{2Wh} \cdot \left[ 1 - \frac{a}{2h} \right] \]

\[ \left. \tau \right|_{\alpha} \approx 4 \mu \bar{u}_{ef} \cdot \frac{\pi}{2Wh} \]

And therefore:

\[ F_D \approx 4\pi a^2 \cdot 4 \mu \bar{u}_{ef} \cdot \frac{\pi}{2Wh} \]

Or

\[ F_D \approx 16\pi a^2 \cdot \mu \bar{u}_{ef} \cdot \frac{\pi}{W(H-h)} \]

Stoke’s law

For laminar flow with very small Reynolds number, stoke’s law for drag forces yields:

\[ F_{Stoke} = 6\pi \mu a u_{ef} \bigg|_{\psi=h-a} \]

From Appendix (?) we have:

\[ u_{ef} (\psi) = \frac{3}{2} \frac{\bar{u}_{ef}}{h^2} \cdot \left[ h^2 - \psi^2 \right] \]

By setting \( \psi = h - a \):
Produced Water Re-Injection

\[
\begin{align*}
    u_{cf}(h-a) &= \frac{3}{2} \frac{\bar{u}_{cf}}{h^2} \left[ h^2 - (h-a)^2 \right] \\
    u_{cf}(h-a) &= \frac{3}{2} \frac{\bar{u}_{cf}}{h^2} \left[ 2ah - a^2 \right] \\
    u_{cf}(h-a) &= 3 \frac{\bar{u}_{cf}}{h} \left[ a - \frac{a^2}{2h} \right] \\
    u_{cf}(h-a) &\approx \frac{3 \bar{u}_{cf}}{h} \frac{a}{h} 
\end{align*}
\]

And therefore,

\[
F_d \approx \frac{18 \mu \bar{u}_{cf}}{h} \cdot \pi a^2
\]

**General Form of the Tangential Drag Force**

From above mentioned equations a general form can be proposed for tangential drag force which can be written in the form of:

\[
F_d = \alpha \pi a^2 \frac{\mu \bar{u}_{cf}}{2h}
\]

\[
F_d = \alpha \pi a^2 \frac{\mu \bar{u}_{cf}}{(H-h)}
\]

**E.1.2 Normal Drag Force**

The normal drag force acts on a particle due to permeate velocity and can be calculated using Stoke’s law:

\[
F_r = 6\pi \mu a u_p
\]

**E.1.3 Electrostatic Forces**

According to DLVO theory there are two main electrostatic forces acting on a particle: double layer forces and Van der Waals forces.

**E.1.4 Frictional Force**

Friction force is the Coulomb frictional force, which is proportional to the normal forces acting on the particle.

\[
F_f = f \cdot F_N
\]

Here \( f \) is the Coulomb friction coefficient.

**E.2 Solving for the Force Balance Inequality**

During a crossflow filtration process particle will be deposited if friction force overcomes the tangential drag force. Therefore, while the tangential force overcomes the friction force, cake reaches its critical or equilibrium thickness and forces satisfy the inequality:
Using the above mentioned equations:
\[
\alpha \pi a^2 \frac{\mu u_{ef}}{(H-h)} = f \left( F_e + 6 \pi \mu a u_p + F_G \right) 
\] (E.23)

Case A: Both Electrostatic and Net Gravity forces are negligible
\[
\alpha \pi a^2 \frac{\mu u_{ef}}{(H-h)} = f \left( 6 \pi \mu a u_p \right) 
\] (E.24)
\[
\frac{u_{ef}}{u_p} = \frac{6 \gamma (H-h)}{a} 
\] (E.25)

Where,
\[
\gamma = \frac{f}{\alpha} 
\] (E.26)

Case B: Net Gravity Force is not negligible but Electrostatic Forces are negligible
\[
\alpha \pi a^2 \frac{\mu u_{ef}}{(R-h)} = f \left( 6 \pi \mu a u_p + \frac{4}{3} \pi a^3 \Delta \rho g \right) 
\] (E.27)
\[
\frac{u_{ef}}{u_p} = (\gamma_1 u_p + \gamma_2)(H-h) 
\] (E.28)

Where, \( \gamma_1 = \frac{6f}{\alpha a} \) and \( \gamma_2 = \frac{4a \Delta \rho g}{3 \mu a} \).

Case C: Electrostatic and net Gravity Forces are not negligible

E.3 Derivation of Equations at Steady State Condition

Before we go to complicated cases, we start with steady state equations, which means filter cake has reached its critical height and no deposition occurs.
In this case concentration, velocities and thickness of cake are only functions of $x$ and dependency on time is neglected due to equilibrium in forces and cake build up.

![Figure E-2](image)

### Mass Balance for Fluid

Neglecting accumulation, we write mass balance for fluid in the control volume:

$$
\left( u_{cf} (H-h)W \rho_f \right)_{in} = \left( u_{cf} (H-h)W \rho_f \right)_{out} + \left( \frac{u_{p,1} + u_{p,2}}{2} \rho_f \right) (\Delta x W) \tag{E.29}
$$

Since velocities and height of the cake are just dependent on $x$ we can write:

$$
\begin{align*}
  u_{cf, out} &= u_{cf, in} + \left( \frac{\partial u_{cf, in}}{\partial x} \right) \Delta x \\
  u_{p,2} &= u_{p,1} + \left( \frac{\partial u_{p,1}}{\partial x} \right) \Delta x
\end{align*} \tag{E.30}
$$

Also with introducing $\ell = H - h$, it can be written:

$$
\begin{align*}
  \ell_{out} &= \ell_{in} + \left( \frac{\partial \ell}{\partial x} \right) \Delta x \\
  C_{out} &= C_{in} + \left( \frac{\partial C}{\partial x} \right) \Delta x
\end{align*} \tag{E.31}
$$

Replacing (E.30) and (E.31) in (E.29) and assuming that fluid is incompressible and changes in density are negligible:

$$
\begin{align*}
  u_{cf, in} \ell_{in} &= \left( u_{cf, in} + \left( \frac{\partial u_{cf, in}}{\partial x} \right) \Delta x \right) \ell_{in} + \left( \frac{\partial \ell}{\partial x} \right) \Delta x \right) \ell_{out} + \left( \frac{u_{p,1} + u_{p,2}}{2} \right) \left( \frac{\partial u_{p,1}}{\partial x} \right) \Delta x \\
  \left[ u_{cf, in} \left( \frac{\partial \ell}{\partial x} \right) \Delta x \right] + \left( \ell_{in} \left( \frac{\partial u_{cf, in}}{\partial x} \right) \Delta x \right) + \left( \frac{\partial u_{cf, in}}{\partial x} \right) \left( \frac{\partial \ell}{\partial x} \right) \left( \Delta x \right)^2 \right] + \left( u_{p,1} + \left( \frac{\partial u_{p,1}}{\partial x} \right) \Delta x \right) \Delta x = 0
\end{align*} \tag{E.32}
$$
Dividing by $\Delta x$:

$$\left[ u_{f,in} \left( \frac{\partial \ell}{\partial x} \right) + \ell_{in} \left( \frac{\partial u_{f,in}}{\partial x} \right) + \left( \frac{\partial u_{f,in}}{\partial x} \right) \left( \frac{\partial \ell}{\partial x} \right) \right] \Delta x + \left( u_{p,1} + \frac{\partial u_{p,1}}{\partial x} \right) \frac{\Delta x}{2} = 0 \quad (E.34)$$

When $\Delta x \to 0$ third and fourth terms are negligible:

$$\left[ u_{f,in} \left( \frac{\partial \ell}{\partial x} \right) + \ell_{in} \left( \frac{\partial u_{f,in}}{\partial x} \right) \right] + u_{p,1} = 0 \quad (E.35)$$

$$u_p = -\frac{\partial (u_f \ell)}{\partial x} \quad (E.36)$$

And finally we can write:

$$u_p = -\frac{\partial (u_f (H-h))}{\partial x} \quad (E.37)$$

**Mass Balance Equation for Particles**

\[
\left( u_f c(H-h)W_p \right)_{in} - \left( u_f c(H-h)\rho_p W \right)_{out} + \left( \frac{c_{p,1} + c_{p,2}}{2} u_{p,1} + \frac{c_{p,2}}{2} (\rho_p \Delta x W) \right) = \frac{\partial (c_p \rho_p (H-h) \Delta x W)}{\partial t} \quad (E.38)
\]

For steady state case the right side term of (E.38) is zero. Assuming that particles are incompressible and using (E.30) and (E.31):

$$u_{f,in} C_{in} \ell_{in} - \left[ \left( u_{f,in} \ell_{in} + \frac{\partial u_{f,in}}{\partial x} \right) \Delta x \left( C_{in} + \frac{\partial C}{\partial x} \right) \left( \frac{\partial \ell}{\partial x} \right) \right] = 0 \quad (E.39)$$

Dividing by $\Delta x$ and putting $\Delta x \to 0$:

$$\left[ u_{f,in} C_{in} \ell_{in} + \left( u_{f,in} \ell_{in} + \frac{\partial C}{\partial x} \right) + \left( u_{f,in} \ell_{in} + \frac{\partial u_{f,in}}{\partial x} \right) C_{p,1} \right] = 0 \quad (E.40)$$

$$\left[ C_{in} \left( u_{f,in} \ell_{in} + \frac{\partial C}{\partial x} \right) + \left( u_{f,in} \ell_{in} + \frac{\partial u_{f,in}}{\partial x} \right) + C_{p,1} \right] = 0 \quad (E.41)$$

$$C \left( \frac{\partial (u_f \ell)}{\partial x} \right) + u_f \ell \left( \frac{\partial C}{\partial x} \right) + C_p u_p = 0 \quad (E.42)$$

Using (E.36):

$$-Cu_p + u_f \ell \left( \frac{\partial C}{\partial x} \right) + C_p u_p = 0 \quad (E.43)$$
In steady state $C_p = 0$, because the number of particles getting deposited and being removed from cake are identical.

\[-C u_p + \left( u_{cf} \ell \left( \frac{\partial C}{\partial x} \right) \right) = 0\]

\[\left( u_{cf} (H - h) \right) \frac{\partial C}{\partial x} - u_p C = 0\]

\[\frac{\partial C}{\partial x} = \frac{C(x) u_p (x)}{u_{cf} (x) (H - h(x))}\]  \hspace{1cm} (E.44)

Using (E.25), we will come up with:

\[\frac{dC}{C(x)} = \frac{a}{6 (H - h(x))^2} \, dx\]  \hspace{1cm} (E.45)

\[C(x) = Ae^{\int_{0}^{x} \frac{a}{6 (h(x))^2} \, dx}\]  \hspace{1cm} (E.46)

Here $A$ is the integration constant.

**Derivation of Equations for Thickness of the Cake**

(E.37) states that the flux along the experimental setup decreases due to the leakoff of the permeate flux.

\[\frac{\partial Q}{\partial x} = - Wu_p\]  \hspace{1cm} (E.47)

\[Q(x) = u_{cf} (x) W (H - h(x))\]  \hspace{1cm} (E.48)

Permeate velocity can be quantified using Darcy’s law and measuring the pressure decline in y direction:

\[u_p = \frac{k_i \partial p}{\mu \partial y}\]  \hspace{1cm} (E.49)

Here, $k_i$ is the permeability of the medium and $\mu$ viscosity of the fluid carrying the particles. Then for the filter cake it can be written as:

\[\left( p_w - p_o \right) = \frac{u_p \mu h}{k_i} \]  \hspace{1cm} (E.50)

And for the porous medium within the drainage length:

\[\left( p_o - p_l \right) = \frac{u_p \mu l}{k} \]  \hspace{1cm} (E.51)

Adding (E.50) to (E.51):

\[\left( p_w - p_l \right) = u_p \mu \left( \frac{h}{k_i} + \frac{l}{k} \right) \]  \hspace{1cm} (E.52)

From (E.52) another equation can be written for permeate velocity as:
\[ u_p = \left( \frac{p_w - p_i}{\mu} \right) \left( \frac{h + l}{k_c} \right)^{-1} \]  
(E.53)

(E.53) can be re-arranged as:

\[ u_p = \left( \frac{p_w - p_i}{\mu} \right) \left( \frac{k_c}{kh + k_c l} \right) \]  
(E.54)

With introducing

\[ \Theta = \frac{k_c l}{k} \]  
(E.55)

and

\[ \psi = \frac{k_c (p_w - p_i)}{\mu} \]  
(E.56)

Permeate velocity can be written as:

\[ u_p = \frac{\psi}{h + \Theta} \]  
(E.57)

As can be seen from (E.57), permeate velocity, \( u_p \), is a function of thickness of the external filter cake:

\[ u_p = u_p(h(x)) \]

Putting (E.48) and (E.57) into (E.25):

\[ \frac{Q}{W(H - h)} = \frac{6\gamma(H - h)}{a} \]  
(E.58)

Then we can find for Q:

\[ Q = \frac{6\gamma W \psi(H - h)^2}{a} \]  
(E.59)

If we re-arrange it:

\[ \frac{dQ}{dx} = \frac{6\gamma}{a} W \left[ \left( H - h \right)^2 \frac{d\psi}{dx} - 2\psi(H - h) \frac{dh}{dx} \left( h + \Theta \right) - \psi \left( H - h \right)^2 \frac{dh}{dx} \right] \]  
(E.60)

Equating (E.47) and (E.60):

\[ -\psi = \frac{6\gamma}{a} (H - h) \left[ (H - h) \frac{d\psi}{dx} - \psi \left( 2 + \frac{(H - h)}{h + \Theta} \right) \frac{dh}{dx} \right] \]  
(E.61)
Produced Water Re-Injection

\[-1 = \frac{6\gamma}{a} (H-h) \left[ \frac{(H-h)\,dy}{\psi\,dx} - \left(2 + \frac{(H-h)}{h + \Theta}\right)\,dh \right] \]  
\[= \frac{6\gamma}{a} (H-h) \left[ \left(2 + \frac{H-h}{h + \Theta}\right)\,dh - \frac{(H-h)\,dy}{\psi\,dx} \right] - 1 = 0 \]  
\[\text{(E.62)}\]

\[\text{From Error! Reference source not found.:} \]
\[\frac{dp}{dx} = -\frac{12\mu\bar{u}_{cf}}{(H-h)^2} \]  
\[\text{(E.64)}\]

Replacing (E.25) and (E.57) into (E.64) we get:
\[\frac{dp}{dx} = -\frac{72\gamma\mu\psi}{a(H-h)(h + \Theta)} \]  
\[\text{(E.65)}\]

from (E.65) and (E.56) it can be followed that:
\[\frac{1}{\psi}\frac{dy}{dx} = -\frac{72\gamma k_c}{a(H-h)(h + \Theta)} \]  
\[\text{(E.66)}\]

Therefore, (E.63) will change to:
\[\frac{6\gamma}{a} (H-h) \left[ \left(2 + \frac{H-h}{h + \Theta}\right)\,dh + \frac{72\gamma k_c}{a(h + \Theta)} \right] - 1 = 0 \]  
\[\text{(E.67)}\]

Or:
\[\frac{dh}{dx} = \left[ h + \Theta \right] \left[ \frac{a}{6\gamma(H-h)} - \frac{72\gamma k_c}{a(h + \Theta)} \right] \]  
\[\text{(E.68)}\]

Using (E.25) and (E.57), (E.68) becomes:
\[\frac{dh}{dx} = \left[ h + \Theta \right] \left[ \frac{u_p}{\bar{u}_{cf}} - \frac{72\gamma k_c}{a} \frac{\psi}{u_p} \right] \]  
\[\text{(E.69)}\]

**Calculation of cake thickness in the right boundary**

Combining (E.25) and (E.57) yields:
\[\bar{u}_{cf} = \frac{6\gamma(H-h)}{a} \frac{\psi}{h + \Theta} \]  
\[\text{(E.70)}\]

Putting (E.70) in (E.48) follows:
\[Q = \frac{6\gamma W}{a} (H-h)^2 \frac{\psi}{h + \Theta} \]  
\[\text{(E.71)}\]

If we solve (E.71) for \(h\) we will get:
\[h = H + \frac{Qa \pm \sqrt{Qa(Qa + 24\gamma W \psi (H + \Theta))}}{12\gamma W \psi} \]  
\[\text{(E.72)}\]

(E.72) gives the thickness of cake for given flow rates. This equation can be used to calculate the thickness in the right boundary.
E.4 Derivation of Equations for Transient Case (No Erosion)

Mass Balance Equation for fluid in suspension

\[
\left(\bar{u}_{cf} (H - h) W \rho_f \right)_{in} - \left(\bar{u}_{cf} (H - h) W \rho_f \right)_{out} + \left(\frac{u_{p,1} + u_{p,2}}{2}\right)(\rho_f \Delta x W) + \frac{\partial}{\partial t} \left(\rho_f \varphi_c (H - h) \Delta x W\right)
\]

(E.73)

Assuming that fluid is incompressible and repeating the same procedure as we did for steady state case, we will get the following equation:

\[
-\frac{\partial}{\partial x}\left(\bar{u}_{cf} (H - h)\right) = u_p + \varphi_c \frac{\partial}{\partial x} (H - h)
\]

(E.74)

Mass Balance Equation for Particles

\[
\left(\bar{u}_{cf} C (H - h) W \rho_p \right)_{in} - \left(\bar{u}_{cf} C (H - h) W \rho_p \right)_{out} = \left(\frac{u_{p,1} + u_{p,2}}{2}, C_{p,1}, C_{p,2}\right) (\rho_p \Delta x W) + \frac{\partial}{\partial t} (\rho_p (H - h) \Delta x W)
\]

(E.75)

Assuming that particles are incompressible and all number of deposited and entering particles into the system are identical we will end up with:

\[
-\frac{\partial}{\partial x}\left(\bar{u}_{cf} C (H - h)\right) = u_p C + \frac{\partial}{\partial t} \left[C (H - h)\right]
\]

(E.76)

Or:

\[
C \left(u_p + \frac{\partial}{\partial t} (H - h)\right) + (H - h) u_c \frac{\partial C}{\partial x} + u_p C = -C \frac{\partial}{\partial t} (H - h) - (H - h) \frac{\partial C}{\partial t}
\]

\[
C \frac{\partial}{\partial x} \left[(H - h) u_c\right] + (H - h) u_c \frac{\partial C}{\partial x} + u_p C = -C \frac{\partial}{\partial t} (H - h) - (H - h) \frac{\partial C}{\partial t}
\]

(E.77)

Putting (E.74) into (E.77) gives:

\[
-\frac{\partial}{\partial x} \left[\begin{array}{c}
C \left(u_p + \frac{\partial}{\partial t} (H - h)\right) + (H - h) u_c \frac{\partial C}{\partial x} + u_p C
\end{array}\right] = -C \frac{\partial}{\partial t} (H - h) - (H - h) \frac{\partial C}{\partial t}
\]

(E.78)

With simplifying (E.78) we will get:

\[
u_c \frac{\partial C}{\partial x} + \frac{\partial C}{\partial t} = 0
\]

(E.79)
E.5 General Case: Considering both Deposition and Erosion

In filtration the rate of cake height growth can be obtained from mass balance for particles. Mass balance for the particle mass flow in Figure E-3 can be written as:

\[
\dot{V}_{p, in} - \dot{V}_{p, back} = \frac{dV_p}{dt}
\]  
(E.80)

The problem is divided into two parts. We derive two separate equations for deposited and eroded or back transported particles and finally add them together.

**Deposition Rate**

Assuming that volumetric concentration is too small compare to the solidity of the medium and density of particles is constant, for the deposited part we can write:

\[
\dot{V}_{p, in} = \frac{dV}{dt}
\]  
(E.81)

The particle flux to the filter cake is given by:

\[
\dot{V}_{p, in} = C_u_p
\]  
(E.82)

Replacing (E.82) into (E.81) we can write:

\[
A_f(u_p,C) = \frac{d((1 - \varphi_c)h^*A_f)}{dt}
\]  
(E.83)

Here \(h^*\) is the thickness of cake and \(\varphi_c\) is the porosity of cake. In reality porosity of the cake is changing with time; however for the simplicity we ignore the changes in porosity of the cake, so:

\[
(1 - \varphi_c)\frac{dh^*}{dt} = u_p,C
\]  
(E.84)

**Back Transport of the Particles (Cake Erosion)**
\[ \dot{V}_{p, \text{back}} = -\frac{dV^*}{dt} \]  

(E.85)

The main parameter which causes the back transport of the particles into the system is crossflow velocity \( \omega \).

\[ C_u \backslash \text{back} = \omega \overline{u}_{cf} \]  

(E.86)

\( \alpha \) may be called erosion coefficient and will be calculated later.

Again with the same assumptions for back transport or erosion rate we get:

\[ (1 - \varphi_e) \frac{dh^*}{dt} = \omega \overline{u}_{cf} \]  

(E.87)

Here, \( h^* \) is the eroded height of the cake and \( \varphi_e \) is the porosity of cake.

Finally if we add (E.84) to (E.87) it follows that:

\[ (1 - \varphi_e) \left[ \frac{d(h^* - h^*)}{dt} \right] = C_u u_p - \omega \overline{u}_{cf} \]  

(E.88)

Actual height of the cake is the difference between two heights:

\[ h = h^* - h \]  

(E.89)

\[ (1 - \varphi_e) \frac{dh}{dt} = C_u u_p - \omega \overline{u}_{cf} \]  

(E.90)

### Calculation of Erosion Coefficient

When cake reaches to equilibrium the rate of deposition and erosion is equal. It means that the left hand side of (E.88) is zero and as a result \( \alpha \) can be calculated by following equation:

\[ \omega = \left( \frac{u_p C}{\overline{u}_{cf}} \right)_{SS} \]  

(E.91)

and for erosion rate we can write:

\[ m_{\text{back}} = \left( \frac{u_p C}{\overline{u}_{cf}} \right)_{SS} u_{cf} \]  

(E.92)

Putting (E.91) into (E.90) the rate of cake growth will be given by:

\[ (1 - \varphi_e) \frac{dh}{dt} = C_u u_p + \left( \frac{u_p C}{\overline{u}_{cf}} \right)_{SS} \overline{u}_{cf} \]  

(E.93)

Or,

\[ (1 - \varphi_e) \frac{dh}{dt} = \frac{a}{6\gamma} \left[ \left( \frac{C}{(H - h)} \right) + \left( \frac{C}{(H - h)} \right)_{SS} \right] \]  

(E.94)
E.6 Mass Balance Equations for Suspension (With Erosion)

To write mass balance equations we divide the control volume into two parts, suspension and cake. We write mass balance for each part and then add them together to get the equations.

**Mass Balance Equation for fluid**

**Part I in the suspension**

\[
\left( u_{cf} (H-h)W \rho_f \right)_{in} = \left( u_{cf} (H-h)W \rho_f \right)_{out} + \left( \frac{u_{p,1} + u_{p,2}}{2} \rho_f \Delta x W \right)_{incake} + \left( \frac{\partial}{\partial t} \left( \rho_f (H-h) \Delta x W \right) \right)_{incake}
\]

\[\text{(E.95)}\]

**Part II on the cake**

\[
\left( \frac{u_{p,1} + u_{p,2}}{2} \rho_f \Delta x W \right)_{incake} = \left( \frac{u_{p,1} + u_{p,2}}{2} \rho_f \Delta x W \right)_{outcake} + \left( \frac{\partial}{\partial t} \left[ \rho_f \varphi_c h \Delta x W \right] \right)_{outcake}
\]

\[\text{(E.96)}\]

Assuming that fluid is incompressible and repeating the same procedure as we did for steady state case and replacing (E.96) into (E.95) follows:

\[\frac{\partial}{\partial x} \left[ \bar{u}_{cf} (H-h) \right] = u_p + \frac{\partial}{\partial t} \left[ H-h (1-\varphi_c) \right] \]

\[\text{(E.97)}\]

**Mass Balance Equation for Particles**

**Part I in the suspension**

\[
\left( u_{cf} \rho_p \right)_{in} + \left( \omega u_{cf} \Delta x W \right) =
\left( u_{cf} \rho_p \right)_{out} + \left( \frac{C_{p,1} + C_{p,2}}{2} \frac{u_{p,1} + u_{p,2}}{2} \rho_p \Delta x W \right) + \left( \frac{\partial}{\partial t} \left( \rho_p C (H-h) \Delta x W \rho_p \right) \right)
\]

\[\text{(E.98)}\]

**Part II on the cake**

\[
\left( \frac{C_{p,1} + C_{p,2}}{2} \frac{u_{p,1} + u_{p,2}}{2} \rho_p \Delta x W \right) = \left( \omega u_{cf} \Delta x W \rho_p \right) + \left( \frac{\partial}{\partial t} \left[ \rho_p (1-\varphi_c) h \Delta x W \right] \right)
\]

\[\text{(E.99)}\]

Assuming that particles are incompressible and replacing (E.99) into (E.98) we will get:

\[
\frac{\left( u_{cf} C (H-h) \right)_{in} - \left( u_{cf} C (H-h) \right)_{out}}{\Delta x} = \frac{\partial}{\partial t} \left( C (H-h) \right) + \frac{\partial}{\partial t} \left[ (1-\varphi_c) h \right]
\]

\[\text{(E.100)}\]

\[
- \frac{\partial}{\partial x} \left[ Cu_{cf} (H-h) \right] = \frac{\partial}{\partial t} \left[ (H-h) C + (1-\varphi_c) h \right]
\]

\[\text{(E.101)}\]

Or:

\[
-C \frac{\partial}{\partial x} \left( u_{cf} (H-h) \right) - u_{cf} (H-h) \frac{\partial C}{\partial x} = \frac{\partial}{\partial t} \left[ (H-h) C + (1-\varphi_c) h \right]
\]

\[\text{(E.102)}\]
Re-arranging of (E.102) follows:

\[
(H - h) \frac{\partial C}{\partial t} = -\left[ \frac{\partial}{\partial x} \left[ C \Pi_{cf} (H - h) \right] + (1 - \varphi_e - C) \frac{\partial h}{\partial t} \right]
\]  

(E.103)
Appendix F

SATURATION PROCEDURE

Figure F-1, depicts the saturation unit which is used at the beginning of each experiment to saturate the core. Once that is done successfully this unit is disengaged from the experimental setup.

F.1 Pre-Saturation Preparation of the Setup

Before attempting to saturate the core, the following steps have to be carried out.

1. Thorough clean up of the core holder
2. Thorough drying of the core holder and the attached pipes
3. Insertion of the core with the appropriate end pieces
4. Ensure that all valves are set to the closed position and then connect the relevant pipes
5. Pump the confining oil into the annulus and elevate the pressure to the predetermined confining pressure

Figure F-1: Schematic of the Saturation Unit (Al-Abduwani, internal report 2003)
Thorough cleaning of the core holder involves washing the annulus with ethanol. Thus, washing out the remaining oil film that is coating the exterior of the rubber sleeve as well as the interior of the stainless steel core holder. This is done to ensure that no contamination of the core takes place when it is inserted into the rubber sleeve at a later stage.

Once the annular space has been cleaned up, the interior of the rubber sleeve is to be cleaned with a new batch of ethanol. This is done to remove any sand grains or injected particles from the interior of the rubber sleeve. Ensure that the pressure taps are thoroughly cleaned too.

Using pressurised air, the core holder is to be dried. Similarly the accessible pipes should be cleaned and dried.

(Note that the pipes should have been flushed at the end of the previous experiment as part of the post experiment clean-up. If that was not done, then ensure that you read that section and follow the relevant instructions before commencing)

The core and the core plugs should be inserted into the rubber sleeve. The two end caps of the core holder should be firmly sealed. The core holder can now be placed into the setup and oil should be pumped into its annulus. The pressure is to be elevated to the preset value.

The setup is now ready for the saturation process.

**F.2 Saturation Procedure**

Before commencing with the saturation of the core, thorough preparation of the setup must have been followed as instructed in the previous section.

The first step is to establish vacuum conditions in the two tanks depicted in Figure as well as the area inside the rubber sleeve. Therefore the tubes connecting the pressure taps to the differential pressure gauges have to be checked for leakages. The pipes have to be clean and dry and sealed at the pressure gauge end.

It is also essential that no flow of water takes place. This can be achieved easily if the pressure in the whole system is dropped evenly without creating any pressure differentials that will induce flow.

The beaker in vacuum chamber labelled [Tank 1] should be filled up with a volume of distilled water that is roughly 5 times the volume of the pore-space (this can easily be calculated) and the tubes connecting the saturation unit to the core. This is to ensure that at no point will there be insufficient amount of water. Place a magnet fish in the beaker, which will be used to stir the water to aid the process of deaeration.
Connect the saturation unit to the setup and engage valves B1 and C1 (Figure 1). Ensure that valves H and J are open, while valves I, and M should be closed (Figure ). Needle valve K should be as dilated as possible.

Engage the vacuum pump and ensure that the pressure drops to around –0.95 gauge bars. Maintain at this pressure for 30 minutes while the magnetic stirrer is switched on and is placed under TANK1.

After this time has elapsed, switch off the magnetic stirrer.

Close valve H.

Open valve I, very slightly to allow a pressure differential of 0.05 bars and then close it.

Using needle valve K, regulate the flow of water.

Eventually water will start flowing into [Tank2]. However, this process takes some time (approximately 1.5 hours). At this point the vacuum pump should be switched off, valve H opened up and needle valve K fully dilated. The pressure is now equalised and since [TANK1] and [TANK2] are at the same hydrostatic level no back flow should take place.

Manipulate either valve L or M to slowly increase the pressure of the system to atmospheric pressure.

Close valves A, B and C.

Decouple the saturation unit.

Thus, the saturation process has been completed.

**F.3 Deaeration of the set up**

Some of the pipes between the pump and the core holder will contain some air. Make sure that there is no air in whole system. To that end remove all the existence air in the set up. The below steps should be followed to do the deairation process:

Step 1: Reduce pressure in both tanks down to -1.0 bars. Leave for 2 hours for the water to be deaerated. ([TANK2] is vacuumed. So when relevant valves (B1 & C1) are open it can suck out the existence air in the set up).

Step 2: Now tanks are both vacuumed. To have flow, pressure at the top and the bottom of sample must be different. Therefore close valve J. Use valve M to increase pressure in [Tank1] by increment of 0.05 bars until steady flow is initiated.
Step 3: From the preceding experiments the pressure difference between [Tank1] and [Tank2] must be 0.2 bars. Needless to say, to have flow from bottom to top pressure of [Tank1] should be higher than of [Tank2]. Maintain this pressure difference of 0.2 until about 5 times of pore volume has flowed through the core into [Tank2]. Use valves L and M to increase the pressure in tanks, maintaining a dp of 0.2 bars. Use an incremental of 0.1 bars to do this. Wait about 2 minutes at each interval.

<table>
<thead>
<tr>
<th>[Tank1] (bar)</th>
<th>[Tank2] (bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.8</td>
<td>-1.0</td>
</tr>
<tr>
<td>-0.7</td>
<td>-0.9</td>
</tr>
<tr>
<td>Wait for 2 minutes</td>
<td></td>
</tr>
<tr>
<td>-0.6</td>
<td>-0.8</td>
</tr>
<tr>
<td>Wait for 2 minutes</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Wait for 2 minutes</td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>-0.2</td>
</tr>
<tr>
<td>Wait for 2 minutes</td>
<td></td>
</tr>
</tbody>
</table>

Step 4: To run the experiment pressure in valve C should be atmospheric. In the last saturation step pressure in valve C is -0.2 while in B is 0.0 barg. If pressure in valve B is equal to pressure in valve C, due to the hydrostatic head flow would occur and the saturation would be affected. To prevent this affect, connect peristaltic pump in location S. Switch on peristaltic pump.

Step 5: Increase pressure in [Tank2] up to atmospheric.

Step 6: Wait for a few PVs.

Step 7: Close valves C & A2 and engage valve A3 so that air in the set up can go out through the pipe. Close valve A3 when water comes out. (Figure 4)
Step 8: The chamber above the set up is full of water. This water pushes the air to top of the chamber. To remove this air unscrew the pipe just above the chamber (T in Figure 5) and let it go out.

![Figure 5: Deaeration from the chamber(step 8)](image)

Step 9: Reconnect the joints (T in Figure 4) and open valve C1, then water comes out to [TANK2]. (Figure 6)

![Figure 6: schematic of step 9](image)
Produced Water Re-Injection
Appendix G
UNSUCCESSFUL STORIES WITH LATEX PARTICLES

Figure 0-1: Pressure vs. Time, Green latex particles injection was followed by Red ones.

dp vs. Time

Figure 0-2: Pressure vs. Time, Red particles were injected, then switched to Blue (which changed the slope of the graph), in time 3200 sec we switched back to Red particles again.
Produced Water Re-Injection
Appendix H

CAN HEMATITE BE DETECTED USING CT-SCANNER?

Sensitivity of equation (6.1) to density difference alone

The following equation will be obtained in this case assuming that: \( \mu = \rho_{\text{eff}} \)

\[
\Delta HU = \frac{1000}{\rho_w} \Delta \rho_{\text{eff}} \quad (H.1)
\]

Where after deposition of hematite we will have:

\[
\rho_{\text{eff}}^h = (1 - \phi) \rho_{\text{mr}} + \varepsilon \rho_h + (\phi - \varepsilon) \rho_{\text{air}} \quad (H.2)
\]

\( \varepsilon \) is the volume of deposited hematite particle over the total volume. \( \varepsilon = 0 \) when there is no deposition. Therefore,

\[
\Delta \rho_{\text{eff}} = \rho_{\text{eff}}^h - \rho_{\text{eff}}^0 = \varepsilon' \frac{\rho_{\text{mr}}}{\rho_h} (\rho_h - \rho_{\text{air}}) \quad (H.3)
\]

where,

\( \varepsilon' = \frac{\text{Mass of hematite}}{\text{Mass of matrix rock}} \)

From the previous experiments the amount of deposited hematite is: \( \varepsilon' = 0.01 \). Also using \( \rho_{\text{mr}} = 5400 \text{ kg/m}^3 \) for hematite we will get:

\[
\Delta HU = 26.5
\]

Which is higher than minimum measurable value of the Ct-scanner (=10).

Sensitivity to atomic number alone

In this case:

\[
\mu = \beta Z_e^{3.8} \quad (H.4)
\]

and

\[
\Delta HU = 1000 \left( \frac{\Delta Z_e^{3.8}}{Z_e^{3.8}} \right) \quad (H.5)
\]

For the system of hematite and sandstone (\( SiO_2 \)) effective atomic number can be calculated by:

\[
Z_e = \left( g_{Si} Z_{Si}^{3.8} + g_{Fe} Z_{Fe}^{3.8} + g_{O} Z_{O}^{3.8} + g_{N} Z_{N}^{3.8} \right) \quad (H.6)
\]

or:
\[ Z_e = \left( \frac{N_{Si}Z_{Si}}{G} Z_{Si}^{3.8} + \frac{N_{Fe}Z_{Fe}}{G} Z_{Fe}^{3.8} + \frac{N_{O}Z_{O}}{G} Z_{O}^{3.8} + \frac{N_{N}Z_{N}}{G} Z_{N}^{3.8} \right)^{1/38} \]  \hspace{1cm} (H.7)

where,

\[ G = N_{Si}Z_{Si} + N_{Fe}Z_{Fe} + N_{O}Z_{O} + N_{N}Z_{N} \] \hspace{1cm} (H.8)

with,

\[ N_i = \frac{M_i}{\zeta_i} \] \hspace{1cm} (H.9)

\( N_i \) is the corresponding number of atom \( i \) and

\[ M_i = \frac{m_i}{\zeta_i} \]

\( m_i \) is the mass of constituent \( i \) and \( \zeta_i \) is the molecular mass of constituent \( i \).

For a system of sandstone and hematite porosity of sand can be calculated as:

\[ \phi = \frac{V_h + V_{air}}{V_{mr} + V_h + V_{air}} \] \hspace{1cm} (H.10)

\[ \varepsilon' = \frac{m_h}{m_{mr}} \] \hspace{1cm} (H.11)

where, \( V_{mr} \) is the volume of sand matrix. The mass of corresponding hematite to the chosen arbitrary matrix rock mass is:

\[ m_h = \varepsilon' m_{mr} \] \hspace{1cm} (H.12)

The mass of corresponding air in the chosen control volume can be calculated by:

\[ V_h = \frac{m_h}{\rho_h} = \frac{\varepsilon' m_{mr}}{\rho_h} \]

\[ \phi (V_{mr} + V_h + V_{air}) = V_h + V_{air} \]

\[ V_{air} (1 - \phi) = (\phi - 1)V_h + \phi V_{mr} \]

\[ V_{air} = \frac{\phi V_{mr} - (1 - \phi)V_h}{(1 - \phi)} \]

therefore,

\[ V_{air} = \frac{\phi}{1 - \phi} V_{mr} - V_h \] \hspace{1cm} (H.13)

Using above equations the following equations are obtained to calculate the mass of matrix, air and hematite:

\[ m_{mr} = \text{chosen} \] \hspace{1cm} (H.14)
\[ m_h = \varepsilon' m_{mr} \quad \text{(H.15)} \]

\[ m_{air} = m_{mr} \rho_{air} \left[ \frac{\phi}{\rho_{mr} (1 - \phi)} - \frac{\varepsilon'}{\rho_h} \right] \quad \text{(H.16)} \]

Before doing the experiments \( \varepsilon' = 0 \). From the previous experiments the amount of deposited hematite is: \( \varepsilon' = 0.01 \frac{\text{gr of hematite}}{\text{gr of matrix rock}} \). This will give:

\[ \Delta HU = 46.5 \]

Which is higher than minimum measurable value of the Ct-scanner (=10).
Appendix I

EXPERIMENTAL RESULTS: GRAPHS AND DATA

I.1 Experiment 1

Table I-1: Table of Experiment 1

<table>
<thead>
<tr>
<th>$c_0$ [mg/L]</th>
<th>$Q_{cf}$ [L/hr]</th>
<th>$u_{cf}$ [m/sec]</th>
<th>$Q_p$ [L/hr]</th>
<th>$u_p$ [m/sec]</th>
<th>$u_{cf}/u_p$</th>
<th>start</th>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>5.50</td>
<td>4.41E-3</td>
<td>5.50</td>
<td>4.41E-3</td>
<td>1</td>
<td>00:07:00</td>
<td>01:27:10</td>
</tr>
</tbody>
</table>

Table I-2: Scanning time table of experiment 1

<table>
<thead>
<tr>
<th>Scan No.</th>
<th>Time [hr:min:sec]</th>
<th>Time [sec]</th>
<th>Real time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>scan01</td>
<td>00:08:10</td>
<td>490</td>
<td>70</td>
</tr>
<tr>
<td>scan02</td>
<td>00:11:09</td>
<td>669</td>
<td>249</td>
</tr>
<tr>
<td>scan03</td>
<td>00:14:09</td>
<td>849</td>
<td>429</td>
</tr>
<tr>
<td>scan04</td>
<td>00:19:09</td>
<td>1149</td>
<td>729</td>
</tr>
<tr>
<td>scan05</td>
<td>00:24:09</td>
<td>1449</td>
<td>1029</td>
</tr>
<tr>
<td>scan06</td>
<td>00:34:09</td>
<td>2049</td>
<td>1629</td>
</tr>
<tr>
<td>scan07</td>
<td>00:44:10</td>
<td>2650</td>
<td>2230</td>
</tr>
<tr>
<td>scan08</td>
<td>00:59:09</td>
<td>3549</td>
<td>3129</td>
</tr>
<tr>
<td>scan09</td>
<td>01:14:17</td>
<td>4457</td>
<td>4037</td>
</tr>
<tr>
<td>scan10</td>
<td>01:25:09</td>
<td>5109</td>
<td>4689</td>
</tr>
</tbody>
</table>

Table I-3: Sampling table of experiment 1

<table>
<thead>
<tr>
<th>solution sample</th>
<th>start time</th>
<th>end time</th>
<th>start time [sec]</th>
<th>end time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>00:08:40</td>
<td>00:09:00</td>
<td>520</td>
<td>540</td>
</tr>
<tr>
<td>2</td>
<td>00:11:40</td>
<td>00:12:00</td>
<td>700</td>
<td>720</td>
</tr>
<tr>
<td>3</td>
<td>00:14:40</td>
<td>00:15:00</td>
<td>880</td>
<td>900</td>
</tr>
<tr>
<td>4</td>
<td>00:19:40</td>
<td>00:20:00</td>
<td>1180</td>
<td>1200</td>
</tr>
<tr>
<td>5</td>
<td>00:24:40</td>
<td>00:25:00</td>
<td>1480</td>
<td>1500</td>
</tr>
<tr>
<td>6</td>
<td>00:34:40</td>
<td>00:35:00</td>
<td>2080</td>
<td>2100</td>
</tr>
<tr>
<td>7</td>
<td>00:44:40</td>
<td>00:45:00</td>
<td>2680</td>
<td>2700</td>
</tr>
<tr>
<td>8</td>
<td>00:59:40</td>
<td>01:00:00</td>
<td>3580</td>
<td>3600</td>
</tr>
<tr>
<td>9</td>
<td>01:14:40</td>
<td>01:15:00</td>
<td>4480</td>
<td>4500</td>
</tr>
<tr>
<td>10</td>
<td>01:25:50</td>
<td>01:26:10</td>
<td>5150</td>
<td>5170</td>
</tr>
</tbody>
</table>

Table I-4: Extracted results from experiment 1

<table>
<thead>
<tr>
<th>$C_0$</th>
<th>$\lambda_\infty$ [m$^{-1}$]</th>
<th>$\lambda_0$ [m$^{-1}$]</th>
<th>$\lambda_{SS}$ [m$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.0 \times 10^{-4}$</td>
<td>2.91</td>
<td>86.75</td>
<td>86.56</td>
</tr>
</tbody>
</table>
Figure I-1: Normalised CT numbers with respect to water along the core (Experiment 1)

Figure I-2: Concentration profile along the core (Experiment 1)
Figure I-3: Initial concentration vs. $u_p t$, Experimental data and linear fitting (Experiment 1)

$$y C_0(t) = 0.00013 u_p t + 0.00011$$

Figure I-4: Filtration coefficient vs. inverse of time, experimental data and linear fitting (Experiment 1)

$$y = 390.63x + 4.4609$$
I.2 Experiment 2

Table I-5: Table of Experiment 2

<table>
<thead>
<tr>
<th>c_0 [mg/L]</th>
<th>Q_{cf} [L/hr]</th>
<th>u_{cf} [m/sec]</th>
<th>Q_p [L/hr]</th>
<th>u_p [m/sec]</th>
<th>u_{cf}/u_p</th>
<th>start</th>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>5.50</td>
<td>4.41E-3</td>
<td>2.75</td>
<td>4.41E-3</td>
<td>0.5</td>
<td>00:11:00</td>
<td>01:33:10</td>
</tr>
</tbody>
</table>

Table I-6: Scanning time table of experiment 2

<table>
<thead>
<tr>
<th>Scan No.</th>
<th>Time [hr:min:sec]</th>
<th>Time [sec]</th>
<th>Real time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>scan01</td>
<td>00:12:10</td>
<td>730</td>
<td>70</td>
</tr>
<tr>
<td>scan02</td>
<td>00:15:09</td>
<td>909</td>
<td>249</td>
</tr>
<tr>
<td>scan03</td>
<td>00:18:17</td>
<td>1097</td>
<td>437</td>
</tr>
<tr>
<td>scan04</td>
<td>00:23:12</td>
<td>1392</td>
<td>732</td>
</tr>
<tr>
<td>scan05</td>
<td>00:28:09</td>
<td>1689</td>
<td>1029</td>
</tr>
<tr>
<td>scan06</td>
<td>00:38:12</td>
<td>2292</td>
<td>1632</td>
</tr>
<tr>
<td>scan07</td>
<td>00:48:43</td>
<td>2923</td>
<td>2263</td>
</tr>
<tr>
<td>scan08</td>
<td>01:03:11</td>
<td>3791</td>
<td>3131</td>
</tr>
<tr>
<td>scan09</td>
<td>01:18:16</td>
<td>4696</td>
<td>4036</td>
</tr>
<tr>
<td>scan10</td>
<td>01:32:10</td>
<td>5530</td>
<td>4870</td>
</tr>
</tbody>
</table>

Table I-7: Sampling table of experiment 2

<table>
<thead>
<tr>
<th>solution sample</th>
<th>start time</th>
<th>end time</th>
<th>start time [sec]</th>
<th>end time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>00:12:40</td>
<td>00:13:00</td>
<td>760</td>
<td>780</td>
</tr>
<tr>
<td>2</td>
<td>00:15:50</td>
<td>00:16:10</td>
<td>950</td>
<td>970</td>
</tr>
<tr>
<td>3</td>
<td>00:18:40</td>
<td>00:19:00</td>
<td>1120</td>
<td>1140</td>
</tr>
<tr>
<td>4</td>
<td>00:23:40</td>
<td>00:24:00</td>
<td>1420</td>
<td>1440</td>
</tr>
<tr>
<td>5</td>
<td>00:28:40</td>
<td>00:29:00</td>
<td>1720</td>
<td>1740</td>
</tr>
<tr>
<td>6</td>
<td>00:38:40</td>
<td>00:39:00</td>
<td>2320</td>
<td>2340</td>
</tr>
<tr>
<td>7</td>
<td>00:49:40</td>
<td>00:50:00</td>
<td>2980</td>
<td>3000</td>
</tr>
<tr>
<td>8</td>
<td>01:03:40</td>
<td>01:04:00</td>
<td>3820</td>
<td>3840</td>
</tr>
<tr>
<td>9</td>
<td>01:18:40</td>
<td>01:19:00</td>
<td>4720</td>
<td>4740</td>
</tr>
<tr>
<td>10</td>
<td>01:32:40</td>
<td>01:33:00</td>
<td>5560</td>
<td>5580</td>
</tr>
</tbody>
</table>

Table I-8: Extracted results from experiment 2

<table>
<thead>
<tr>
<th>C_0</th>
<th>\lambda_\infty [m^{-1}]</th>
<th>\lambda_0 [m^{-1}]</th>
<th>\lambda_{ss} [m^{-1}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0 \times 10^{-4}</td>
<td>1.51</td>
<td>82.90</td>
<td>79.710</td>
</tr>
</tbody>
</table>
Figure I-5: Normalised CT numbers with respect to water along the core (Experiment 2)

Figure I-6: Concentration profile along the core (Experiment 2)
Produced Water Re-Injection

Figure I-7: Initial concentration vs. 1/t, Experimental data and linear fitting (Experiment 2)

\[ y = 279.05x + 4.3784 \]

Figure I-8: Filtration coefficient vs. inverse of time, experimental data and linear fitting (Experiment 2)
## I.3 Experiment 3

### Table I-9: Table of Experiment 3

<table>
<thead>
<tr>
<th>c₀ [mg/L]</th>
<th>Qₜ [L/hr]</th>
<th>uₜ [m/sec]</th>
<th>Qₚ [L/hr]</th>
<th>uₚ [m/sec]</th>
<th>uₜ/</th>
<th>ₚ start</th>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>7.33</td>
<td>5.87E-3</td>
<td>2.75</td>
<td>2.20E-3</td>
<td>0.375</td>
<td>00:08:00</td>
<td>01:12:10</td>
</tr>
</tbody>
</table>

### Table I-10: Scanning time table of experiment 3

<table>
<thead>
<tr>
<th>Scan No.</th>
<th>Time [hr:min:sec]</th>
<th>Time [sec]</th>
<th>Real time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>scan01</td>
<td>00:09:10</td>
<td>550</td>
<td>70</td>
</tr>
<tr>
<td>scan02</td>
<td>00:12:14</td>
<td>734</td>
<td>254</td>
</tr>
<tr>
<td>scan03</td>
<td>00:15:09</td>
<td>909</td>
<td>429</td>
</tr>
<tr>
<td>scan04</td>
<td>00:18:10</td>
<td>1090</td>
<td>610</td>
</tr>
<tr>
<td>scan05</td>
<td>00:24:09</td>
<td>1449</td>
<td>969</td>
</tr>
<tr>
<td>scan06</td>
<td>00:30:19</td>
<td>1819</td>
<td>1339</td>
</tr>
<tr>
<td>scan07</td>
<td>00:40:10</td>
<td>2410</td>
<td>1930</td>
</tr>
<tr>
<td>scan08</td>
<td>00:50:10</td>
<td>3010</td>
<td>2530</td>
</tr>
<tr>
<td>scan09</td>
<td>01:00:09</td>
<td>3609</td>
<td>3129</td>
</tr>
<tr>
<td>scan10</td>
<td>01:10:09</td>
<td>4209</td>
<td>3729</td>
</tr>
</tbody>
</table>

### Table I-11: Sampling table of experiment 3

<table>
<thead>
<tr>
<th>solution sample</th>
<th>start time</th>
<th>end time</th>
<th>start time [sec]</th>
<th>end time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>00:09:50</td>
<td>00:10:10</td>
<td>590</td>
<td>610</td>
</tr>
<tr>
<td>2</td>
<td>00:12:40</td>
<td>00:13:00</td>
<td>760</td>
<td>780</td>
</tr>
<tr>
<td>3</td>
<td>00:15:40</td>
<td>00:16:00</td>
<td>940</td>
<td>960</td>
</tr>
<tr>
<td>4</td>
<td>00:18:40</td>
<td>00:19:00</td>
<td>1120</td>
<td>1140</td>
</tr>
<tr>
<td>5</td>
<td>00:24:40</td>
<td>00:25:00</td>
<td>1480</td>
<td>1500</td>
</tr>
<tr>
<td>6</td>
<td>00:30:40</td>
<td>00:31:00</td>
<td>1840</td>
<td>1860</td>
</tr>
<tr>
<td>7</td>
<td>00:40:40</td>
<td>00:41:00</td>
<td>2440</td>
<td>2460</td>
</tr>
<tr>
<td>8</td>
<td>00:50:40</td>
<td>00:51:00</td>
<td>3040</td>
<td>3060</td>
</tr>
<tr>
<td>9</td>
<td>01:00:40</td>
<td>01:01:00</td>
<td>3640</td>
<td>3660</td>
</tr>
<tr>
<td>10</td>
<td>01:10:40</td>
<td>01:11:00</td>
<td>4240</td>
<td>4260</td>
</tr>
</tbody>
</table>

### Table I-12: Extracted results from experiment 3

<table>
<thead>
<tr>
<th>C₀</th>
<th>λ∞ [m⁻¹]</th>
<th>λ₀ [m⁻¹]</th>
<th>λss [m⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.91×10⁻⁴</td>
<td>2.08</td>
<td>63.47</td>
<td>58.40</td>
</tr>
</tbody>
</table>
Figure I-9: Normalised CT numbers with respect to water along the core (Experiment 3)

Figure I-10: Concentration profile along the core (Experiment 3)
Figure I-11: Initial concentration vs. $u_p t$, Experimental data and linear fitting (Experiment 3)

\[ c_0(t) = 0.00014 u_p t + 0.6e^{-0.005} \]

Figure I-12: Filtration coefficient vs. inverse of time, experimental data and linear fitting (Experiment 3)

\[ y = 294.63x + 4.0674 \]
I.4 Experiment 4

Table I-13: Table of Experiment 4

<table>
<thead>
<tr>
<th>$c_0$ [mg/L]</th>
<th>$Q_{cf}$ [L/hr]</th>
<th>$u_{cf}$ [m/sec]</th>
<th>$Q_p$ [L/hr]</th>
<th>$u_p$ [m/sec]</th>
<th>$u_{cf}/u_p$</th>
<th>start</th>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>7.33</td>
<td>5.87E-3</td>
<td>3.67</td>
<td>2.94E-3</td>
<td>0.5</td>
<td>00:12:00</td>
<td>01:56:40</td>
</tr>
</tbody>
</table>

Table I-14: Scanning time table of experiment 3

<table>
<thead>
<tr>
<th>Scan No.</th>
<th>Time [hr:min:sec]</th>
<th>Time [sec]</th>
<th>Real time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>scan01</td>
<td>00:13:10</td>
<td>790</td>
<td>70</td>
</tr>
<tr>
<td>scan02</td>
<td>00:16:11</td>
<td>971</td>
<td>251</td>
</tr>
<tr>
<td>scan03</td>
<td>00:19:09</td>
<td>1149</td>
<td>429</td>
</tr>
<tr>
<td>scan04</td>
<td>00:22:10</td>
<td>1330</td>
<td>610</td>
</tr>
<tr>
<td>scan05</td>
<td>00:28:09</td>
<td>1689</td>
<td>969</td>
</tr>
<tr>
<td>scan06</td>
<td>00:34:10</td>
<td>2050</td>
<td>1330</td>
</tr>
<tr>
<td>scan07</td>
<td>00:44:10</td>
<td>2650</td>
<td>1930</td>
</tr>
<tr>
<td>scan08</td>
<td>00:59:10</td>
<td>3550</td>
<td>2830</td>
</tr>
<tr>
<td>scan09</td>
<td>01:30:03</td>
<td>5403</td>
<td>4683</td>
</tr>
<tr>
<td>scan10</td>
<td>01:55:10</td>
<td>6910</td>
<td>6190</td>
</tr>
</tbody>
</table>

Table I-15: Sampling table of experiment 4

<table>
<thead>
<tr>
<th>solution sample</th>
<th>start time</th>
<th>end time</th>
<th>start time [sec]</th>
<th>end time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>00:13:40</td>
<td>00:14:00</td>
<td>820</td>
<td>840</td>
</tr>
<tr>
<td>2</td>
<td>00:16:40</td>
<td>00:17:00</td>
<td>1000</td>
<td>1020</td>
</tr>
<tr>
<td>3</td>
<td>00:19:40</td>
<td>00:20:00</td>
<td>1180</td>
<td>1200</td>
</tr>
<tr>
<td>4</td>
<td>00:22:40</td>
<td>00:23:00</td>
<td>1360</td>
<td>1380</td>
</tr>
<tr>
<td>5</td>
<td>00:28:40</td>
<td>00:29:00</td>
<td>1720</td>
<td>1740</td>
</tr>
<tr>
<td>6</td>
<td>00:34:40</td>
<td>00:35:00</td>
<td>2080</td>
<td>2100</td>
</tr>
<tr>
<td>7</td>
<td>00:44:40</td>
<td>00:45:00</td>
<td>2680</td>
<td>2700</td>
</tr>
<tr>
<td>8</td>
<td>00:59:40</td>
<td>01:00:00</td>
<td>3580</td>
<td>3600</td>
</tr>
<tr>
<td>9</td>
<td>01:30:40</td>
<td>01:31:00</td>
<td>5440</td>
<td>5460</td>
</tr>
<tr>
<td>10</td>
<td>01:55:40</td>
<td>01:56:00</td>
<td>6940</td>
<td>6960</td>
</tr>
</tbody>
</table>

Table I-16: Extracted results from experiment 4

<table>
<thead>
<tr>
<th>$C_0$</th>
<th>$\lambda_{\infty}$ [m$^{-1}$]</th>
<th>$\lambda_0$ [m$^{-1}$]</th>
<th>$\lambda_{SS}$ [m$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.91 \times 10^{-4}$</td>
<td>1.18</td>
<td>58.46</td>
<td>54.795</td>
</tr>
</tbody>
</table>
Figure I-13: Normalised CT numbers with respect to water along the core (Experiment 4)

Figure I-14: Concentration profile along the core (Experiment 4)
Figure I-15: Initial concentration vs. $u_p t$, Experimental data and linear fitting (Experiment 4)

Figure I-16: Filtration coefficient vs. inverse of time, experimental data and linear fitting (Experiment 4)
I.5 Experiment 6

Table I-17: Table of Experiment 6

<table>
<thead>
<tr>
<th>$c_0$ [mg/L]</th>
<th>$Q_{cf}$ [L/hr]</th>
<th>$u_{cf}$ [m/sec]</th>
<th>$Q_p$ [L/hr]</th>
<th>$u_p$ [m/sec]</th>
<th>$u_{cf}/u_p$</th>
<th>start</th>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>3.33</td>
<td>2.67E-3</td>
<td>5.0</td>
<td>4.00E-3</td>
<td>1.5</td>
<td>00:02:00</td>
<td>01:42:05</td>
</tr>
</tbody>
</table>

Table I-18: Scanning time table of experiment 6

<table>
<thead>
<tr>
<th>Scan No.</th>
<th>Time [hr:min:sec]</th>
<th>Time [sec]</th>
<th>Real time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>scan01</td>
<td>00:03:13</td>
<td>193</td>
<td>73</td>
</tr>
<tr>
<td>scan02</td>
<td>00:06:09</td>
<td>369</td>
<td>249</td>
</tr>
<tr>
<td>scan03</td>
<td>00:10:30</td>
<td>630</td>
<td>510</td>
</tr>
<tr>
<td>scan04</td>
<td>00:15:09</td>
<td>909</td>
<td>789</td>
</tr>
<tr>
<td>scan05</td>
<td>00:20:10</td>
<td>1210</td>
<td>1090</td>
</tr>
<tr>
<td>scan06</td>
<td>00:30:09</td>
<td>1809</td>
<td>1689</td>
</tr>
<tr>
<td>scan07</td>
<td>00:40:09</td>
<td>2409</td>
<td>2289</td>
</tr>
<tr>
<td>scan08</td>
<td>00:55:09</td>
<td>3309</td>
<td>3189</td>
</tr>
<tr>
<td>scan09</td>
<td>01:10:09</td>
<td>4209</td>
<td>4089</td>
</tr>
<tr>
<td>scan10</td>
<td>01:30:11</td>
<td>5411</td>
<td>5291</td>
</tr>
<tr>
<td>Scan11</td>
<td>01:40:10</td>
<td>6010</td>
<td>5890</td>
</tr>
</tbody>
</table>

Table I-19: Sampling table of experiment 6

<table>
<thead>
<tr>
<th>solution sample</th>
<th>start time</th>
<th>end time</th>
<th>start time [sec]</th>
<th>end time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>00:03:40</td>
<td>00:04:00</td>
<td>220</td>
<td>240</td>
</tr>
<tr>
<td>2</td>
<td>00:06:40</td>
<td>00:07:00</td>
<td>400</td>
<td>420</td>
</tr>
<tr>
<td>3</td>
<td>00:10:50</td>
<td>00:11:10</td>
<td>650</td>
<td>670</td>
</tr>
<tr>
<td>4</td>
<td>00:15:40</td>
<td>00:16:00</td>
<td>940</td>
<td>960</td>
</tr>
<tr>
<td>5</td>
<td>00:20:40</td>
<td>00:21:00</td>
<td>1240</td>
<td>1260</td>
</tr>
<tr>
<td>6</td>
<td>00:30:40</td>
<td>00:31:00</td>
<td>1840</td>
<td>1860</td>
</tr>
<tr>
<td>7</td>
<td>00:40:40</td>
<td>00:41:00</td>
<td>2440</td>
<td>2460</td>
</tr>
<tr>
<td>8</td>
<td>00:55:40</td>
<td>00:56:00</td>
<td>3340</td>
<td>3360</td>
</tr>
<tr>
<td>9</td>
<td>01:10:40</td>
<td>01:11:00</td>
<td>4240</td>
<td>4260</td>
</tr>
<tr>
<td>10</td>
<td>01:30:40</td>
<td>01:31:00</td>
<td>5440</td>
<td>5460</td>
</tr>
<tr>
<td>11</td>
<td>01:40:40</td>
<td>01:41:00</td>
<td>6040</td>
<td>6060</td>
</tr>
</tbody>
</table>

Table I-20: Extracted results from experiment 6

<table>
<thead>
<tr>
<th>$C_0$</th>
<th>$\lambda_m$ [m$^{-1}$]</th>
<th>$\lambda_0$ [m$^{-1}$]</th>
<th>$\lambda_{ss}$ [m$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6.36 \times 10^{-4}$</td>
<td>2.74</td>
<td>78.20</td>
<td>76.420</td>
</tr>
</tbody>
</table>
Figure I-17: Normalised CT numbers with respect to water along the core (Experiment 6)

Figure I-18: Concentration profile along the core (Experiment 6)
Figure I-19: Initial concentration vs. $u_p t$, Experimental data and linear fitting (Experiment 6)

$y = 543.77x + 4.337$

Figure I-20: Filtration coefficient vs. inverse of time, experimental data and linear fitting (Experiment 6)

$y = 543.77x + 4.337$
I.6 Experiment 7

Table I-21: Table of Experiment 7

<table>
<thead>
<tr>
<th>$c_0$ [mg/L]</th>
<th>$Q_{cf}$ [L/hr]</th>
<th>$u_{cf}$ [m/sec]</th>
<th>$Q_p$ [L/hr]</th>
<th>$u_p$ [m/sec]</th>
<th>$u_{cf}/u_p$</th>
<th>start</th>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>3.67</td>
<td>2.94E-3</td>
<td>1.83</td>
<td>1.4E-3</td>
<td>0.5</td>
<td>00:03:00</td>
<td>02:51:10</td>
</tr>
</tbody>
</table>

Table I-22: Scanning time table of experiment 7

<table>
<thead>
<tr>
<th>Scan No.</th>
<th>Time [hr:min:sec]</th>
<th>Time [sec]</th>
<th>Real time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>scan01</td>
<td>00:04:09</td>
<td>249</td>
<td>69</td>
</tr>
<tr>
<td>scan02</td>
<td>00:07:09</td>
<td>429</td>
<td>249</td>
</tr>
<tr>
<td>scan03</td>
<td>00:10:10</td>
<td>610</td>
<td>430</td>
</tr>
<tr>
<td>scan04</td>
<td>00:15:09</td>
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<td>729</td>
</tr>
<tr>
<td>scan05</td>
<td>00:20:10</td>
<td>1210</td>
<td>1030</td>
</tr>
<tr>
<td>scan06</td>
<td>00:30:09</td>
<td>1809</td>
<td>1629</td>
</tr>
<tr>
<td>scan07</td>
<td>00:40:09</td>
<td>2409</td>
<td>2229</td>
</tr>
<tr>
<td>scan08</td>
<td>01:00:28</td>
<td>3628</td>
<td>3448</td>
</tr>
<tr>
<td>scan09</td>
<td>01:25:10</td>
<td>5110</td>
<td>4930</td>
</tr>
<tr>
<td>scan10</td>
<td>01:52:41</td>
<td>6761</td>
<td>6581</td>
</tr>
<tr>
<td>scan11</td>
<td>02:20:55</td>
<td>8455</td>
<td>8275</td>
</tr>
<tr>
<td>scan12</td>
<td>02:50:09</td>
<td>10209</td>
<td>10029</td>
</tr>
</tbody>
</table>

Table I-23: Sampling table of experiment 7

<table>
<thead>
<tr>
<th>solution sample</th>
<th>start time</th>
<th>end time</th>
<th>start time [sec]</th>
<th>end time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>00:04:40</td>
<td>00:05:00</td>
<td>280</td>
<td>300</td>
</tr>
<tr>
<td>2</td>
<td>00:07:40</td>
<td>00:08:00</td>
<td>460</td>
<td>480</td>
</tr>
<tr>
<td>3</td>
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<td>00:11:00</td>
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<td>660</td>
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<tr>
<td>4</td>
<td>00:15:40</td>
<td>00:16:00</td>
<td>940</td>
<td>960</td>
</tr>
<tr>
<td>5</td>
<td>00:20:40</td>
<td>00:21:00</td>
<td>1240</td>
<td>1260</td>
</tr>
<tr>
<td>6</td>
<td>00:30:40</td>
<td>00:31:00</td>
<td>1840</td>
<td>1860</td>
</tr>
<tr>
<td>7</td>
<td>00:40:40</td>
<td>00:41:00</td>
<td>2440</td>
<td>2460</td>
</tr>
<tr>
<td>8</td>
<td>01:00:50</td>
<td>01:01:10</td>
<td>3650</td>
<td>3670</td>
</tr>
<tr>
<td>9</td>
<td>01:25:40</td>
<td>01:26:00</td>
<td>5140</td>
<td>5160</td>
</tr>
<tr>
<td>10</td>
<td>01:53:00</td>
<td>01:53:20</td>
<td>6780</td>
<td>6800</td>
</tr>
<tr>
<td>11</td>
<td>02:21:20</td>
<td>02:01:40</td>
<td>8480</td>
<td>7300</td>
</tr>
<tr>
<td>12</td>
<td>02:50:40</td>
<td>02:51:00</td>
<td>10240</td>
<td>10260</td>
</tr>
</tbody>
</table>

Table I-24: Extracted results from experiment 7

<table>
<thead>
<tr>
<th>$C_0$</th>
<th>$\lambda_\infty$ $[m^{-1}]$</th>
<th>$\lambda_0$ $[m^{-1}]$</th>
<th>$\lambda_{ss}$ $[m^{-1}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4.27 \times 10^{-4}$</td>
<td>2.04</td>
<td>75.20</td>
<td>75.045</td>
</tr>
</tbody>
</table>
Figure I-21: Normalised CT numbers with respect to water along the core (Experiment 7)

Figure I-22: Concentration profile along the core (Experiment 7)
Figure I-23: Initial concentration vs. $u_p t$, Experimental data and linear fitting (Experiment 7)

Figure I-24: Filtration coefficient vs. inverse of time, experimental data and linear fitting (Experiment 7)
I.7 Breakthrough Curves

![Effluent breakthrough Curve Exp1](image1)

**Figure I-25: Effluent Breakthrough Curve for Experiment 1**

![Effluent Breakthrough 2](image2)

**Figure I-26: Effluent Breakthrough curve for Experiment 2**
Figure I-27: Effluent Breakthrough curve for experiment 3

Figure I-28: Effluent Breakthrough curve for experiment 4
I.8 Permeability Decline Curves

Figure I-30: Permeability decline curve for Experiment 1
Figure I-31: Permeability decline curve for Experiment 2

Figure I-32: Permeability decline curve for Experiment 3
**Figure I-33: Permeability decline curve for Experiment 1**

**Figure I-34: Permeability decline curve for Experiment 5**
Appendix J

DESCRIPTISATION OF THE EQUATIONS AND SENSITIVITY OF CAKE THICNESS TO DIFFERENT PARAMETERS

Figure J-1: The equations were solved in boundaries

To discretise the equation the channel was divided into n segments (cells). The equations were solved for boundaries.

J.1 Discretisation of Steady State Equations

- **Permeate Velocity**

\[ u_p = \frac{\psi}{h + \Theta} \quad (J.1) \]
\[ u_p' = \frac{\psi'}{h' + \Theta} \quad (J.2) \]

- **Crossflow Velocity**

\[ u_{cf}' = \frac{6\gamma}{a} \left(H - h'_i\right) u_{p,i}' \quad (J.3) \]

or using:

\[ -\frac{\partial}{\partial x} \left( u_{cf} \left(H - h\right) \right) = u_p \quad (J.4) \]
\[ u_{cf,i+1}' = \frac{\left(u_{cf,i}' \left(H - h'_i\right)\right) - u_{p,i}' \Delta x}{H - h'_{i+1}} \quad (J.5) \]

- **Cake Thickness**

\[ \frac{dh}{dx} = \left[ \frac{h + \Theta}{h + 2\Theta + H} \right] \left[ \frac{u_p}{u_{cf}} - \frac{72\gamma k_c}{a} \frac{72\gamma k_c}{a} \right] \quad (J.6) \]
Produced Water Re-Injection

\[ h'_{i+1} = h'_{i} + \Delta x \left[ \frac{h'_{i} + \Theta}{h'_{i} + 2\Theta + H} \left[ \frac{u_{p_i}'}{u_{c_i}'} - \frac{72\gamma k_{c}}{a} u_{p_i}' \right] \right] \]  

(J.7)

J.2 Discretisation of Transient Equations

- **Permeate Velocity**

\[ u_p = \frac{\psi}{h + \Theta} \]  

(J.8)

\[ u_{p_i}' = \frac{\psi_{i}'}{h_{i} + \Theta} \]  

(J.9)

- **Crossflow Velocity**

\[ - \frac{\partial}{\partial x} \left( u_{c_i} (H - h) \right) = u_p \frac{\partial h}{\partial t} \]  

(J.10)

\[ u_{c_{i+1}} = \left[ \left( \frac{h_{i+1} - h_{i}}{\Delta t} - u_{p_i}' \right) \Delta x + \left( u_{c_{i+1}} (H - h_{i+1}) \right) \right] \]  

(J.11)

- **Cake Thickness Growth**

\[ (1 - \varphi_c) \frac{dh}{dt} = u_p C - \omega u_{c_i} \]  

(J.12)

\[ h(x, t + \Delta t) = h(x, t) + \left[ \frac{\Delta t}{1 - \varphi_c} \right] \left( u_p (x, t) C(x, t) - \omega u_{c_i} (x, t) \right) \]  

(J.13)

\[ h'_{i+1} = h'_{i} + \left[ \frac{dt}{(1 - \varphi_c)} \left( u_{c_{i+1}}' C_{i+1}' - \omega u_{c_{i+1}}' \right) \right] \]  

(J.14)

with

\[ \omega = MEAN \left[ \left( \frac{u_p C}{u_{c_i}} \right) \right] \]  

(J.15)

\[ (1 - \varphi_c) \frac{dh}{dt} = u_p C - \omega u_{c_i} \frac{\bar{u}_{c_i}}{H - h} \]  

(J.16)
\[ h(x, t + \Delta t) = h(x, t) + \left[ \frac{\Delta t}{(\Delta t)(1-\varphi_{t})} \left( \bar{u}_p(x, t) C(x, t) - \omega \frac{u_{c}(x, t)}{H - h(x, t)} \right) \right] \] (J.17)

\[ h_t^{i+1} = h_t^i + \left[ \frac{dt}{(1-\varphi_{t})} \left( \bar{u}_p(x, t) C_t^j - \omega \frac{u_{c}^{j}}{H - h_t^j} \right) \right] \] (J.18)

With:

\[ \omega = MEAN \left[ \frac{a C}{6 \gamma} \right] \] (J.19)

if we assume that the back transport can be written as:

\[ u_{back} = \omega \frac{u_{c}^{m}}{(H - h)^{\gamma}} \] (J.20)

then:

\[ (1-\varphi_{t}) \frac{dh}{dt} = \bar{u}_p C - \omega \frac{u_{c}^{m}}{(H - h)^{\gamma}} \] (J.21)

and:

\[ \omega = MEAN \left[ \frac{a (H - h)^{\gamma-1} C}{6 \gamma u_{c}^{\gamma-1}} \right] \] (J.22)
Figure J-2: Schematic description of the Matlab subroutine for steady state case profiles

1. Start
2. Initial guess
   \[ P_{r+1} = P_{r+1} = P_{wp} \]
3. Calculate the cake thickness:
   \[ \psi = \frac{k_c(P - P_a)}{\mu} \]
4. Solving for left boundary:
   \[ h = H + \frac{Q_a \pm Q_a(Q_a + 24\gamma\psi(H + \Theta))}{12\gamma\psi} \]
5. Calculate the cake thickness:
   \[ h'_{l+1} = h'_{l} + \Delta x \left[ h'_{l} + \Theta \right] \left[ \frac{u'_{p_{l}} - 72\gamma k_c \psi_{l}}{a} \right] \]
6. Back solve for the pressure:
   \[ \frac{dp}{dx} = \frac{72\gamma \psi}{a(H - h)(h + \Theta)} \]
7. Is this Error acceptable?
   - Yes: Calculate \[ u_p, u_{df}, c \]
   - No: \[ Error = P_{r+1} - P_{wp} \]
8. END
J.3 Sensitivity analysis

- Particle size

![Graph showing sensitivity of steady state cake thickness to particle size](image)

Figure J-3: Sensitivity of the steady state cake thickness to particle size
Produced Water Re-Injection

- **Back Pressure**

![Figure J-4: Sensitivity of the steady state cake thickness to back pressure](image)

- **Cake Permeability**

![Figure J-5: Sensitivity of the steady state cake thickness to permeability of the cake](image)
• Permeability of the Porous Media

Figure J-6: Sensitivity of the steady state cake thickness to the permeability of porous media

• Gamma

Figure J-7: Sensitivity of the steady state cake thickness to gamma
ACKNOWLEDGMENT

I am grateful for the God who gave me the ability to carry on this work.

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